NEW ESSENTIALS
OF
BUSINESS ARITHMETIC

VAN TUYL

[illegible]

3. Perform muliplication and division in order in which they appear.

Jack

premiter = 2l + 2W

1. an Equation is a statement of a quality.

$2a + 3B$

2. To add in algebra add coeffici of like terms –

~~Jack Dode~~

$\frac{1}{2}a +$

How to write Equations

1. unknown ha is [illegible] by X or n

2. Find a statement showe by wote it is to be increased, decrd, multor di

3. [illegible]

Jack

$ xx
1.20

Jack Dudeck
Portland, Mich.
Grade 9.
1980 No Zero

$x = 5$

1. $3x = 15$
If even no. are divided by same quotien are equal

A. N.

I $\frac{1}{3}x = 8$
If equal nos are x by some products are equal.

C & M

II $x + 7 = 15$
if equal nos are subtracted from same remainders are even.

V $x - 3 = 5$
If equal numbers are added to same the sumes are equal

says you

Closed for repairs.

NEW ESSENTIALS OF BUSINESS ARITHMETIC

BY

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W. P. 16

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PREFACE

New Essentials of Business Arithmetic has been prepared to meet the needs of those schools which offer a course in business arithmetic that is not so comprehensive nor so difficult as the course provided for in the author's New Complete Business Arithmetic.

This text aims to present the subject of business arithmetic in such a manner that those who use it shall be able to acquire a knowledge of the principles governing business calculations, together with a reasonable degree of skill in the use of such principles.

Placing the subject of aliquot parts at the beginning of the text is a departure from the old standard sequence of topics which has met with instant approval of both teachers and pupils who have used Essentials of Business Arithmetic or Complete Business Arithmetic. The importance of this subject cannot be overestimated either for its practical value or as a means of arousing the interest of pupils in the work as a whole. The angle of approach to a subject is vital. The "aliquot part" approach has been approved by large numbers of teachers.

The following is a partial list of features:

The chapter on aliquot parts, as applied to billing, trade discount, and simple interest is placed early in the text.

Special drill exercises are introduced for developing facility and accuracy in aliquot parts, simple interest, addition, percentage, etc.

An abundance of material is provided for both oral and written work.

Common and decimal fractions are treated interchangeably, as they are used in business.

Emphasis is placed on the fundamental principles of arithmetic rather than on set rules for the solution of problems.

The explanations are clear and full.

Many of the problems have informational value.

The treatment of such important topics as pay rolls, graphs, trade discount, etc. is in accord with modern business practice.

The sales basis as well as the cost basis for determining profits is fully explained and illustrated.

Five sets of Speed and Written Tests are provided at various stages of the work to determine the student's progress in the subject.

The statements made in this text are strictly in accord with the best business practice of the day. Many business men have freely given advice concerning customs and problems arising in their experience.

The treatment of the subject matter is correct. The wide use and approval by teachers, of the author's Essentials of Business Arithmetic and Complete Business Arithmetic give assurance that from a pedagogical standpoint this text meets every requirement.

The author wishes to express his grateful appreciation to the large number of business men, teachers and associates for their advice and criticism without which this volume would fall short of its present standard of merit.

George H. Van Tuyl.

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NEW ESSENTIALS OF
BUSINESS ARITHMETIC

READING AND WRITING NUMBERS

1. Numbers are expressed in three ways: (1) by written words, (2) by letters (Roman system), and (3) by figures (Arabic system).

ROMAN NOTATION

2. In the Roman system of writing numbers seven letters are used. They are:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

3. Writing numbers by the Roman system is based on the following principles:

PRINCIPLES: **1.** *Repeating a letter repeats its value.*

2. *The value of a letter written after a letter of greater value is to be added to the larger value.*

3. *The value of a letter written before a letter of greater value is to be subtracted from the greater value.*

4. *A horizontal bar over a letter increases its value one thousand times.*

NOTES. (1) Only whole numbers can be written in the Roman system of notation.

(2) Roman notation is of little practical value, being used only to record dates and chapters in books, dates on buildings, numbers on a watch face, etc.

4. Roman Notation Illustrated

II	= 2	XIV	= 14	LIX	= 59	CCCLXVI	= 366
III	= 3	XVI	= 16	LXXIV	= 74	DCXCVI	= 696
IV or IIII*	= 4	XIX	= 19	LXXXIX	= 89	MCXIX	= 1119
VI	= 6	XXIII	= 23	XC	= 90	MCM	= 1900
IX	= 9	XXIV	= 24	CX	= 110	$\overline{\text{MCMX}}$	= 1,910,000

5. 1. Write in Roman notation:

12	85	336	598	10,000
18	94	472	1077	15,000
21	111	505	1384	100,000
47	128	666	1527	150,000
63	247	872	1898	1,000,000

2. Read the following:

XVIII	LXIX	CDLIX	$\overline{\text{MC}}$
XXIX	XCIX	$\overline{\text{D}}$CD	MDCCCXCIX
XXXI	CIX	DCCCLXXXIX	MDCCXXVIII
XLIV	CLXXII	CM	$\overline{\text{V}}$CCLXXX
XLIX	CCCLX	MCMIX	$\overline{\text{L}}$

ABABIC NOTATION

6. The **Arabic system** of notation makes use of ten characters or figures, and a period or decimal point. They are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and .

7. With these figures and the decimal point any required number can be written.

8. For convenience in reading numbers, the figures are sometimes divided into groups of three, in this way:

2,564,879,598.

The reason for this grouping is best understood by observing the following table:

---

* IIII is used on timepieces.

TABLE SHOWING HOW TO READ NUMBERS

ten-trillions	trillions	hundred-billions	ten-billions	billions	hundred-millions	ten-millions	millions	hundred-thousands	ten-thousands	thousands	hundreds	tens	units
8	9,	7	2	4,	5	6	3,	9	0	0,	1	0	6

The number is read, "eighty-nine trillion, seven hundred twenty-four billion, five hundred sixty-three million, nine hundred thousand, one hundred six."

9. In like manner read the following numbers:

1. 21,460
2. 34,829
3. 127,724
4. 306,200
5. 425,002
6. 800,005
7. 962,024
8. 872,463
9. 900,047
10. 1,456,789
11. 3,248,764
12. 5,002,005
13. 27,250,820
14. 20,300,600
15. 65,897,249
16. 185,496,728
17. 593,972,847
18. 15,647,989,721

10. Write the following numbers, using the Arabic system:

1. Seven hundred forty-three.
2. Nine thousand fourteen.
3. Eleven thousand one hundred four.
4. Fifteen thousand twenty.
5. Seventy-six thousand ninety-nine.
6. Fourteen hundred eighty-eight.
7. One hundred sixty thousand four hundred.
8. Eight hundred thousand nine hundred ninety.
9. Two million sixteen thousand seventeen.
10. Six hundred eleven million fourteen thousand eighty-six.

UNITED STATES MONEY

11. Any number of cents less than one hundred is read as cents: thus, $.45 or 45 ¢ is read "forty-five cents," but any value equal to or greater than one hundred cents is read as dollars, or as dollars and cents: thus, $8.75 or 875 ¢ would be read "eight dollars, seventy-five cents." A period, or point, separates the number of dollars from the number of cents.

12. Read the following:

$3.17
$4.28
$9.15
$10.20

$12.40
$16.50
$85.90
$104.26

$826.52
$1624.43[*]
$1837.16
$2487.49

$1476.83
$1982.76
$14,728.41
$146,789.09

13. Write the following in figures:

1. One hundred sixteen dollars, fifty-nine cents.
2. Three hundred nine dollars, seventy cents.
3. Twelve hundred twenty-one dollars, eighty-eight cents.
4. Twenty-four hundred forty-three dollars, nine cents.
5. Fifty-seven hundred six dollars, thirty-three cents.

14. Money is the generally accepted measure of value.

15. Each nation has its own system of money or coinage.

The principal nations of the world have adopted gold as the standard of monetary value.

16. **United States money** consists of the coins, notes, and certificates authorized by Congress to be used as money. It is a decimal system.

TABLE

10 mills = 1 cent, ¢
10 cents = 1 dime, d.
10 dimes = 1 dollar, $
10 dollars = 1 eagle

(The mill is not a coin.)

17. There are eleven different kinds of money in circulation in the United States: gold coins, silver dollars, subsidiary silver, nickels, pennies, gold certificates, silver certificates, Treasury notes, United States notes (greenbacks), Federal Reserve notes and Federal Reserve Bank Notes.

---

[*] In reading $1624.43 say "sixteen hundred twenty-four dollars, forty-three cents," not "one thousand six hundred twenty-four dollars, forty-three cents." One reason for saying it and writing it as suggested is that it takes less time and space.

18. The gold dollar, containing 25.8 grains of standard gold .9 fine, is the unit of value.

19. The silver dollar weighs 15.988 times as much as the gold dollar. That is, the ratio of gold to silver is 15.988 to 1 (commonly called 16 to 1).

20. DENOMINATIONS, WEIGHT, AND FINENESS OF THE COINS OF THE UNITED STATES

GOLD				
DENOMINATION		FINE GOLD CONTAINED (GRAINS)	ALLOY CONTAINED (GRAINS)	WEIGHT (GRAINS)
One dollar not coined (since 1890)		23.22	2.58	25.80
Quarter eagle	($ 2.50)	58.05	6.45	64.5
Half eagle	($ 5.00)	116.10	12.90	129.
Eagle	($10.00)	232.20	25.80	258.
Double eagle	($20.00)	464.40	51.60	516.
		SILVER		
		PURE SILVER CONTAINED		
One dollar		371.25	41.25	412.5
Half dollar		173.61	19.29	192.9
Quarter dollar		86.805	9.645	96.45
Dime		34.722	3.858	38.58
		MINOR		
		PURE COPPER CONTAINED		
Five cents*	(nickel)	57.87	19.29	77.16
One cent†	(copper)	45.60	2.40	48.

NOTE. The alloy in gold and silver coins neither adds to nor detracts from the value of the coins. Its purpose is to harden the coin, thus reducing the loss by abrasion.

---

* 75% copper and 25% nickel.

† 95% copper and 5% tin and zinc.

ALIQUOT PARTS

21. An **Aliquot part** of a number is a number that is contained in it an integral number of times.

Thus, 2, 4, 5, 10, 20, 25, and 50 are aliquot parts of 100; that is, 2 = $\frac{1}{50}$ of 100, 25 = $\frac{1}{4}$ of 100, etc.

NOTE. Other fractional parts of 100 are called **aliquant parts** of 100. For convenience, all fractional parts of 100 will be treated as aliquot parts.

HALVES AND QUARTERS

22. 1. What part of 100 cents, or $1.00, is 25 ¢? 50 ¢? 75 ¢?

2. At 25 ¢ a yard, how many yards can be bought for $1.00?

3. If 4 yd. cost $1.00, how much will 8 yd. cost? 12 yd.? 16 yd.?

4. How much will 20 yd. cost at 25 ¢ a yard? at 50 ¢ a yard? at 75 ¢ a yard?

5. Compare 25 ¢ with 50 ¢; with 75 ¢. 50 ¢ with 75 ¢; with 25 ¢. 75 ¢ with 25 ¢; with 50 ¢.

6. Compare the number of yards that can be bought for 50 ¢ and for 25 ¢; for 25 ¢ and for 75 ¢; for 75 ¢ and for 50 ¢.

7. State a rule for finding the cost of any number of articles at 25 ¢ each; at 50 ¢ each; at 75 ¢ each.

23. Make all extensions mentally, and find the total value of each of the following:

1.

16 yd. @ 25 ¢
24 yd. @ 25 ¢
30 yd. @ 50 ¢
16 yd. @ 75 ¢
40 yd. @ 50 ¢

2.

32 yd. @ 50 ¢
48 yd. @ 25 ¢
54 yd. @ 50 ¢
60 yd. @ 75 ¢
80 yd. @ 75 ¢

3.

120 yd. @ 25 ¢
140 yd. @ 50 ¢
150 yd. @ 25 ¢
160 yd. @ 75 ¢
126 yd. @ 25 ¢

4.

13 yd. @ 25¢
21 yd. @ 50¢
39 yd. @ 25¢
31 yd. @ 75¢
48 yd. @ 75¢

5.

3486 lb. @ 25¢
1984 lb. @ 50¢
4821 lb. @ 25¢
8448 lb. @ 75¢
1331 lb. @ 50¢

6.

128 lb. @ 75¢
324 lb. @ 50¢
233 lb. @ 25¢
199 lb. @ 75¢
247 lb. @ 25¢

Make up and find the total value of each of two or more groups of quantities like the above as directed by the teacher.

EIGHTHS

24. 1. $12\frac{1}{2}$¢ is what part of 25¢? 2. $\frac{1}{2}$ of $\frac{1}{4}$ is how much?

3. $12\frac{1}{2}$¢ is what part of $1?

4 25¢ is how many eighths of a dollar?

5. 25¢ + $12\frac{1}{2}$¢ = how many cents?

6. $37\frac{1}{2}$¢ is what part of a dollar?

7. What is the sum of $\frac{1}{2}$ and $\frac{1}{8}$? 50¢ + $12\frac{1}{2}$¢ = ?

8. $62\frac{1}{2}$¢ is what part of a dollar? 9. 75¢ + $12\frac{1}{2}$¢ = ?

10. $\$\frac{7}{8}$ = how many cents?

11. What part of a dollar is $12\frac{1}{2}$¢? $37\frac{1}{2}$¢? $62\frac{1}{2}$¢? $87\frac{1}{2}$¢?

12. What is the cost of 16 lb. of raisins at $12\frac{1}{2}$¢ a pound? of 16 baskets of potatoes at $37\frac{1}{2}$¢ a basket? of 16 yd. of cloth at $62\frac{1}{2}$¢ per yard? of 16 bu. of apples at $87\frac{1}{2}$¢ a bushel?

25. Using the aliquot-part method, find the total value of each of the following:

1.

24 lb. @ $37\frac{1}{2}$¢
32 lb. @ $12\frac{1}{2}$¢
40 lb. @ $62\frac{1}{2}$¢
56 lb. @ $37\frac{1}{2}$¢

2.

48 lb. @ $12\frac{1}{2}$¢
64 lb. @ 25¢
72 lb. @ $37\frac{1}{2}$¢
88 lb. @ $87\frac{1}{2}$¢

3.

120 yd. @ $12\frac{1}{2}$¢
164 yd. @ $12\frac{1}{2}$¢
200 yd. @ $37\frac{1}{2}$¢
240 yd. @ $12\frac{1}{2}$¢

Make two or more groups like the above and find the total value of each.

SIXTEENTHS

26. 1. $6\frac{1}{4}$ ¢ is what part of $12\frac{1}{2}$ ¢? of 25 ¢? of 50 ¢? of $1?

2. 3 times $6\frac{1}{4}$ ¢ is how many cents?

3. $18\frac{3}{4}$ ¢ is what part of a dollar?

4. 25 ¢ or $\$\frac{1}{4}$ is how many sixteenths of a dollar?

5. 25 ¢ + $6\frac{1}{4}$ ¢ are how many cents?

6. $31\frac{1}{4}$ ¢ is what part of a dollar?

7. $\frac{1}{2}-\frac{1}{16}=?$ $43\frac{3}{4}$ ¢ is what part of a dollar?

8. $\frac{1}{2}+\frac{1}{16}=?$ $56\frac{1}{4}$ ¢ = what part of a dollar?

9. In like manner find how many cents there are in $\$\frac{11}{16}$, $\$\frac{13}{16}$, and $\$\frac{15}{16}$ by comparing $6\frac{1}{4}$ ¢ ($\$\frac{1}{16}$) with 75 ¢ and $1.

27. Study the following table and compare values with one another.

	HALVES	QUARTERS	EIGHTHS	SIXTEENTHS
1.				$.06\frac{1}{4}$
2.			$.12\frac{1}{2}$	
3.				$.18\frac{3}{4}$
4.		.25		
5.				$.31\frac{1}{4}$
6.			$.37\frac{1}{2}$	
7.				$.43\frac{3}{4}$
8.	.50			
9.				$.56\frac{1}{4}$
10.			$.62\frac{1}{2}$	
11.				$.68\frac{3}{4}$
12.		.75		
13.				$.81\frac{1}{4}$
14.			$.87\frac{1}{2}$	
15.				$.93\frac{3}{4}$
16.	1.00	1.00	1.00	1.00

28. In the above table, note that the number of sixteenths is indicated in the left hand column. Thus, $.43\frac{3}{4}$ is on the 7th line, hence $.43\frac{3}{4}$ is $\frac{7}{16}$. $.62\frac{1}{2}$ is on the 10th line, hence it is $\frac{10}{16}$ or $\frac{5}{8}$, etc.

Note that .31¼, being on the 5th line, is $\frac{5}{16}$; $\frac{5}{16}$ is the sum of $\frac{4}{16}$ (¼) + $\frac{1}{16}$ (¼ of $\frac{4}{16}$). That is, to find the value of 492 yd. at 31¼ ¢ solve thus:

$492		=value at $1
123		=value at .25
30	75	=value at .06¼
$153	75	=value at $ .31¼

Find the value at $.25 ($\frac{4}{16}$) and then at $.06¼ ($\frac{1}{16}$) and add the results.

If, in the above illustration, the price had been $.18¾, the operation would have been the same, except the value at $.06¼ would be subtracted from the value at $.25.

Again, from the table, note that .81¼ is on the 13th line, hence .81¼ equals $\frac{13}{16}$. $\frac{13}{16} = \frac{8}{16} + \frac{4}{16} + \frac{1}{16}$, or .81¼ = .50 + .25 + .06¼.

Hence, to find the value of 578 yd. at .81¼ ¢, solve thus:

$578		=value at $1.
289		=value at .50
144	50	=value at .25
36	13	=value at .06¼
$469	63	=value at $ .81¼

To multiply by .93¾ ($\frac{15}{16}$), simply deduct $\frac{1}{16}$.

Similarly, analyze .56¼; .43¾; .68¾, and apply the above method of solution to the following exercises, unless results can be determined mentally.

29. Find the total value of each of the following:

1.

32 yd. @ 6¼ ¢
48 yd. @ 12½ ¢
64 yd. @ 18¾ ¢
96 yd. @ 6¼ ¢
160 yd. @ 43¾ ¢
80 yd. @ 31¼ ¢
136 yd. @ 56¼ ¢
72 yd. @ 18¾ ¢

2.

80 yd. @ 31¼ ¢
128 yd. @ 37½ ¢
144 yd. @ 56¼ ¢
96 yd. @ 93¾ ¢
72 yd. @ 37½ ¢
112 yd. @ 37½ ¢
152 yd. @ 31¼ ¢
84 yd. @ 18¾ ¢

3.

47 yd. @ 25 ¢
53 yd. @ 12½ ¢
58 yd. @ 6¼ ¢
63 yd. @ 50 ¢
75 yd. @ 37½ ¢
104 yd. @ 18¾ ¢
168 yd. @ 87½ ¢
192 yd. @ 75 ¢

4.

194 yd. @ 87½¢
64 yd. @ 18¾¢
552 yd. @ 81¼¢
104 yd. @ 62½¢
4264 yd. @ 6¼¢

5.

39 yd. @ 56¼¢
96 yd. @ 31¼¢
1046 yd. @ 87½¢
52 yd. @ 43¾¢
1972 yd. @ 93¾¢

6.

108 yd. @ 68¾¢
3922 yd. @ 37½¢
3360 yd. @ 18¾¢
5497 yd. @ 81¼¢
2849 yd. @ 56¼¢

Thirds and Sixths

30. 1. What part of a dollar is 33⅓¢? 66⅔?
2. What is ½ of ⅓? ½ of 33⅓¢ is how many cents?
3. 16⅔¢ = what part of a dollar?
4. 3 times 16⅔¢ = how many cents? what part of a dollar?
5. 4 times 16⅔¢ = how many cents? what part of a dollar?
6. $1 − 16⅔¢ = how many cents? what part of a dollar?

31. Find total value of each group:

1.

48 lb. @ 16⅔¢
54 lb. @ 66⅔¢
30 lb. @ 83⅓¢

2.

66 yd. @ 33⅓¢
76 yd. @ 25¢
76 yd. @ 12½¢

3.

57 yd. @ 66⅔¢
72 yd. @ 16⅔¢
90 yd. @ 83⅓¢

4.

64 lb. @ 6¼¢
96 lb. @ 33⅓¢
88 lb. @ 37½¢
91 lb. @ 66⅔¢

5.

48 qt. @ 33⅓¢
72 qt. @ 16⅔¢
96 qt. @ 12½¢
60 qt. @ 6¼¢

6.

120 yd. @ 37½¢
126 yd. @ 16⅔¢
132 yd. @ 83⅓¢
144 yd. @ 33⅓¢

Write and find the total value of two or more groups like the above.

Twelfths

32. 1. ⅓ of 25¢ = how many cents?
2. ⅓ of ¼ = ? 8⅓¢ = what part of a dollar?
3. 50¢ − 8⅓¢ = how many cents?
4. $\frac{1}{2} - \frac{1}{12} = ?$ $41⅔¢ = what part of a dollar?
5. 50¢ + 8⅓¢ = how many cents? what part of a dollar?
6. $1 − 8⅓¢ = how many cents? what part of a dollar?

33. Study the following table and compare values with one another.

	HALVES	THIRDS	QUARTERS	SIXTHS	TWELFTHS
1.					.08⅓
2.				.16⅔	
3.			.25		
4.		.33⅓		5	
5.					.41⅔
6.	.50				
7.					.58⅓
8.		.66⅔			
9.			.75		
10.				.83⅓	
11.					.91⅔
12.	1.00	1.00	1.00	1.00	1.00

34. Analyze .41⅔.

.41⅔, being on the 5th line, is $\frac{5}{12}$. $\frac{5}{12}$ is $\frac{1}{12}$ less than $\frac{6}{12}$ ($\frac{1}{2}$). That is .41⅔ equals .50 less ⅙ of .50.

Similarly, analyze .58⅓.

35. Using the above suggested method of solution where advantageous, find the total value of the following groups:

1.
36 yd. @ 8⅓ ¢
48 yd. @ 16⅔ ¢
24 yd. @ 41⅔ ¢
60 yd. @ 58⅓ ¢
72 yd. @ 83⅓ ¢

2.
66 yd. @ 33⅓ ¢
84 yd. @ 41⅔ ¢
120 yd. @ 91⅔ ¢
42 yd. @ 58⅓ ¢
54 yd. @ 16⅔ ¢

3.
96 yd. @ 83⅓ ¢
84 yd. @ 41⅔ ¢
72 yd. @ 91⅔ ¢
60 yd. @ 8⅓ ¢
132 yd. @ 58⅓ ¢

NOTE. Many articles are bought by the dozen and sold by the piece. A knowledge of the "twelfths" is essential in determining the price of one.

4.
75 lb. @ 8⅓ ¢
88 lb. @ 16⅔ ¢
90 lb. @ 41⅔ ¢
96 lb. @ 58⅓ ¢
84 lb. @ 66⅔ ¢

5.
24 yd. @ 58⅓ ¢
36 yd. @ 41⅔ ¢
60 yd. @ 91⅔ ¢
48 yd. @ 16⅔ ¢
72 yd. @ 33⅓ ¢

6.
140 lb. @ 33⅓ ¢
160 lb. @ 91⅔ ¢
240 lb. @ 58⅓ ¢
360 lb. @ 41⅔ ¢
118 lb. @ 8⅓ ¢

REVIEW

36. Using the shortest solution, find the total cost of:

1.

54 yd. @ 16⅔¢ 9
40 yd. @ 25¢
36 yd. @ 33⅓¢
84 yd. @ 50¢
75 yd. @ 66⅔¢
96 yd. @ 12½¢
48 yd. @ 8⅓¢
36 yd. @ 16⅔¢
42 yd. @ 25¢

2.

120 yd. @ 62½¢
240 yd. @ 87½¢
160 yd. @ 66⅔¢
360 yd. @ 91⅔¢
164 yd. @ 37½¢
66 yd. @ 41⅔¢
68 yd. @ 43¾¢
75 yd. @ 58⅓¢
111 yd. @ 33⅓¢

3.

33 yd. @ 12½¢
41 yd. @ 8⅓¢
52 yd. @ 6¼¢
63 yd. @ 12½¢
89 yd. @ 33⅓¢
113 yd. @ 75¢
97 yd. @ 62½¢
116 yd. @ 87½¢
121 yd. @ 56¼¢

4.

76 yd. @ 37½¢
92 yd. @ 31¼¢
83 yd. @ 43¾¢
78 yd. @ 41⅔¢
107 yd. @ 6¼¢
113 yd. @ 62½¢
114 yd. @ 83⅓¢
131 yd. @ 16⅔¢
125 yd. @ 25¢
143 yd. @ 6¼¢

5.

78 lb. @ 18¾¢
93 lb. @ 25¢
82 lb. @ 33⅓¢
13 lb. @ 31¼¢
9 lb. @ 37½¢
27 lb. @ 41⅔¢
32 lb. @ 43¾¢
84 lb. @ 50¢
79 lb. @ 56¼¢
97 lb. @ 58⅓¢

6.

87 lb. @ 6¼¢
94 lb. @ 8⅓¢
104 lb. @ 12½¢
231 lb. @ 16⅔¢
321 lb. @ 18¾¢
88 lb. @ 93¾¢
136 lb. @ 91⅔¢
316 lb. @ 83⅓¢
28 lb. @ 81¼¢
7 lb. @ 68¾¢

7.

570 lb. @ 93¾¢
629 lb. @ 91⅔¢
328 lb. @ 87½¢
245 lb. @ 8⅓¢
446 lb. @ 68¾¢
928 lb. @ 75¢
476 lb. @ 18¾¢
846 lb. @ 56¼¢
382 lb. @ 58⅓¢
593 lb. @ 31¼¢

8.

234 lb. @ 87½¢
859 lb. @ 43¾¢
720 lb. @ 81¼¢
924 lb. @ 33⅓¢
634 lb. @ 93¾¢
210 lb. @ 58⅓¢
446 lb. @ 12½¢
888 lb. @ 62½¢
356 lb. @ 56¼¢
926 lb. @ 31¼¢

9.

448 lb. @ 56¼¢
234 lb. @ 6¼¢
756 lb. @ 81¼¢
335 lb. @ 91⅔¢
543 lb. @ 58⅓¢
636 lb. @ 62½¢
927 lb. @ 66⅔¢
360 lb. @ 83⅓¢
448 lb. @ 91⅔¢
864 lb. @ 68¾¢

SEVENTHS, NINTHS, AND FIFTEENTHS

37. In addition to the aliquot parts of a dollar already found, there are a few others of which use is occasionally made: viz. sevenths, ninths, and fifteenths.

1. What is $\frac{1}{7}$ of a dollar? $\frac{2}{7}$? $\frac{3}{7}$?
2. What is $\frac{1}{9}$ of a dollar? $\frac{2}{9}$? $\frac{4}{9}$?
3. What is $\frac{1}{15}$ of a dollar? $\frac{2}{15}$?

NOTE. These lists will not be made complete, as there is seldom any use for others than those given above.

38. Make the extensions mentally, and find total value of each group:

1.	2.	3.
21 lb. @ $14\frac{2}{7}$ ¢	42 lb. @ $42\frac{6}{7}$ ¢	33 lb. @ $14\frac{2}{7}$ ¢
28 lb. @ $28\frac{4}{7}$ ¢	54 lb. @ $44\frac{4}{9}$ ¢	45 lb. @ $28\frac{4}{7}$ ¢
36 lb. @ $11\frac{1}{9}$ ¢	60 lb. @ $13\frac{1}{3}$ ¢	83 lb. @ $6\frac{2}{3}$ ¢
45 lb. @ $6\frac{2}{3}$ ¢	84 lb. @ $14\frac{2}{7}$ ¢	96 lb. @ $11\frac{1}{9}$ ¢
48 lb. @ $6\frac{1}{4}$ ¢	91 lb. @ $28\frac{4}{7}$ ¢	87 lb. @ $12\frac{1}{2}$ ¢

Write two or more groups and find total value of each.

39. In the preceding exercises the price has been an aliquot part of \$1. The same principle applies when the quantity is an aliquot part of 100 units of the article involved.

The cost of 48 yd. @ $12\frac{1}{2}$ ¢ = the cost of $12\frac{1}{2}$ yd. @ 48 ¢.

(a) 48 yd. @ $12\frac{1}{2}$ ¢ = $\frac{1}{8}$ of 48 = 6, hence \$6, cost.
(b) 100 yd. @ 48 ¢ = \$48.
$12\frac{1}{2}$ yd. = $\frac{1}{8}$ of 100 yd.
Hence, $12\frac{1}{2}$ yd. @ 48 ¢ cost $\frac{1}{8}$ of \$48 = \$6, cost.

40. Find total value of each of the following groups:

1.	2.	3.
25 yd. @ 32 ¢	$16\frac{2}{3}$ yd. @ 42 ¢	$6\frac{1}{4}$ lb. @ 16 ¢
$33\frac{1}{3}$ yd. @ 45 ¢	$18\frac{3}{4}$ yd. @ 56 ¢	~~$8\frac{1}{3}$ lb. @ 24 ¢~~
50 yd. @ 78 ¢	$31\frac{1}{4}$ yd. @ 80 ¢	$12\frac{1}{2}$ lb. @ 32 ¢
$37\frac{1}{2}$ yd. @ 48 ¢	$62\frac{1}{2}$ yd. @ 72 ¢	$16\frac{2}{3}$ lb. @ 42 ¢
75 yd. @ 80 ¢	$93\frac{3}{4}$ yd. @ 96 ¢	$18\frac{3}{4}$ lb. @ 48 ¢

4.

25 lb. @ 43¢
31¼ lb. @ 56¢
33⅓ lb. @ 58¢
37½ lb. @ 68¢
43¾ lb. @ 96¢

5.

78 yd. @ 33⅓¢
16⅔ yd. @ 33¢
56¼ yd. @ 44¢
58⅓ yd. @ 66¢
38 yd. @ 62½¢

6.

43 yd. @ 66⅔¢
68¾ yd. @ 32¢
81¼ yd. @ 16¢
83⅓ yd. @ 72¢
80 yd. @ 93¾¢

7.

89 lb. @ 6⅔¢
93 lb. @ 6¼¢
87 lb. @ 12½¢
76 lb. @ 16⅔¢
116 lb. @ 25¢

8.

160 yd. @ 75¢
200 yd. @ 87½¢
240 yd. @ 93¾¢
25 yd. @ $1.20
50 yd. @ $1.60

9.

1148 yd. @ 33⅓¢
2682 yd. @ 37½¢
81¼ yd. @ 80¢
68¾ yd. @ 96¢
75 yd. @ 72¢

APPLICATION OF THE PRINCIPLES OF ALIQUOT PARTS

41. 1. Multiply 48 by 10; by 100; by 1000.

2. What did you do to multiply by 10? by 100? by 1000?

3. 2½ is what part of 10? 25 is what part of 100? 250 is what part of 1000?

4. Multiply 48 by 2½; by 25; by 250.

First multiply 48 by 10: that is, annex one cipher, thus, 480. Since the given multiplier is ¼ of 10, the required product is ¼ of 480, or 120.

In like manner to multiply by 25, first annex two ciphers to 48, thus, 4800, and divide by 4, because 25 is ¼ of 100, etc.

5. 3⅓ is what part of 10?
33⅓ is what part of 100?
333⅓ is what part of 1000?

6. Multiply 144 by 3⅓.

Since 3⅓ is ⅓ of 10, first annex one cipher (multiply by 10), thus, 1440, and divide by 3. The required product is 480.

7.

1¼ is what part of 10?
12½ is what part of 100?
125 is what part of 1000?

8.

1⅔ is what part of 10?
16⅔ is what part of 100?
166⅔ is what part of 1000?

Complete

11,345.8

42. Copy and complete the following table:

	1	10	100	1000
⅛'s	.12½	1¼	12½	125
¼'s				
⅙'s				
⅓'s				

43. Make the extensions mentally and find the total value of the items in each group:

1.

24 A.	@ $25
18 cows	@ $25
72 sheep	@ $2½
16 plows	@ $12½
32 horses	@ $125
160 bu.	@ $1¼
330 bu.	@ $1.66⅔
48 wagons	@ $125

2.

25 A.	@ $96
33⅓ bu.	@ $1.08
166⅔ bu.	@ $.84
1250 bu.	@ $.64
2500 bu.	@ $.78
1600 bu.	@ $1.25
16⅔ bu.	@ $1.44
3800 lb.	@ $.12½

3.

125 bu.	@ $1.50
75 wagons	@ $83⅓
6¼ lb.	@ $.44
2500 lb.	@ $.39
12½ bu.	@ $.98
35 horses	@ $250
175 sheep	@ $12.50
50 cows	@ $137.50

4.

43¾ bu.	@ $.64
520 lb.	@ $.18¾
66 lb.	@ $.58⅓
750 plows	@ $16
87½ bu.	@ $1.84
1500 bu.	@ $2.60
78 wagons	@ $166⅔
438 lb.	@ $.06¼

5.

8⅓ bu.	@ $1.50
29 cows	@ $125
59 sheep	@ $16⅔
500 lb.	@ $.19
35 desks	@ $75
66⅔ bu.	@ $.90
580 lb.	@ $.18¾
25 lb.	@ $9

6.

225 bu.	@ $2.50
40 horses	@ $250
124 desks	@ $50
43¾ lb.	@ $.36
1500 lb.	@ $.44
17 cows	@ $125
464 sheep	@ $18¾
28 chairs	@ $33⅓

X

TRADE DISCOUNT

44. All the information about aliquot parts in the preceding pages is directly applicable to Trade Discount.

45. The expression "per cent" (symbol %) means hundredths. That is, 25% = $\frac{25}{100}$ = $\frac{1}{4}$.

46. Find the net cost of the following, allowing a discount of $37\frac{1}{2}$%:

66 lb. @ $66\frac{2}{3}$¢ = \$44
80 lb. @ 40¢ = \$32
120 lb. @ 75¢ = \$90
240 lb. @ $37\frac{1}{2}$¢ = \$90   \$256
Less $37\frac{1}{2}$% ($\frac{1}{4}$ + $\frac{1}{2}$ of $\frac{1}{4}$)   96   \$160

The items are extended mentally by taking $\frac{2}{3}$ of 66; $\frac{2}{5}$ of 80; $\frac{3}{4}$ of 120; and $\frac{3}{8}$ of 240. By adding, the gross cost is found to be \$256. $37\frac{1}{2}$% discount = a discount of $\frac{3}{8}$ ($\frac{1}{4}$ + $\frac{1}{8}$). $\frac{3}{8}$ of \$256 = \$96, which deducted, leaves \$160, the net cost.

47. Find the net cost of each of the following:

1.
72 lb. @ $16\frac{2}{3}$¢ 12
48 lb. @ $6\frac{1}{4}$¢ 3
32 lb. @ $12\frac{1}{2}$¢ 4
56 lb. @ $37\frac{1}{2}$¢ 21
Less $12\frac{1}{2}$%

2.
72 lb. @ 25¢
96 lb. @ $33\frac{1}{3}$¢
90 lb. @ $44\frac{4}{9}$¢
80 lb. @ $87\frac{1}{2}$¢
Less $18\frac{3}{4}$%

3.
36 doz. @ 75¢
48 doz. @ $12\frac{1}{2}$¢
60 doz. @ 25¢
72 doz. @ $37\frac{1}{2}$¢
Less $16\frac{2}{3}$%

4.
48 doz. @ $6\frac{1}{4}$¢ 35
64 doz. @ \$1.25
24 doz. @ 84¢
$37\frac{1}{2}$ doz. @ \$2.40
Less $33\frac{1}{3}$%

5.
64 lb. @ $12\frac{1}{2}$¢
240 lb. @ $33\frac{1}{3}$¢
480 lb. @ $56\frac{1}{4}$¢
320 lb. @ $6\frac{1}{4}$¢
Less 10%

6.
125 lb. @ 48¢
375 lb. @ 72¢
320 lb. @ $62\frac{1}{2}$¢
250 lb. @ 36¢
Less $12\frac{1}{2}$%

7.
196 doz. @ \$1.25
26 doz. @ 75¢
86 doz. @ $12\frac{1}{2}$¢
50 doz. @ 74¢
Less 25%

8.
326 doz. @ $62\frac{1}{2}$¢
112 doz. @ 75¢
84 doz. @ $62\frac{1}{2}$¢
2500 lb. @ $6\frac{1}{4}$¢
Less $37\frac{1}{2}$%

9.
841 lb. @ $12\frac{1}{2}$¢
963 lb. @ $16\frac{2}{3}$¢
1047 lb. @ 25¢
141 lb. @ $33\frac{1}{3}$¢
Less 20%

FURTHER SUGGESTIONS FOR SHORTENING MULTIPLICATION

48. In addition to the so-called Aliquot Parts of 100, the use and application of which have been explained, there are many other numbers that are a fractional part more or less than the aliquot parts. By making use of this fact the work of multiplying by numbers that are not aliquot parts of 100 may be shortened. For example, $13\frac{1}{3}$ ¢ is not an aliquot part of a dollar, yet to multiply by $.13\frac{1}{3}$ requires no actual multiplication.

1. Multiply 120 by $.13\frac{1}{3}$.

$$.13\frac{1}{3} = .10 + (\tfrac{1}{3} \text{ of } .10)$$

Therefore, to multiply 120 by $.13\frac{1}{3}$, point off one place in 120, thus, 12.0, and add $\frac{1}{3}$ of 12, which gives a result of 16.

2. Multiply 36 by $.23\frac{1}{3}$.

$$\begin{array}{ll}
.23\frac{1}{3} & = 20 + (\tfrac{1}{6} \text{ of } .20) = \tfrac{2}{10} + (\tfrac{1}{6} \text{ of } \tfrac{2}{10}) \\
\underline{3.6} & \underline{= \tfrac{1}{10} \text{ of } 36.} \\
7.2 & = \tfrac{2}{10} \text{ of } 36. \\
\underline{1.2} & = \tfrac{1}{6} \text{ of } \tfrac{2}{10} \text{ of } 36 \\
8.4 & = .23\tfrac{1}{3} \text{ of } 36
\end{array}$$

Moving the point one place to the left in 36 gives $\frac{1}{10}$ of 36; multiplying by 2 gives $\frac{2}{10}$ of 36. To find the remaining part take $\frac{1}{6}$ of 7.2, which is 1.2. Add this to 7.2, giving 8.4, the desired result.

3. To multiply by $.13\frac{3}{4}$, find value at $.12\frac{1}{2}$ and add $\frac{1}{10}$ of the result.

That is, to find the value of 104 yd. at $13\frac{3}{4}$ ¢, solve thus:

$$\begin{array}{r|l l}
\$104 & & = \text{value at } \$1 \\
\hline
13 & & = \text{value at } \quad .12\tfrac{1}{2} \\
1 & 30 & = \text{value at } \quad .01\tfrac{1}{4} \ (\tfrac{1}{10} \text{ of } \$.12\tfrac{1}{2}) \\
\hline
14 & 30 & = \text{value at } \$ \ .13\tfrac{3}{4}
\end{array}$$

4. To multiply by $.27\frac{1}{2}$, find the value at .25 and add $\frac{1}{10}$ of the result.

That is, to find the value of 146 yd. at $27\frac{1}{2}$ ¢, solve thus:

$$\begin{array}{r|l l}
\$146 & & = \text{value at } \$1 \\
\hline
36 & 50 & = \text{value at } \quad .25 \\
3 & 65 & = \text{value at } \quad .02\tfrac{1}{2} \\
\hline
\$ \ 40 & 15 & = \text{value at } \$ \ .27\tfrac{1}{2}
\end{array}$$

5. To multiply by .33¾, point off 1 place, multiply by 3, and add ⅛ of the result. (A vertical line takes the place of the decimal points.)

Thus 426 × $.33¾:

$ 42	6	= value at $.10
127	8	= value at .30
15	975	= value at .03¾ (⅛ of .30)
143	775	
143	78	= value at $.33¾

6. To multiply by .35, find the value at .25 and at .10, and add the results.

7. To multiply by .45, find the value at .50, and deduct ⅒ of the result.

8. To multiply by .83⅓; .87½; .91⅔; .93¾ or other fractional values which lack but one part of unity, deduct the one part from the value at unity.

Thus, 137 × $.83⅓.

$137		= value at $1.
22	83	= value at .16⅔
114	17	= value at .83⅓

Note. This plan of solution avoids the necessity of retaining the fraction in the value at $.16⅔. .83⅓ is 5 times .16⅔. If $22.83 is multiplied by 5, the result is $114.15, which is incorrect, due to the dropping of the fraction ⅓ in the value, $22.83.

SUGGESTIONS FOR CLOSE OBSERVATION AND PRACTICE

9. To multiply by .02½, point off 1 place and divide by 4.
10. To multiply by .03⅓, point off 1 place and divide by 3.
11. To multiply by .05, point off 1 place and divide by 2.
12. To multiply by .07½, point off 1 place and deduct ¼ of the result.
13. To multiply by .09, point off 1 place and deduct ⅒ of the result.
14. To multiply by .10, point off 1 place.
15. To multiply by .11, point off 1 place and add ⅒ of the result.

16. To multiply by .11$\frac{1}{4}$, point off 1 place and add $\frac{1}{8}$ of the result.

17. To multiply by .11$\frac{1}{2}$, divide by 8, and deduct .01 of the original number.

18. To multiply by .13$\frac{1}{2}$, divide by 8 and add .01 of the original number.

19. To multiply by .15, point off 1 place and add $\frac{1}{2}$ of the result.

20. To multiply by .17$\frac{1}{2}$, point off 1 place, add $\frac{1}{2}$ of the result and $\frac{1}{2}$ of the second result. (.17$\frac{1}{2}$ = .10 + .05 + .02$\frac{1}{2}$)

21. To multiply by .18, point off 1 place, multiply by 2 and deduct $\frac{1}{10}$ of the result.

22. To multiply by .19, point off 1 place, multiply by 2 and deduct .01 of original number.

23. To multiply by. 22$\frac{1}{2}$, divide by 4 and deduct $\frac{1}{10}$ of the result.

24. To multiply by .23$\frac{3}{4}$, divide by 4 and deduct $\frac{1}{20}$ of the result.

25. To multiply by .24, divide by 4 and deduct .01 of the original number.

26. To multiply by .26, divide by 4 and add .01 of the original number.

27. To multiply by .26$\frac{2}{3}$, point off 1 place and add $\frac{1}{6}$ of the original number. (.26$\frac{2}{3}$ = .10 + .16$\frac{2}{3}$)

28. To multiply by .35, point off 1 place and add $\frac{1}{4}$ of the original number, etc.

These few suggestions are made here in the hope that the student, after becoming familiar with them, will be led to discover many other new and related combinations for himself.

49. Find total value of each group:

#

1.	2.	3.
48 yd. @ 7$\frac{1}{2}$¢	90 yd. @ 18¢	7$\frac{1}{2}$ yd. @ 36¢
56 yd. @ 9¢	96 yd. @ 22¢	11$\frac{1}{2}$ yd. @ 48¢
64 yd. @ 13$\frac{1}{2}$¢	110 yd. @ 22$\frac{1}{2}$¢	16 yd. @ 45¢
80 yd. @ 11$\frac{1}{2}$¢	116 yd. @ 27$\frac{1}{2}$¢	17$\frac{1}{2}$ yd. @ 62¢
88 yd. @ 17$\frac{1}{2}$¢	124 yd. @ 35¢	18 yd. @ 65¢

37.97

#

4.

55 yd. @ 44 ¢
55 yd. @ 36 ¢
65 yd. @ 33 ¢
51 yd. @ 27 ¢
39 yd. @ 55 ¢

5.

132 yd. @ 22 ¢
141 yd. @ 22½ ¢
123 yd. @ 17½ ¢
163 yd. @ 27½ ¢
172 yd. @ 27 ¢

6.

219½ yd. @ 33 ¢
313¼ yd. @ 35 ¢
317½ yd. @ 44 ¢
273½ yd. @ 45 ¢
124¾ yd. @ 55 ¢

7.

24 lb. @ 2½ ¢
24 lb. @ 3⅓ ¢
48 lb. @ 5 ¢
44 lb. @ 7½ ¢
56 lb. @ 9 ¢

8.

123 lb. @ 10 ¢
113 lb. @ 11 ¢
120 lb. @ 11¼ ¢
140 lb. @ 12 ¢
150 lb. @ 13⅓ ¢

9.

84 lb. @ 15 ¢
92 lb. @ 17½ ¢
64 lb. @ 22 ¢
72 lb. @ 23¾ ¢
44 lb. @ 24 ¢

10.

54 lb. @ 26⅔ ¢
88 lb. @ 27½ ¢
144 lb. @ 31¼ ¢
160 lb. @ 33¾ ¢
176 lb. @ 35 ¢

11.

37 by 11
28 by 13¾
44 by 18¾
48 by 22
56 by 22½

12.

140 by 31¼
150 by 37½
160 by 43¾
180 by 45
190 by 47½

13.

465 yd. @ 18 ¢
532 yd. @ 27½ ¢
651 yd. @ 33¾ ¢
576 yd. @ 45 ¢
733 yd. @ 36 ¢

14.

563 yd. @ 11¼ ¢
871 yd. @ 13¾ ¢
728 yd. @ 17½ ¢
193 yd. @ 22 ¢
292 yd. @ 22½ ¢

15.

364 lb. @ 2½ ¢
573 lb. @ 3⅓ ¢
492 lb. @ 5 ¢
532 lb. @ 7½ ¢
657 lb. @ 9 ¢

16.

654 bu. @ $2.50
832 bu. @ $1.25
960 bu. @ $.87½
884 bu. @ $1.12½
762 bu. @ $3.37½

17.

847 bu. @ $1.15
933 bu. @ $1.35
538 bu. @ $2.25
854 bu. @ $1.55
732 bu. @ $3.25

18.

496 A. @ $125
534 A. @ $112.50
678 A. @ $87.50
742 A. @ $250
232 A. @ $137.50

RAPID CALCULATION DRILL CHART

50. Suggestion to Teachers. Let the pupils choose sides as in a spelling match. Read the following exercises to them and have each student in turn state the results. Keep a record of the credits for each side, or let pupils choose from opposite side when a credit is made. Other ways of using the chart will be devised by the teacher to suit the particular needs of the class.

16 yd. @ 25 ¢
32 yd. @ 25 ¢
24 yd. @ 25 ¢
32 yd. @ 50 ¢
44 yd. @ 25 ¢
52 yd. @ 25 ¢
48 yd. @ 50 ¢
36 yd. @ 25 ¢
44 yd. @ 75 ¢
24 yd. @ 75 ¢
36 yd. @ 50 ¢
48 yd. @ 75 ¢
60 yd. @ 75 ¢
64 yd. @ 25 ¢
68 yd. @ 25 ¢
64 yd. @ 50 ¢
84 yd. @ 25 ¢
88 yd. @ 50 ¢
88 yd. @ 75 ¢
96 yd. @ 75 ¢
92 yd. @ 50 ¢
25 yd @ 16 ¢
50 yd. @ 24 ¢
25 yd. @ 36 ¢
75 yd. @ 44 ¢
25 yd. @ 56 ¢
50 yd. @ 63 ¢
25 yd. @ 22 ¢
25 yd. @ 54 ¢

43 yd. @ 25 ¢
49 yd. @ 50 ¢
56 yd. @ 75 ¢
38 yd. @ 25 ¢
41 yd. @ 25 ¢
65 yd. @ 50 ¢
99 yd. @ 25 ¢
97 yd. @ 50 ¢
91 yd. @ 25 ¢
90 yd. @ 50 ¢
89 yd. @ 25 ¢
32 yd. @ 12½ ¢
48 yd. @ 12½ ¢
32 yd. @ 37½ ¢
24 yd. @ 37½ ¢
16 yd. @ 62½ ¢
24 yd. @ 62½ ¢
32 yd. @ 87½ ¢
48 yd. @ 37½ ¢
52 yd. @ 12½ ¢
64 yd. @ 37½ ¢
64 yd. @ 62½ ¢
72 yd. @ 12½ ¢
80 yd. @ 37½ ¢
88 yd. @ 62½ ¢
36 yd. @ 12½ ¢
44 yd. @ 12½ ¢
48 yd. @ 62½ ¢
56 yd. @ 37½ ¢

37½ yd. @ 48 ¢
62½ yd. @ 32 ¢
12½ yd. @ 56 ¢
37½ yd. @ 72 ¢
37½ yd. @ 64 ¢
32 yd. @ 6¼ ¢
48 yd. @ 6¼ ¢
64 yd. @ 18¾ ¢
64 yd. @ 31¼ ¢
32 yd. @ 31¼ ¢
24 yd. @ 6¼ ¢
48 yd. @ 31¼ ¢
32 yd. @ 18¾ ¢
16 yd. @ 6¼ ¢
32 yd. @ 56¼ ¢
48 yd. @ 56¼ ¢
6¼ yd. @ 32 ¢
18¾ yd. @ 32 ¢
6¼ yd. @ 48 ¢
6¼ yd. @ 64 ¢
18¾ yd. @ 32 ¢
18 yd. @ 33⅓ ¢
24 yd. @ 33⅓ ¢
27 yd. @ 66⅔ ¢
33 yd. @ 66⅔ ¢
36 yd. @ 33⅓ ¢
42 yd. @ 33⅓ ¢
48 yd. @ 66⅔ ¢
21 yd. @ 66⅔ ¢

RAPID CALCULATION DRILL CHART

12 yd. @ 83⅓¢
78 yd. @ 16⅔¢
72 yd. @ 83⅓¢
43 yd. @ 16⅔¢
54 yd. @ 83⅓¢
32 yd. @ 16⅔¢
27 yd. @ 16⅔¢
60 yd. @ 83⅓¢
84 yd. @ 16⅔¢
24 yd. @ 83⅓¢
30 yd. @ 83⅓¢
42 yd. @ 16⅔¢
36 yd. @ 83⅓¢
42 yd. @ 83⅓¢
48 yd. @ 33⅓¢
48 yd. @ 16⅔¢
45 yd. @ 16⅔¢
51 yd. @ 16⅔¢
54 yd. @ 16⅔¢
60 yd. @ 83⅓¢
66 yd. @ 83⅓¢
72 yd. @ 16⅔¢
24 yd. @ 8⅓¢
36 yd. @ 83⅓¢
48 yd. @ 41⅔¢
24 yd. @ 41⅔¢
36 yd. @ 41⅔¢
48 yd. @ 8⅓¢
60 yd. @ 41⅔¢
60 yd. @ 58⅓¢
72 yd. @ 8⅓¢
84 yd. @ 8⅓¢
24 yd. @ 58⅓¢
36 yd. @ 58⅓¢
12 yd. @ 8⅓¢
12 yd. @ 41⅔¢

24 A. @ $33⅓
32 A. @ $25
63 A. @ $33⅓
40 A. @ $25
32 A. @ $125
24 A. @ $25
16 A. @ $250
24 bu. @ $1.33⅓
36 bu. @ $2.50
44 bu. @ $1.25
52 bu. @ $2.50
48 bu. @ $1.25
56 T @ $12.50
64 T. @ $12.50
72 T. @ $12.50
16 oz. @ 2½¢
24 oz. @ 2½¢
21 oz. @ 3⅓¢
28 oz. @ 2½¢
32 oz. @ 2½¢
36 oz. @ 1⅔¢
33 oz. @ 3⅓¢
48 oz. @ 1¼¢
56 oz. @ 2½¢
12 oz. @ 3⅓¢
16 oz. @ 1¼¢
32 oz. @ 1¼¢
32 oz. @ 2½¢
48 oz. @ 1⅔¢
48 oz. @ 3⅓¢
66 oz. @ 3⅓¢
48 oz. @ 25¢
56 yd. @ 12½¢
64 yd. @ 37½¢
72 yd. @ 62½¢
84 yd. @ 25¢

72 lb. @ 33⅓¢
66 lb. @ 66⅔¢
54 lb. @ 66⅔¢
72 lb. @ 8⅓¢
60 lb. @ 75¢
66 lb. @ 83⅓¢
84 lb. @ 16⅔¢
90 lb. @ 16⅔¢
96 lb. @ 66⅔¢
48 yd. @ 58⅓¢
60 yd. @ 41⅔¢
32 yd. @ 62½¢
48 yd. @ 56¼¢
39 yd. @ 33⅓¢
72 yd. @ 83⅓¢
27 oz. @ 3⅓¢
26 oz. @ 2½¢
96 oz. @ 1¼¢
32 oz. @ 2½¢
96 oz. @ 2½¢
96 oz. @ 3⅓¢
96 bu. @ $1.25
24 bu. @ $1.66⅔
80 T. @ $6¼
72 T. @ $8⅓
42 T. @ $12½
30 T. @ $12.50
42 T. @ $16⅔
72 bu. @ $1.66⅔
24 bu. @ $2.50
72 yd. @ 33⅓¢
64 yd. @ 75¢
32 yd. @ 56¼¢
40 yd. @ 62½¢
36 yd. @ 41⅔¢
20 lb. @ 12½¢

ALIQUOT PARTS APPLIED TO DIVISION

51. 1. At 25¢ a yard, how many yards of cloth can be bought for \$1? for \$2? for \$10? for \$25?

2. At $37\frac{1}{2}$¢ a yard, how many yards can be bought for \$1? for \$2? for \$5? for \$15?

$37\frac{1}{2}$¢ = \$$\frac{3}{8}$. \$1 ÷ \$$\frac{3}{8}$ = $\frac{8}{3}$; $\frac{8}{3}$ yd., or $2\frac{2}{3}$ yd., can be bought for \$1

Since $\frac{8}{3}$ yd., or $2\frac{2}{3}$ yd., can be bought for \$1, 2 times $2\frac{2}{3}$ yd., or $5\frac{1}{3}$ yd., can be bought for \$2, and 5 times $2\frac{2}{3}$ yd., or $13\frac{1}{3}$ yd., can be bought for \$5, etc.

NOTE. $1 \div \frac{3}{8} = \frac{8}{3}$. $\frac{8}{3}$ is called the reciprocal of $\frac{3}{8}$. The reciprocal of any fraction is the fraction inverted.

Hence, to divide by an aliquot part, multiply by the reciprocal of the fractional equivalent.

52. Find the number of yards that can be bought in each case:

TOTAL COST	COST OF ONE YARD	TOTAL COST	COST OF ONE YARD
1. \$15.00	\$ $.33\frac{1}{3}$	16. \$ 91.00	\$ $.58\frac{1}{3}$
2. 36.00	.75	17. 100.00	$.62\frac{1}{2}$
3. 75.00	$.62\frac{1}{2}$	18. 110.00	$.66\frac{2}{3}$
4. 25.00	$.31\frac{1}{4}$	19. 121.00	$.68\frac{3}{4}$
5. 13.00	$.14\frac{2}{7}$	20. 129.00	.75
6. 16.00	$.08\frac{1}{3}$	21. 130.00	$.81\frac{1}{4}$
7. 27.00	$.18\frac{3}{4}$	22. 135.00	$.83\frac{1}{3}$
8. 29.00	.25	23. 59.50	$.87\frac{1}{2}$
9. 35.00	$.31\frac{1}{4}$	24. 66.00	$.91\frac{2}{3}$
10. 41.00	$.33\frac{1}{3}$	25. 75.00	$.93\frac{3}{4}$
11. 42.00	$.37\frac{1}{2}$	26. 45.00	1.25
12. 47.50	$.41\frac{2}{3}$	27. 93.00	1.50
13. 49.00	$.43\frac{3}{4}$	28. 90.00	$1.87\frac{1}{2}$
14. 57.00	.50	29. 105.00	2.50
15. 81.00	$.56\frac{1}{4}$	30. 440.00	$3.33\frac{1}{3}$

53. To find the cost of goods sold by the 100, 1000, or ton.

1. Find the cost of 168 lb. of feed @ \$1.10 per cwt.

\$1.68 = cost at \$1 per cwt.
.17 = cost at .10 per cwt.
\$1.85 = cost at \$1.10 per cwt.

First point off 2 places in the number of pounds, thus, 168 lb. = 1.68 cwt. At \$1 per hundredweight, the cost would be \$1.68. At \$.10 per hundredweight, the cost would be \$.168 or \$.17. Hence, at \$1.10 per hundredweight, the cost would be the sum of \$1.68 and \$.17, or \$1.85.

2. At $1.87 per C. find the cost of 634 bolts.

634 = 6.34 C.
6.34 × $1.87 = $11.86

634 bolts = 6.34 hundreds. Since 1 hundred costs $1.87, 6.34 hundreds cost 6.34 times $1.87, or $11.86.

3. At $15 per thousand, find the cost of 648 ft. of lumber.

$6.48 = cost at 1 ¢ per foot.
3.24 = cost at $\frac{1}{2}$ ¢ per foot.
$9.72 = cost at $15 per M.

$15 per M = $1\frac{1}{2}$ ¢ per foot. At 1 ¢ a foot, 648 ft. cost $6.48, and at $\frac{1}{2}$ ¢ a foot, the cost is $\frac{1}{2}$ of $6.48, or $3.24. Hence, at $1\frac{1}{2}$ ¢ a foot or $15 per M, the cost is $6.48 + $3.24, or $9.72.

4. At $16.40 per M, find the cost of 13,400 bricks.

13400 = 13.4 thousands.
13.4 × $16.40 = $219.76

The number of M is found by pointing off 3 places in the number 13,400, giving 13.4 M.

At $16.40 per M, 13.4 M cost 13.4 × $16.40, or $219.76

5. $13.50 per ton = ($6 + $\frac{1}{8}$ of $6) per 1000 lb.
Hence 2780 lb. at $13.50 per ton =
$16.68 at $6 per 1000 lb.
2.09 at $.75 per 1000 lb.
$18.77 at $6.75 per 1000 lb., or $13.50 per ton.

6. $1.35 per 100 lb. = (1 ¢ + $\frac{1}{4}$ ¢ + $\frac{1}{10}$ ¢) per pound.

Hence 580 lb. at $1.35 per 100 lb. =
$5.80 at 1 ¢ per pound
1.45 at $\frac{1}{4}$ ¢ per pound
.58 at $\frac{1}{10}$ ¢ per pound
$7.83 at $1.35 per 100 lb.

7. $31.25 per 1000 lb. = ($25 + $\frac{1}{4}$ of $25) per 1000 lb.
Hence 6720 lb. at $31.25 per 1000 lb. =
$168.00 at $25 per 1000 lb.
42.00 at $6.25 per 1000 lb.
$210.00 at $31.25 per 1000 lb.

54. Devise a short method of finding the value of:

8. 4370 lb. coal at $6.75 per ton.
9. 5860 lb. hay at $15.75 per ton.
10. 22,470 brick at $11.25 per 1000.

11. 528 lb. meal at $1.75 per 100 lb.
12. 3156 lb. feed at $27 per ton.
13. 17,640 ft. lumber at $23.75 per 1000 ft.
14. 13,260 ft. lumber at $33.75 per 1000 ft.

55. Solve mentally, if possible, the following:

Find the cost of:

1.	220 lb. bran	@ $1.35 per cwt.	@ $1.40 per cwt.
2.	350 lb. feed	@ 2.50 per cwt.	@ 2.25 per cwt.
3.	675 lb. nails	@ 5.50 per cwt.	@ 5.75 per cwt.
4.	875 lb. salt	@ .80 per cwt.	@ .88 per cwt.
5.	950 posts	@ 6.25 per C.	@ 6.75 per C.
6.	1100 posts	@ 7.25 per C.	@ 7.70 per C.
7.	1250 posts	@ 8.00 per C.	@ 7.20 per C.
8.	1550 lb. lead	@ 7.50 per cwt.	@ 6.37½ per cwt.
9.	38,440 lb. pig iron	@ 1.50 per cwt.	@ 1.62½ per cwt.
10.	24,680 lb. sugar	@ 5.50 per cwt.	@ 5.75 per cwt.
11.	1875 ft. lumber	@ 35.00 per M.	@ 45.00 per M.
12.	1450 ft. lumber	@ 25.00 per M.	@ 22.00 per M.
13.	23,360 brick	@ 18.75 per M.	@ 17.50 per M.
14.	14,500 shingles	@ 5.62½ per M.	@ 5.25 per M.
15.	346 ft. lumber	@ 33.00 per M.	@ 31.25 per M.
16.	23,465 ft. lumber	@ 37.50 per M.	@ 33.75 per M.
17.	47,860 brick	@ 11.25 per M.	@ 12.50 per M.
18.	12,375 brick	@ 13.75 per M.	@ 15.00 per M.
19.	7850 shingles	@ 6.75 per M.	@ 7.50 per M.
20.	319 ft. lumber	@ 45.00 per M.	@ 55.00 per M.
21.	2660 lb. hay	@ 18.00 per T.	@ 22.50 per T.
22.	3474 lb. hay	@ 26.00 per T.	@ 24.00 per T.
23.	3096 lb. straw	@ 14.50 per T.	@ 13.75 per T.
24.	7276 lb. straw	@ 13.75 per T.	@ 12.50 per T.
25.	1680 lb. feed	@ 24.60 per T.	@ 27.90 per T.
26.	4288 lb. feed	@ 26.00 per T.	@ 27.00 per T.
27.	13,890 lb. coal	@ 13.75 per T.	@ 12.50 per T.
28.	47,976 lb. coal	@ 13.50 per T.	@ 11.25 per T.
29.	1890 lb. coal	@ 9.50 per T.	@ 8.80 per T.
30.	560 lb. coal	@ 12.50 per T.	@ 13.50 per T.

I = prt.

SIMPLE INTEREST

ALIQUOT PARTS APPLIED TO SIMPLE INTEREST

56. Simple interest is treated in this part of the text for two reasons: first, because it is required early in the course by all students who are at the same time pursuing the study of bookkeeping, and second, to correlate it with the work in "Aliquot Parts."

57. **Interest** is the price paid for the use of money.

58. The **principal** is the sum for the use of which interest is paid.

59. For ordinary interest calculations, a year is considered as having 12 mo. of 30 da. each, or 360 da.

The expression "6% interest" or "interest at 6%" means,

The interest on $1 for 1 year is $.06. 2 months is $\frac{1}{6}$ of a year, therefore,

The interest on $1 for 2 months (60 days) is $.01. Since the interest on $1 at 6% for 60 days is $.01,

on $5 it is $.05
on $40 it is $.40
on $175 it is $1.75
on $1000 it is $10.00, etc.

Hence, to find the interest for 2 mo. or 60 days, at 6%, on any sum of money, move the decimal point TWO places to the left.

60. Find the interest on $450.50 for 60 da. at 6%.

$4|50.50 = $4.51, int. at 6% for 60 da.

Since at 6% for 60 da. the interest on $1 is $.01, the interest on $450.50 is $.01 for each dollar, or $4.5050, which equals $4.51. (Answers in interest problems are always given in the nearest whole cent.)

61. Find the interest at 6% on each of the following for 60 da.:

1. $630
2. $750
3. $365
4. $846
5. $932
6. $1036
7. $2465
8. $3675
9. $4270
10. $824.75
11. $968.66
12. $732.45

62. In the preceding pages on aliquot parts 100 was the basis of comparison. In interest solutions 60 is the basis. Memorize the following aliquot parts of 60. (Compare with the parts of an hour.)

30 = $\frac{1}{2}$ of 60
20 = $\frac{1}{3}$ of 60
15 = $\frac{1}{4}$ of 60
12 = $\frac{1}{5}$ of 60
10 = $\frac{1}{6}$ of 60
6 = $\frac{1}{10}$ of 60
5 = $\frac{1}{12}$ of 60
4 = $\frac{1}{15}$ of 60
3 = $\frac{1}{20}$ of 60

63. What combinations of 60 da. and aliquot parts of 60 da. will best give (*a*) 45 da.? (*b*) 27 da.? (*c*) 85 da.?

(*a*) 60 da.
15 da. = $\frac{1}{4}$ of 60
45 da.

(*b*) 60 da.
20 da. = $\frac{1}{3}$ of 60
6 da. = $\frac{1}{10}$ of 60
1 da. = $\frac{1}{6}$ of 6
27 da.

(*c*) 60 da.
20 da. = $\frac{1}{3}$ of 60
5 da. = $\frac{1}{4}$ of 20
85 da.

64. Find combinations of 60 da. and aliquot parts of 60 da. that will best amount to:

1. 55 da.
2. 40 da.
3. 33 da.
4. 35 da.
5. 25 da.
6. 27 da.
7. 32 da.
8. 26 da.
9. 66 da.
10. 75 da.
11. 72 da.
12. 80 da.
13. 87 da.
14. 93 da.
15. 84 da.
16. 90 da.
17. 11 da.
18. 14 da.
19. 16 da.
20. 19 da.
21. 21 da.
22. 76 da.
23. 96 da.
24. 105 da.
25. 110 da.
26. 135 da.
27. 116 da.
28. 101 da.

65. 1. Find the interest at 6% on $540 for 22 da.

$5	40	= int. for 60 da.
1	80	= int. for 20 da.
	18	= int. for 2 da.
1	98	= int. for 22 da.

22 da. is composed of the aliquot parts 20 da. and 2 da.

$5.40 is the interest for 60 da.

20 da. is $\frac{1}{3}$ of 60 da., hence the interest for 20 da. is $\frac{1}{3}$ of $5.40, or $1.80. The remaining 2 da. is $\frac{1}{10}$ of 20 da., and the interest for 2 da. is $\frac{1}{10}$ of $1.80, or $.18.

The interest for 22 da. is $1.98.

2. Find the interest at 6% on $1716.75 for 87 da.

	$17	1675	= int. for 60 da.
	5	7225	= int. for 20 da.
	1	7167	= int. for 6 da.
		2861	= int. for 1 da.
	$24	8928	= int. for 87 da.
or	$24	89	= int. for 87 da.

87 da. = 60 da. + 20 da. + 6 da. + 1 da. Find 60 da. interest, which is $17.1675 and add to it $\frac{1}{3}$ of itself (for 20 da.), $\frac{1}{10}$ of itself (for 6 da.), and $\frac{1}{6}$ of the interest for 6 da. (for 1 da.) which gives $24.89, the interest for 87 da.

NOTES. (1) The vertical line takes the place of the decimal points in the solution.

(2) The interest for fractional parts of the time should be kept correct to the 4th decimal place to insure accuracy in the cents column.

66. Find the interest at 6% on:

1. $260 for 30 da.
2. $360 for 45 da.
3. $480 for 50 da.
4. $560 for 36 da.
5. $840 for 26 da.
6. $720 for 25 da.
7. $960 for 40 da.
8. $900 for 44 da.
9. $1260 for 70 da.
10. $1600 for 90 da.
11. $1800 for 80 da.
12. $2000 for 75 da.
13. $2100 for 72 da.
14. $2200 for 96 da.
15. $2600 for 105 da.
16. $3000 for 16 da.
17. $4200 for 30 da.
18. $4800 for 33 da.
19. $5100 for 45 da.
20. $8000 for 45 da.
21. $9000 for 50 da.
22. $9600 for 90 da.
23. $1000 for 75 da.
24. $1340 for 120 da.
25. $1600 for 150 da.
26. $1800 for 210 da.
27. $3000 for 180 da.
28. $4000 for 240 da.
29. $4500 for 320 da.
30. $4800 for 160 da.
31. $5000 for 150 da.
32. $6300 for 160 da.
33. $6600 for 140 da.
34. $7200 for 115 da.
35. $7500 for 100 da.
36. $8400 for 99 da.

67. 1. What is the interest on $100 at 6% for 6 da.?

6 da. is what part of 60 da.?
What is the interest on $100 at 6% for 60 da.?
What is the interest on $100 at 6% for 6 da.?
What is the quickest way of finding the interest on $100 at 6% for 6 da.?
What is the quickest way of finding the interest for 6 da. at 6% on any sum of money?

2. Find the interest on $1460 at 6% for (a) 6 days; (b) 9 days.

(a)
$1|460 = interest for 6 da.

(b)
$1|460 = interest for 6 da.
  |73 = interest for 3 da.
$2|19 = interest for 9 da.

(a) Move the decimal point three places to the left. The result is the interest at 6% for 6 days.

(b) After finding the interest at 6% for 6 days, find the interest for 3 days by taking ½ of the interest for 6 days. Adding the interest for 6 days and the interest for 3 days gives the interest for 9 days.

Having the interest for 6 da., how may the interest be found for

1 da.?
2 da.?
3 da.?
4 da.?
5 da.?
7 da.?
8 da.?
9 da.?
10 da.?
18 da.?
24 da.?
36 da.?

#1.30

68. Find the interest at 6% on the following:

1. $400 for 6 da.
2. $800 for 6 da.
3. $1000 for 9 da.
4. $1200 for 8 da.
5. $1400 for 18 da.
6. $1500 for 7 da.
7. $1800 for 24 da.
8. $1560 for 36 da.
9. $1280 for 42 da.
10. $2000 for 39 da.
11. $1250 for 48 da.
12. $1125 for 24 da.
13. $1440 for 19 da.
14. $1280 for 21 da.
15. $1480 for 24 da.
16. $1600 for 27 da.
17. $2100 for 7 da.
18. $2400 for 3 da.
19. $3600 for 2 da.
20. $4200 for 1 da.
21. $9000 for 4 da.

69. Find the total interest at 6% on:

1.
$420 for 18 da.
$540 for 20 da.
$640 for 21 da.
$350 for 13 da.
$432 for 15 da.
$128 for 16 da.

2.
$630 for 11 da.
$750 for 9 da.
$780 for 26 da.
$960 for 36 da.
$1200 for 49 da.
$840 for 50 da.

3.
$1300 for 66 da.
$1560 for 72 da.
$1480 for 78 da.
$1260 for 84 da.
$1450 for 90 da.
$1660 for 91 da.

4.
$1728 for 95 da.
$1890 for 98 da.
$1820 for 100 da.
$1675 for 108 da.
$1440 for 110 da.
$1360 for 120 da.

5.
$800 for 31 da.
$900 for 41 da.
$1000 for 48 da.
$460 for 27 da.
$390 for 19 da.
$260 for 16 da.

6.
$320 for 24 da.
$390 for 2 da.
$410 for 12 da.
$812 for 15 da.
$915 for 20 da.
$990 for 10 da.

7.
$828 for 45 da.
$936 for 15 da.
$832 for 75 da.
$648 for 70 da.
$444 for 80 da.
$555 for 40 da.

8.
$736 for 105 da.
$888 for 110 da.
$960 for 115 da.
$822 for 150 da.
$440 for 130 da.
$375 for 88 da.

9.
$482.75 for 12 da.
$321.50 for 18 da.
$428.48 for 15 da.
$320.10 for 20 da.
$516.74 for 24 da.
$608.31 for 30 da.

10.
$597.30 for 36 da.
$321.47 for 40 da.
$218.38 for 42 da.
$712.80 for 6 da.
$811.75 for 10 da.
$928.30 for 30 da.

11.
$731.45 for 13 da.
$806.40 for 14 da.
$318.25 for 11 da.
$208.14 for 10 da.
$327.38 for 16 da.
$445.50 for 19 da.

12.
$673.87 for 21 da.
$832.49 for 22 da.
$738.27 for 25 da.
$387.39 for 29 da.
$873.93 for 5 da.
$783.79 for 3 da.

13.
$720.30 for 25 da.
$628.50 for 44 da.
$731.20 for 14 da.
$906.75 for 9 da.
$887.63 for 12 da.
$674.70 for 13 da.

14.
$475 for 26 da.
$938 for 33 da.
$840 for 17 da.
$729 for 19 da.
$325 for 39 da.
$480 for 47 da.

15.
$425 for 38 da.
$576 for 67 da.
$545 for 88 da.
$750 for 112 da.
$800 for 99 da.
$235 for 81 da.

16.
$329.60 for 32 da.
$572.40 for 38 da.
$635.70 for 52 da.
$596.75 for 17 da.
$831.50 for 12 da.
$938.75 for 31 da.

17.
$800 for 37 da.
$900 for 44 da.
$1200 for 39 da.
$1500 for 53 da.
$1800 for 67 da.
$2000 for 78 da.

18.
$450 for 28 da.
$480 for 21 da.
$540 for 19 da.
$660 for 23 da.
$720 for 32 da.
$840 for 40 da.

19.
$700 for 57 da.
$1100 for 83 da.
$1300 for 96 da.
$1700 for 47 da.
$1600 for 81 da.
$1400 for 102 da.

20.
$521.60 for 23 da.
$387.60 for 32 da.
$573.20 for 41 da.
$753.30 for 55 da.
$839.70 for 64 da.
$931.75 for 20 da.

21.
$220 for 12 da.
$330 for 15 da.
$560 for 18 da.
$640 for 24 da.
$720 for 30 da.
$960 for 10 da.

22.
$830 for 6 da.
$870 for 9 da.
$910 for 12 da.
$530 for 15 da.
$780 for 45 da.
$880 for 42 da.

23.
$380 for 46 da.
$780 for 40 da.
$190 for 57 da.
$350 for 51 da.
$870 for 18 da.
$880 for 24 da.

24.
$540 for 82 da.
$840 for 12 da.
$670 for 91 da.
$900 for 29 da.
$1200 for 39 da.
$1400 for 90 da.

INTEREST DRILL CHART

70. Use this chart as suggested on page 27.

Find the interest at 6% on:

$800 for 30 da.
$900 for 20 da.
$600 for 15 da.
$700 for 30 da.
$640 for 15 da.
$720 for 20 da.
$480 for 10 da.
$540 for 6 da.
$400 for 9 da.
$500 for 12 da.
$800 for 15 da.
$720 for 20 da.
$960 for 30 da.
$920 for 20 da.
$860 for 30 da.
$700 for 60 da.
$600 for 10 da.
$500 for 15 da.
$400 for 75 da.
$500 for 90 da.
$600 for 80 da.
$400 for 90 da.
$800 for 75 da.
$900 for 80 da.
$600 for 90 da.
$440 for 90 da.
$780 for 30 da.
$860 for 30 da.
$460 for 30 da.
$560 for 15 da.
$660 for 20 da.
$680 for 15 da.
$240 for 120 da.

$360 for 15 da.
$320 for 15 da.
$150 for 30 da.
$250 for 120 da.
$300 for 30 da.
$400 for 6 da.
$600 for 1 da.
$800 for 6 da.
$420 for 20 da.
$350 for 12 da.
$250 for 12 da.
$750 for 20 da.
$450 for 20 da.
$150 for 20 da.
$550 for 12 da.
$1200 for 15 da.
$1000 for 90 da.
$1400 for 90 da.
$1500 for 80 da.
$1200 for 80 da.
$1200 for 70 da.
$1500 for 72 da.
$1800 for 30 da.
$1800 for 80 da.
$1600 for 15 da.
$1600 for 75 da.
$1800 for 90 da.
$2000 for 60 da.
$2200 for 30 da.
$2400 for 15 da.
$3600 for 20 da.
$3400 for 30 da.
$3600 for 80 da.

$4400 for 15 da.
$4200 for 10 da.
$4300 for 6 da.
$4700 for 12 da.
$4500 for 20 da.
$4600 for 30 da.
$5100 for 20 da.
$5200 for 15 da.
$4400 for 90 da.
$4500 for 80 da.
$4200 for 70 da.
$4400 for 120 da.
$4000 for 80 da.
$3000 for 240 da.
$3800 for 30 da.
$3300 for 10 da.
$3400 for 15 da.
$5400 for 10 da.
$4750 for 6 da.
$4900 for 12 da.
$4700 for 12 da.
$2500 for 12 da.
$2600 for 15 da.
$2800 for 45 da.
$2400 for 50 da.
$3500 for 72 da.
$3600 for 55 da.
$4800 for 55 da.
$5600 for 30 da.
$3500 for 12 da.
$3700 for 30 da.
$6600 for 15 da.
$2600 for 18 da.

ADDITION

71. Addition is the process of finding one number or quantity equal in value to two or more other numbers or quantities.

72. The several numbers or quantities to be added are called **addends.**

73. Investigation shows that the most frequently used arithmetical operation is simple addition. It is important that it be done rapidly and accurately.

The following suggestions will be found helpful.

ORAL DRILL EXERCISES

74. 1. Count by 3's from 1 to 100; from 2 to 101.

2. Count by 4's from 1 to 101; from 2 to 102; from 3 to 103.

3. Count by 5's beginning with 0; 1; 2; 3; 4, until 100 is passed.

4. Count by 6's beginning with 0; 1; 2; 3; 4; 5, until 100 is passed.

5. Count by 7's beginning with 0; 1; 2; 3; 4; 5; 6, until 100 is passed.

6. Count by 8's beginning with 0; 1; 2; 3; 4; 5; 6; 7, until 100 is passed.

7. Count by 9's beginning with 0; 1; 2; 3; 4; 5; 6; 7; 8, until 100 is passed.

8. Count by 10's beginning with 0; 1; 2; 3; 4; 5; 6; 7; 8; 9; until 100 is passed.

9. Count by 11's beginning with 0; 1; 2; 3; 4; 5; 6; 7; 8; 9; 10, until 100 is passed.

10. Count by 12's beginning with 0; 1; 2; 3; 4; 5; 6; 7; 8; 9; 10; 11, until 100 is passed.

RAPID DRILL EXERCISES

75. Drill on the following until you can state the results immediately:

9	8	7	6	9	4	5	3	7	9
1	8	4	2	1	3	5	6	1	2
2	1	1	3	3	3	5	2	4	2
7	1	5	5	6	4	4	2	5	6

9	8	7	6	9	9	7	8	9	6
1	2	3	4	5	2	4	4	7	3
9	8	7	6	5	8	6	6	3	7

17	18	24	39	43	67	76	43	61	55
2	3	5	2	8	4	2	2	1	3
3	3	4	6	1	4	7	4	7	5
5	6	1	2	1	2	1	4	2	2

38	47	64	89	79	97	48	53	89	62
9	8	3	4	3	6	5	1	9	8
1	2	7	6	7	4	5	9	1	2

56	67	83	47	82	69	43	67	79	88
2	3	6	3	8	1	4	2	3	4
4	2	3	4	1	4	4	6	6	1
4	5	1	3	1	5	2	2	1	5

81	47	74	82	71	89	74	90	27	44
6	3	4	4	6	2	6	8	3	4
2	4	5	3	1	2	1	1	4	4
2	3	1	3	3	6	3	1	3	2

76. Two important aids to accuracy and speed in addition work are: (1) to make the figures neatly and of uniform size; and (2) to write them exactly in columns.

Add:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
64	59	94	56	49	59	67	61	67	56
28	63	89	38	84	63	38	49	84	73
39	87	21	27	47	72	79	72	98	87
131									

11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
69	19	64	56	59	53	48	53	43	53
76	66	83	79	32	87	72	89	72	87
87	74	39	96	27	98	28	72	29	91
24	47	77	87	87	82	63	68	38	76

21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
43	74	92	94	69	88	63	48	56	45
82	91	28	76	78	72	79	27	74	97
79	43	74	87	86	38	64	68	83	86
87	87	68	92	97	76	87	93	92	73
64	96	93	47	74	43	99	27	76	49
47	78	72	83	83	89	76	64	57	36
82	47	46	97	72	96	64	56	83	17
96	64	87	48	87	74	87	74	92	85
0	550	360	6.4	66	6	119	7	693	

77. The rapidity of addition may be increased by adding figures by groups. Note the groups of 10 in the following. Add rapidly:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
4	7	4	6	3	9	7	4	5	6
6	3	3	3	4	1	3	4	3	2
8	9	3	1	3	4	6	2	2	2
2	1	7	4	5	2	2	5	4	3
7	5	2	4	2	3	2	4	2	5
3	5	1	2	3	1	8	1	4	2

11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
37	49	67	56	56	47	31	77	47	49
84	36	84	34	74	39	63	69	83	84
97	83	97	73	64	28	39	83	76	39
26	92	32	91	59	37	87	65	64	73
84	78	43	79	76	73	79	27	96	42
97	46	88	65	85	81	73	39	89	87
43	87	79	82	93	79	47	47	97	65
		496					401	552	

CHECKING THE WORK

78. **Checking** is a means of testing the accuracy of the work. Accuracy can be obtained by frequently checking the results.

79. The best method of checking addition is to add the columns in the reverse order. If the column is first added from the bottom up, check by adding from the top down.

80. Many bookkeepers and accountants use the method shown in the following illustrative solution:

Add:

(1st)		(Check)
	3967	
26	8234	27
31	7987	32
32	4372	31
27	5976	26
30536	30536	30536

By adding the units' column, a sum of 26 is obtained, which is written as shown. Next, by adding the tens' column, the sum is found to be 31, which is written as in the illustration. The sums of the hundreds' and thousands' columns are 32 and 27, respectively, which are written as here shown.

Beginning with the thousands' column and adding in the opposite direction we find the sum to be 27, which is written as shown in the column marked "Check." The sum of the hundreds', tens', and units' columns, which are 32, 31, and 26, respectively, are written as in the illustration. By adding these several sums the result is seen to agree with the first result obtained. Hence the work is probably correct.

81. Casting out nines.

147 =	12 =	3
229 =	13 =	4
331 =	7 =	7
707	14 =	14
	5 =	5

The sum of the digits 1, 4, and 7 is 12, equal to one 9 and 3 over. The sum of the digits in the second addend is 13, equal to one 9 and 4 over; and the sum in the third addend is 7, equal to no 9's and 7 over. The sum of the several remainders, 3, 4, and 7, is 14, equal to one 9 and 5 over. The sum of the digits in the sum, 707, is 14, equal to one 9 and 5 over. Hence the addition is probably correct.

NOTES. (1) The digits of 147 are the separate numbers 1, 4, and 7.

(2) The remainders 3, 4, 7, and 5 are the sums of the digits in 12, 13, 7, and 14, respectively. 12 is equal to one 9 with a remainder of 3. The division is unnecessary, however, as the excess (remainder) can always be found by adding the digits.

(3) The remainders 3, 4, and 7 are generally called the "excess of nines," and the principle is stated thus: "The excess of nines in a sum is equal to the excess in the sum of the excesses."

(4) Casting out nines is not an absolute test of accuracy. Transposition of figures cannot be detected by this check. nor can the omission or addition of 9's or ciphers be discovered.

S. E. J. D.

82. Casting out elevens.

$$\begin{array}{r} 475 = 2\\ 382 = 8\\ 479 = 6\\ \hline 1336 = 5 \end{array}$$

Find the sum of the digits in the odd places (beginning at units) and from it deduct the sum of the digits in the even places. If the sum of the digits in the even places exceeds the sum of those in the odd places, add 11, or some multiple of 11, to the sum of the digits in the odd places.

Thus,

$$\left.\begin{array}{ll} 5+4-7 & =2\\ 2+3+11-8 & =8\\ 9+4-7 & =6\\ \hline 6+3-3-1 & =5 \end{array}\right\} \begin{array}{l} 6+8+2=16;\\ 16 = \text{one 11 and 5 over.} \end{array}$$

NOTE. Casting out elevens will detect errors not located by the nine check, as indicated in note (4), p. 41.

83. Add the following, and check the results:

1.	2.	3.	4.	5.	6.
2874	8727	8237	9371	6738	3796
3957	3469	6489	6487	9674	3987
6498	5487	5387	5238	5976	4598
7237	3245	6959	9764	8238	3294
4596	5894	6432	5437	9783	5786
3872	7978	8643	6087	4598	5487
5295	5673	9782	9121	1359	7372
9638	9178	9176	4327	9385	9765

84. Each of the following columns should be added in fifteen seconds. Drill on them until correct results can be obtained in that time.

1.	2.	3.	4.	5.	6.
4789	7968	4894	4893	4389	1531
3847	7324	3762	3469	5767	4574
5963	8759	5437	9276	9247	8276
8769	4365	9864	1498	6389	9832
5271	9859	4767	9763	9127	4976
8728	7312	5943	7987	8784	2872
9592	8764	8976	6477	4728	3193
7352	5231	4382	9816	2854	8715
5897	9827	4767	8125	1645	2810
7684	5876	5723	7784	5196	4567

7.	8.	9.	10.	11.
32478672	42876597	45967248	12345678	24681357
45972861	27638976	53876289	98765432	91352468
56972764	45972489	93856214	87654321	57913546
36824579	72510764	72387276	34567892	94872468
59472876	63897123	10468217	45678923	74692789

12.	13.	14	15.	16.
76945897	54763247	57689727	47897684	98753214
34872469	98674382	68972543	32076483	76543216
89778976	76948269	74202604	82478978	97531246
69877978	63897476	70876437	68764376	89765382
45987678	48769531	97687248	98765432	74974768

85. Each of the following columns should be added in one minute Drill on them until it can be done with facility and accuracy.

	1.	2.	3.	4.	5.
1.	34568	93876	69784	38976	94768
2.	72487	47876	72864	72487	48694
3.	95438	83297	14897	92764	38972
4.	64876	45872	32571	12476	66666
5.	45987	76978	89724	53897	89764
6.	98321	32589	89724	69748	92787
7.	87643	13489	97247	48724	77777
8.	97987	87943	83971	93697	48938
9.	34786	76897	72481	54328	55555
10.	49876	37643	92482	97648	48743
11.	24689	28971	76893	63872	33333
12.	39872	59761	64762	59763	98697
13.	97246	97246	59769	97687	88888
14.	72489	38972	43872	53792	46479
15.	93247	89726	74672	47867	99999
16.	59876	97642	94897	83897	72687
17.	72489	57921	58932	46876	22222
18.	72979	38761	72468	93872	48769

SUGGESTION. The number of problems in the above columns can be increased by requiring pupils to find the sum of numbers 1 to 10 inclusive; 2 to 11; 3 to 12; 4 to 13; etc.

6. The following is a list of the twenty largest cities in the United States in order of size, according to the last census. Find their total population. New York, N. Y., 5,620,048; Chicago, Ill., 2,701,705; Philadelphia, Pa., 1,823,779; Detroit, Mich., 993,678; Cleveland, O., 796,841; St. Louis, Mo., 772,897; Boston, Mass., 748,060; Baltimore, Md., 733,826; Pittsburgh, Pa., 588,343; Los Angeles, Cal., 576,673; Buffalo, N. Y., 506,775; San Francisco, Cal., 506,676; Milwaukee, Wis., 457,147; Washington, D. C., 437,571; Newark, N. J., 414,524; Cincinnati, O., 401,247; New Orleans, La., 387,219; Minneapolis, Minn., 380,582; Kansas City, Mo., 324,410; Seattle, Wash., 315,312.

86. Problems like the following will be interesting and helpful for rapid addition drills:

```
  32347
  46943
  -----
```

The teacher dictates two numbers of five figures each, the student writing them as shown in the margin.

```
  32347
  46943
  79290√
 126233
 205523
 331756√
 537279
 869035
1406314√
2275349
-------
5910069
```

The remaining numbers in the column are found by the student in this manner:

The third number is the sum of the first two numbers; the fourth number is the sum of the second and third numbers; the fifth number is the sum of the third and fourth numbers; etc. When there are ten numbers the whole column is added. As the column is written, every third number is checked, as here shown, so as to tell quickly when the tenth number is written. With a little practice, students should find the final total in one minute and thirty seconds from the time the second number is dictated.

Extend columns to ten numbers and add:

1. 47568
   56473

7,5[illegible],964

2. 65748
   65342

9,3[illegible],226

3. 82736
   54637

4. 29384
   75867

5. 73645
   92834

6. 29384
   94857

7. 74632
   96879

8. 85769
   87965

9. 84754
   85764

10. 35798
    98765

11. 12345
    67892

12. 98647
    86793

✓

13. 57463
65374

14. 21364
56784

15. 39485 ✓
23876

16. 39264
23876

17. 36482 ✓
84756

18. 12935
94857

19. 39486 ✓
21345

20. 39485
31495

21. 94857 ✓
73645

22. 73645
12385

23. 92836 ✓
84756

24. 82734
93754

25. 12938 ✓
91542

26. 45678
64532

27. 67891 ✓
45768

28. 38475
93847

29. 37465 ✓
43265

30. 48576
19867

31. 95843 ✓
41567

32. 38475
85769

33. 64573 ✓
75643

34. 43789
59764

35. 97856 ✓
89543

36. 34768
56789

37. 74635 ✓
95864

38. 53426
63545

38. 47586 ✓
75648

40. 29731
63549

41. 75312 ✓
21357

42. 15486
35943

43. 52565 ✓
43464

44. 98991
75728

HORIZONTAL ADDITION

87. Addition from left to right is important on invoices, time sheets, sales sheets, statistical reports, etc.

88. Add from left to right. Check by adding from right to left.

X 1. 4, 8, 7, 6, 3, 2, 7, 9.
2. 7, 9, 6, 4, 5, 8, 9, 7.
3. 6, 4, 7, 9, 8, 7, 9, 5, 6.
4. 15, 18, 29, 73, 96, 47, 85, 93, 88.
5. 57, 93, 84, 96, 82, 78, 49, 56, 97.
6. 72, 89, 37, 43, 87, 92, 46, 34, 73.
7. 67, 83, 96, 38, 72, 47, 93, 89, 92. X

9,355,5

Add the following by columns and from left to right. Check the result by adding the horizontal and vertical totals.

8.	9.	10.
54+78+72=	38+28+54=	74+37+45=
36+29+83=	27+36+38=	56+78+58=
85+34+96=	54+79+29=	89+56+79=
92+56+87=	93+87+98=	79+33+46=
+ + =	+ + =	+ + =

11.	12.	13.
382+592+848=	568+827+584=	382+916+538=
961+796+796=	729+968+763=	976+693+872=
729+816+382=	384+817+972=	438+821+279=
384+723+794=	754+927+475=	728+572+186=
+ + =	+ + =	+ + =

14. Find the total sales in all departments for each month, and the total sales in each department for the year. Check the work by adding the two sets of totals.

SUMMARY OF MONTHLY SALES

	CLOTHING	DRESS GOODS	GLOVES	HATS	SHOES	TOTALS
Jan. . .	~~$16,427.85~~	$12,427.83	~~$3,472.50~~	~~$5,472.60~~	$7,592.50	
Feb. . .	~~15,138.20~~	11,428.50	2,896.75	4,854.30	6,438.90	
Mar. . .	18,278.26	12,736.25	3,124.60	~~4,472.60~~	~~5,372.50~~	
Apr. . .	~~17,493.87~~	~~10,964.78~~	~~2,148.75~~	~~4,134.50~~	~~5,796.25~~	
May . .	~~15,382.97~~	9,974.63	2,004.50	~~3,834.75~~	~~5,176.85~~	
June . .	~~14,582.76~~	~~9,864.59~~	~~1,995.70~~	~~3,564.75~~	~~6,487.50~~	
July . .	~~13,996.80~~	~~9,645.72~~	~~1,875.50~~	~~3,472.25~~	~~6,193.85~~	
Aug. . .	~~14,592.73~~	~~8,796.21~~	1,850.60	3,678.50	~~5,040.45~~	
Sept. . .	~~15,827.64~~	9,793.13	1,924.30	4,075.65	6,384.90	
Oct. . .	~~18,973.90~~	~~12,897.08~~	~~2,394.60~~	~~4,763.83~~	~~6,876.75~~	
Nov. . .	22,738.75	14,594.77	3,548.75	5,276.90	7,564.55	
Dec. . .	28,693.72	18,798.99	3,875.50	5,798.75	8,097.55	
Totals .						

Give the horizontal, vertical and grand totals of the following:

SALES	UNIFORMS	LEGGINGS	CAPS	TOTALS
15. Monday . . .	$1249.96	$28.75	$368.42	
Tuesday . . .	1585.98	392.52	793.65	
Wednesday . .	1896.47	87.50	715.26	
Thursday . .	807.96	92.75	156.91	
Friday	1193.53	136.50	653.65	
Saturday . .	3168.49	148.25	876.52	
Totals . . .	9902,39	886,27	3564,41	

SALES	DRESS GOODS	SHOES	NOTIONS	TOTALS
16. Monday . . .	$87.68	$18.75	$16.83	
Tuesday . . .	125.76	26.82	9.76	
Wednesday .	235.67	76.85	23.82	
Thursday . .	89.62	89.75	48.98	
Friday	123.98	110.60	75.67	
Saturday . .	982.76	234.50	163.97	
Totals . . .	1645,47	557,27	339,03	

MONTHS	BOOKS	SHOES	MILLINERY	DRY GOODS	TOTALS
17. July . . .	$4653.90	$6543.87	$5346.75	$8235.53	
August . .	3253.76	4324.80	7636.65	9845.75	
September .	4324.45	7342.98	4898.90	7342.89	
October . .	8780.19	8756.65	3756.98	6547.87	
November .	3738.98	3655.45	8765.89	8453.00	
December .	7544.45	7298.90	9755.35	9879.00	
Totals .	32295,73	37922,65	40160,52	50304,04	

POSTAL REVENUE AT ONE POST OFFICE

	FIRST CLASS	SECOND CLASS	THIRD CLASS	PARCEL POST	SPECIAL DELIVERY	REGISTRY	INSURANCE	TOTALS
18. M. . .	$106.22	$23.15	$13.24	$43.41	$3.60	$4.50	$6.80	
T. . .	145.61	33.43	21.13	57.57	4.50	5.30	7.75	
W. . .	131.99	19.22	14.56	49.92	3.10	1.30	4.80	
Th. . .	201.13	31.52	22.10	28.88	4.70	3.90	1.50	
F. . .	187.67	20.78	31.57	65.72	7.70	7.50	2.50	
S. . .	149.78	31.56	43.44	38.48	4.80	6.50	3.45	
Totals	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	

18. In the following Abstract Sales Book find (1) the total daily sales by adding the sales on account and the sales for cash; (2) the total weekly sales by adding the total daily sales for each week; and (3) the total monthly sales by adding the several weekly totals.

ABSTRACT SALES BOOK

19.

Date		From Sales Sheet No.	Sales On Account		Sales for Cash		Total Daily		Total Weekly		Total Monthly	
Aug.	1	122	132	16	87	85						
	2	123	278	32	115	64						
	3	124	225	83	79	69						
	4	125	196	93	107	86						
	5	126	209	87	45	90						
	6	127	367	64	128	72						
	8	128	119	81	58	27						
	9	129	109	54	69	27						
	10	130	99	82	72	33						
	11	131	116	93	81	57						
	12	132	122	11	76	22						
	13	133	295	60	117	09						
	15	134	98	90	63	76						
	16	135	113	13	71	21						
	17	136	119	17	69	80						
	18	137	97	55	92	96						
	19	138	164	73	97	85						
	20	139	315	76	118	24						
	22	140	98	76	72	13						
	23	141	101	27	78	17						
	24	142	111	12	91	16						
	25	143	127	31	105	16						
	26	144	172	85	116	23						
	27	145	417	81	193	82						
	29	146	107	88	69	28						
	30	147	163	91	121	11						
	31	148	136	54	99	83						

Addition drill chart.

89. This chart is intended to help the pupil learn to add numbers by groups of two and three figures each. The light lines are to divide the columns into such groups of figures. For class drill these or similar columns should be written on large sheets of paper so that they may be read across the room. As the teacher points to any group the pupils will state the sum at sight. Then an individual column should be added two figures at a time. Drill in horizontal addition may be had here also.

The teacher will easily discover many ways of using this chart. It should be used frequently.

82	93	84	76	583	928	93	84	72	93	847	563	18	39	48	57	392	837
92	83	74	82	938	472	92	83	74	83	928	361	29	38	47	38	293	847
19	38	29	20	384	738	29	38	47	28	394	872	29	38	47	28	394	872
12	93	48	29	283	746	28	39	48	27	364	827	29	38	47	26	272	534
12	83	94	82	728	374	27	38	49	28	374	625	12	83	94	82	736	482
29	38	47	26	263	983	29	38	47	28	293	847	21	29	38	47	283	949
29	38	47	26	341	284	29	38	47	48	392	834	83	74	29	38	472	634
19	38	47	58	493	847	39	48	57	38	394	857	83	74	63	84	958	374
93	84	75	84	736	485	39	48	57	38	394	857	13	84	95	93	847	562
92	83	74	73	829	341	84	93	84	75	638	495	38	39	48	57	362	635
29	38	47	26	938	472	29	28	37	46	583	928	28	37	49	28	374	628
92	83	74	63	839	485	38	47	56	48	398	468	93	84	75	13	849	599
93	84	75	63	829	384	39	83	72	63	848	374	29	38	47	46	382	937
29	38	47	46	583	928	12	34	31	29	384	746	29	38	47	46	273	846
38	39	48	57	465	849	93	84	75	64	849	384	48	57	94	85	748	598
93	84	74	65	849	948	93	84	75	64	839	485	83	94	87	56	384	791
93	84	75	63	829	383	83	94	84	75	638	291	93	84	75	62	837	469
93	84	85	73	648	476	83	74	93	84	726	384	94	83	72	63	948	728
92	83	74	65	839	284	38	47	29	38	472	834	93	84	75	69	384	756
12	83	74	65	537	283	93	84	75	64	839	485	28	37	49	28	374	637

SUBTRACTION

90. Subtraction is the process of finding a number or quantity equal to the difference between two other numbers or quantities.

91. In expressions involving a parenthesis ( ), as $16-(5+7)$, the operations indicated within the parenthesis must be performed before the operation indicated by the sign before the parenthesis. Thus, in the example given, 5 and 7 must be added first, and their sum, 12, taken from 16.

The vinculum —— has the same effect as the parenthesis. Thus, $16-\overline{5+7}=4$.

ORAL DRILL EXERCISE

92. 1. Count backwards by 2's beginning with 100; 99.

2. Count backwards by 3's beginning with 100; 99; 98.

3. Count backwards by 4's beginning with 100; 99; 98; 97.

4. Count backwards by 5's beginning with 100; 99; 98; 97; 96.

5. Count backwards by 6's beginning with 100; 99; 98; 97; 96; 95.

6. Count backwards by 7's beginning with 100; 99; 98; 97; 96; 95; 94.

7. Count backwards by 8's beginning with 100; 99; 98; 97; 96; 95; 94; 93.

8. Count backwards by 9's beginning with 100; 99; 98; 97; 96; 95; 94; 93; 92.

9. Count backwards by 10's beginning with 100; 99; 98; 97; 96; 95; 94; 93; 92; 91.

10. Count backwards by 11's beginning with 100; 99; 98; 97; 96; 95; 94; 93; 92; 91; 90.

11. Count backwards by 12's beginning with 100; 99; 98; 97; 96; 95; 94; 93; 92; 91; 90; 89.

93. Subtract quickly:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
27	23	24	27	22	23	25	27	28	26
6	8	12	9	4	7	9	8	9	7
								19	[illegible]

11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
32	35	38	32	39	37	36	34	33	31
19	17	19	17	18	18	19	17	16	15
13		[illegible]	15	21	19	17	17		

21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
54	59	58	54	56	52	51	57	53	55
36	32	39	37	38	39	34	38	37	38
	17	19	17	18	13	17	19	16	17

RAPID DRILL EXERCISE

94. Subtract:

1.	2.	3.	4.	5.	6.	7.
890467	740231	947204	943876	740023	630704	845700
387589	674678	678327	427628	287204	470568	687354

8.	9.	10.	11.	12.	13.	14.
472907	907240	548724	324876	597860	938701	300470
293879	689382	279876	128987	407976	764902	298784

15.	16.	17.	18.	19.	20.	21.
487246	697287	320765	596872	729684	864200	975310
398472	472349	162782	372849	537315	795301	682405

22.	23.	24.	25.	26.	27.	28.
108724	398764	587639	438076	987654	438762	487296
106897	297876	395301	278649	321076	287964	245498

29.	30.	31.	32.	33.	34.	35.
568397	497872	495287	863829	397286	327693	978531
298729	387692	278469	438297	276397	278798	299729

CHECKING THE WORK

95. The best method of checking the work in subtraction is to add the remainder to the subtrahend. If the sum is equal to the minuend, the work is correct.

From 1476 take 938.

1476
938
538 CHECK. 938+538=1476.

Check the work on the preceding page.

THE "MAKING CHANGE" OR AUSTRIAN METHOD

96. This method of subtraction consists in adding to the amount of the purchase enough change to make the sum equal to the amount paid.

1. A man buys groceries to the value of $1.38, and gives a two-dollar bill in payment. How much change should he receive?

The cashier in making change will return to the customer 2 cents, a ten-cent piece, and a half dollar (saying, "$1.38, 40, 50, $2.00," which means $1.38+$.02=$1.40; $1.40+$.10=$1.50; and $1.50+$.50=$2.00).

Of course, other coins than those mentioned might be returned by the cashier, but it is customary to make change in the largest coins possible.

NOTE. This method is used by many in ordinary subtraction problems.

2. From 1231 take 573.

1231
573 3+8=11; write 8.
7+1 (carried)+5=13; write 5.
658 5+1 (carried)+6=12; write 6.

97. What change should be given for a two-dollar bill if the following purchases were made? for a five-dollar bill?

Name the coins or bills given as change. Use the largest denominations possible.

1. $1.56
2. 1.27
3. .87
4. .58
5. 1.33
6. 1.64
7. 1.29
8. $1.17
9. .96
10. .88
11. 1.49
12. 1.68
13. 1.21
14. 1.11
15. $ .72
16. .49
17. .27
18. 1.67
19. 1.76
20. 1.87
21. 1.19
22. $1.82
23. 1.54
24. 1.43
25. .40
26. 1.32
27. 1.41
28. .99

This method of subtraction has an important application in all problems in which the sum of several numbers is to be taken from another number.

29. A man deposits $1625 in a bank and draws checks against it for $122, $245, and $347. Find his balance.

$1625
122
245
347
911

7+5+2=14; 14+1=15. The units' figure of 14 is 1 less than the units' figure of the minuend; hence 1 is added to the 14 to make its units' figure equal to the units' figure of the minuend. The 1 is written as the first or units' figure of the difference. Carry 1 (the tens' figure in 15) and add the tens' column. 1+4+4+2=11; 11+1=12. As before, write 1 in the remainder, and carry 1. Add the hundreds' column. 1+3+2+1=7; 7+9=16. Write 9 in the remainder. The bank balance, then, is $911.

Find the balance of the following bank accounts:

	Total Deposits		Checks Drawn
30.	$ 785.00	. . . .	$ 76.00, $45.00.
31.	$ 427.50	. . . .	$ 37.50, $25.00. $100.00.
32.	$1287.70	. . . .	$128.50, $97.60, $ 52.70.
33.	$2576.28	. . . .	$528.75, $ 7.50, $923.65.
34.	$3789.10	. . . .	$227.50, $ 1.75, $ 85.00, $931.80

35. Find the balance of the following account:

Withdrawal — J. McLOUGHLIN — Investment Gain

1924 Jan.	10		3	75	1924 Jan.	3		1	4250	
						31	Loss and Gain	2	365	50

36. Find the loss or gain on merchandise:

Cost — MERCHANDISE — Returns

1924 Jan.	4		1	1764	50	1924 Jan.	5		2	950	75
							20		8	574	50
							31	*Inventory*		*874*	*75*

37. Find the balance of cash:

RECEIVED						CASH					PAID OUT
1924						1924					
Jan.	3		1	4250		Jan.	3		1	250	
	5		2	950	75		4		1	1764	50
	20		8	574	50		10		3	75	
							12		4	500	

38. Find the loss:

COST						EXPENSE					RETURNS
1924						1924					
Jan.	3		1	250		*Jan.*	*31*	*Inventory*		*480*	
	12		4	500							

THE COMPLEMENT METHOD

98. The **complement** of a number is the difference between that number and the unit of a next higher order.

3 is the complement of 7, because 3 is the difference between 7 and 10; 31 is the complement of 69, because 31 is the difference between 69 and 100; etc.

99. The **complement method** of **subtraction** is based on the principle illustrated in the following examples:

1. From 23 take 8.

$$23-8=(23+2)-(8+2)=25-10=15.$$

The complement of 8 is 2.

If 2 is added to the minuend and is then deducted by adding it to the subtrahend also, it is evident that the value of the remainder will be unchanged. Hence, in the above example we think 23, 25, 15.

2. From 173 take 96.

$$173-96=(173+4)-(96+4)=177-100=77.$$

The complement of 96 is 4.

As stated in the preceding example, if 4 is added to both minuend and subtrahend, the remainder is unchanged. Think 173, 177, 77.

This method of subtraction is specially convenient when one number is to be deducted from the sum of several other numbers.

From the sum of 827, 136, and 472, take 658.

827
136
472
658
777

Arrange the numbers as shown. The complement of 658 is 342. (The complement should be carried in the mind, not written on the paper.) Add the complement to the several numbers of the minuend, and deduct 100, (that is, 1 unit of the next higher order than the subtrahend). The result is 777.

100. By the method just explained solve the following:

1. 48+27+63−85
2. 39+48+97−64
3. 149+164+387−456
4. 827+639+564−387
5. 964+763+816−978
6. 839+572+634−857
7. 768+173+824−643
8. 246+369+741−927
9. 329+417+726−843
10. 274+976+834−748
11. 1238+1376+2473−3764
12. 3872+4391+7684−8727

13. The following statement shows the earnings and the expenses of a certain railroad company for two months Find the total earnings and expenses; the increase or decrease of each class of earnings and expenses for November as compared with October; and the net increase or decrease.

	November	October	Increase	Decrease
Earnings				
Coal traffic	$850,456.25	$550,321.30		
Merchandise	625,382.40	582,498.22		
Passengers	227,421.36	231,375.60		
Express	16,172.44	17,275.85		
Mails	11,328.38	10,531.28		
Miscellaneous	24,275.60	25,386.24		
Total earnings				
Expenses				
Maintenance of road	$150,381.35	$146,327.88		
Maintenance of equipment	192,571.90	207,364.76		
Transportation	602,892.70	570,192.60		
General	60,428.75	63,175.80		
Total expenses				
Net earnings				

14. In the following abstract purchase book find, (1) the amount of cash paid for each purchase, (2) the total amount of all invoices, the total amount of discount, and the total amount paid for merchandise for the month, (3) check the work by subtracting the total discount from the total amount of invoices to see if it agrees with the amount paid.

ABSTRACT PURCHASE BOOK

Date 19—		L.F.	Name	Address	Terms	When Due		When Paid		Amt of Inv		Discount		Amt. Paid	
Aug	1		J. J. Noonan	Chicago	5/10	Aug.	11	Aug.	11	525	60	26	28		
	2		B. B. Drew	N. Y.	2% Cash	Aug.	2	Aug.	2	475		9	50		
	3	✓	Geo. Ames & Co.	Buffalo	Cash	Aug.	3		3	125					
	5		Leo Drake	City	3% Cash		5		6	284	60	8	54		
	6	✓	R. Carter	Bklyn.	Cash		6		8	175					
	8		S. B. Cooper	Denver	5/30	Sept.	7	Sept.	7	891	07	44	55		
	10		W. Wilson	St. Louis	3/10	Aug.	20	Aug.	21	418	24	12	55		
	12		E. B. Young	Ottawa	5/30	Sept.	11	Sept.	12	932	75	46	64		
	14	✓	A. H. Ross	Hartford	Cash	Aug.	14	Aug.	14	231	50				
	16		C. G. Allen	Albany	n/30	Sept.	15	Sept.	15	185	60				
	19		F. Carson	Atlanta	3/30		18			564	70	16	94		
	23		H. H. Pierce	Kan. City	5/30		22			897	84	44	89		
	28		P. C. Andrews	Chicago	2/10		7			617	50	12	35		
	31	✓	R. O. Williams	N. Y.	Cash	Aug.	31	Aug.	31	236	85				
			Mdse. Purchases for Aug.												

MULTIPLICATION

101. Multiplication is the process of taking one number as many times as there are units in another.

102. The **multiplier** and the **multiplicand** are called **factors** of the **product**.

103. To multiply by 10 or a multiple of 10.

1. Multiply 4586 by 10, by 100, by 1000.

$$4586\times 10=45860$$
$$4586\times 100=458600$$
$$4586\times 1000=4586000$$

Hence, to multiply by 10, 100, 1000, etc., annex as many zeros to the multiplicant as there are zeros in the multiplier.

2. Multiply 345 by 30; by 400; by 5000.

$$345\times 30=10350$$
$$345\times 400=138000$$
$$345\times 5000=1725000$$

Hence, to multiply by 30, 400, 5000, etc., multiply the multiplicand by the 3, 4, or 5, etc., and annex as many zeros to the product as there are zeros in the multiplier.

3. Multiply 4300 by 6000.

$$43\times 6=258;\text{ hence }25800000.$$

Omit all zeros on the right of the factors, and multiply the remaining numbers. To the result annex as many zeros as were omitted from both factors.

104. Make the following multiplications mentally where possible:

1. 472 by 10
2. 563 by 100
3. 596 by 1000
4. 738 by 10
5. 920 by 100
6. 6472 by 1000

7. 5345 by 10
8. 824 by 300
9. 9300 by 500
10. 7250 by 6000
11. 82,400 by 5000
12. 93,875 by 100
13. 92,700 by 2000
14. 4300 by 700
15. 89,600 by 4000
16. 97,620 by 100
16. 83,472 by 10
18. 94,800 by 800
19. 7000 by 5000
20. 8900 by 900
21. 9800 by 3000

105. To multiply by 25, 50, 75, etc.

1. Multiply 413 by 25; by 50; by 75.

4)41300
10325

25 is $\frac{1}{4}$ of 100. Multiplying by 25 gives a product $\frac{1}{4}$ as great as the product obtained by multiplying by 100.

Hence, to multiply by 25, annex two zeros to the multiplicand (that is, multiply by 100) and divide by 4.

2)41300
20650

50 is $\frac{1}{2}$ of 100.

Hence, to multiply by 50, annex two zeros to the multiplicand and divide by 2.

4)41300
10325
3
30975

75 is $\frac{3}{4}$ of 100.

Hence, to multiply by 75, annex two zeros to the multiplicand, divide by 4, and multiply the result by 3.

2. Multiply 475 by 125; by 250.

8)475000
59375

125 is $\frac{1}{8}$ of 1000. To multiply by 1000, annex three zeros. The product is now 8 times as large as it should be. Therefore, divide by 8 to obtain the correct result.

4)475000
118750

In like manner, 250 is $\frac{1}{4}$ of 1000. To multiply by 250, annex three zeros to the multiplicand and divide by 4.

106. Find the products mentally where possible:

400

1. 24×25
2. 64×25
3. 88×25
4. 94×50
5. 53×50
6. 87×50
7. 99×75
8. 103×25
9. 137×50
10. 246×75
11. 824×125
12. 938×250
13. 728×500
14. 1498×125
15. 2387×25
16. 7296×250
17. 8974×750
18. 4976×75
19. 8397×250
20. 17986×125

107. To find the product of two factors when each ends in 5.

1. Multiply 45 by 45. 45×45=2025.

To find the square of numbers ending in 5, write 25 for the right-hand two figures of the product. Next add 1 to the tens' figure (4 + 1 = 5 in this example), multiply by the other tens' figure (4 × 5 = 20), and write the product next to the 25 already written. The result is 2025.

15×15

25

95 × 115

1125

2. Multiply 35 by 55. 35×55=1925.

If the sum of the tens' figures is even, write 25 as the first part of the product. The sum of the tens' figures (5 + 3 = 8) is even. Take ½ of the sum of the tens' figures (½ of 8 = 4) and add it to the product of the tens' figures multiplied together. 5 × 3 = 15; 15 + 4 = 19. Write 19 in the product, making 1925.

3. Multiply 45 by 115. 45×115=5175.

The sum of the tens here (4 + 11 = 15) is odd; hence write 75 as the first part of the product. Take ½ the sum of the tens' figures (½ of 15 = 7½), disregard the fraction, and add the half sum to the product of the tens' figures multiplied together. (4 × 11) + 7 = 51. Write 51 in the product, making 5175.

108. Practice on the following until you can read the products at sight:

1. 15×15
2. 25×25
3. 35×35
4. 45×45
5. 55×55
6. 65×65
7. 75×75
8. 85×85
9. 95×95
10. 85×65
11. 85×55
12. 45×65
13. 55×75
14. 85×35
15. 95×85
16. 65×55
17. 45×35
18. 75×85
19. 95×15
20. 85×45
21. 65×75
22. 125× 45
23. 125×125
24. 105× 35
25. 95×105
26. 105×105
27. 115×115
28. 115× 55

109. To square any number of two figures.

1. Multiply 53 by 53. 53×53=2809.

53 is 3 more than 50; 53 plus 3 is 56. Hence, 53×53=(50×56) +(3×3)=2809.

$[(53)^2=(50+3)^2=(50\times50)+(2\times3\times50)+(3\times3)=(50\times50)+(6\times50)+(3\times3)=(50\times56)+9=2809.]$

2. Multiply 69 by 69. 69×69=(70×68)+1 =4761

69 is 1 *less* than 70. 69 *minus* 1=68. Hence, 69×69=(70×68) +(1×1)=4761.

$[(69)^2=(70-1)^2=(70\times70)-(2\times1\times70)+(1\times1)=(70\times70)-(2\times70)+(1\times1)=(68\times70)+1=4761.]$

110. Write the squares of all the numbers from 40 to 60.

111. To find the product of any two numbers of two figures each when the units or the tens are alike.

1. Multiply 42 by 72. $42 \times 72 = 3024$.

$2 \times 2 = 4$; write 4. Take the sum of the tens' figures, and multiply by one of the units' figures. $(7 + 4) \times 2 = 22$; write 2, and carry 2. Take the product of the tens, and add 2. $7 \times 4 + 2 = 30$; write 30, making 3024.

2. Multiply 37 by 34. $37 \times 34 = 1258$.

$4 \times 7 = 28$; write 8, carry 2. Take the sum of the units' figures, and multiply by one of the tens' figures. $(7 + 4) \times 3 = 33$; $33 + 2 = 35$; write 5, carry 3. Take the product of the tens' figures, and carry the 3; $3 \times 3 + 3 = 12$. Therefore, the product is 1258.

NOTE. The student should take careful notice of the fact that when the tens are alike, the units' figures are added, and *vice versa*.

112. Multiply:

1. 43 by 33
2. 24 by 44
3. 52 by 62
4. 67 by 27
5. 83 by 53
6. 69 by 29
7. 74 by 84
8. 56 by 86
9. 37 by 32
10. 48 by 45
11. 58 by 54
12. 88 by 87
13. 93 by 98
14. 69 by 63
15. 67 by 69
16. 78 by 76
17. 39 by 49
18. 58 by 57
19. 67 by 63
20. 72 by 92
21. 86 by 46
22. 55 by 57
23. 39 by 38
24. 46 by 36

113. To multiply by 11 or by a multiple of 11.

1. Multiply 287 by 11.

$287 \times 11 = 3157$.

$$\begin{array}{r} 287 \\ 11 \\ \hline 287 \\ 287 \\ \hline 3157 \end{array}$$

By referring to the solution in the margin, it is seen that the first figure in the product is 7, the units' figure of the multiplicand; the second figure of the product is the sum of the units and the tens; the third is the sum of the tens and the hundreds, including the carrying figure, and the fourth is the hundreds' figure of the multiplicand plus the carrying figure.

The principle involved in the preceding example applies to problems in which it is desired to multiply by a multiple of 11.

2. Multiply 329 by 44. 329×44=14476.

44 is 4 times 11. Make the multiplication mentally by 11, and multiply the result by 4 at the same time. In multiplying by 11 the first figure of the product would be 9, which multiplied by 4 gives 36. Write 6, and carry 3.

Continue the multiplication by 11: 9 + 2 = 11; 4 × 11 = 44, which with 3 to carry, makes 47; write 7, carry 4. 2 + 3 = 5; 4 × 5 = 20, which, with 4 to carry, makes 24; write 4, carry 2. 4 × 3 + 2 = 14; write 14, making 14476.

114. Multiply, mentally, each of the following numbers by 11:

1. 54
2. 35
3. 43
4. 78
5. 96
6. 58
7. 39
8. 76
9. 244
10. 372
11. 628
12. 579
13. 396
14. 927
15. 489
16. 767
17. 4872
18. 3976
19. 8498
20. 2976
21. 4507
22. 5240
23. 3007
24. 9080

115. Multiply each of the above numbers by 22; 33; 55; 77; 88.

116. To multiply by 21, 31, etc., by 101, 201, etc., and by 129, 561, etc.

1. Multiply 287 by 41.

$$\begin{array}{r} 287 \\ 1148 \\ \hline 11767 \end{array} \qquad 41$$

In multiplying by a number having a figure 1 in the unit's place, write the multiplier to one side of the multiplicand instead of under it. This plan saves re-writing the figures of the multiplicand as part of the product because to multiply by 1 is merely to repeat the figures which have already been written. Multiply by the ten's figure, 4, and write the result as indicated.

2. Multiply 458 by 601.

$$\begin{array}{r} 458 \\ 2748 \\ \hline 275258 \end{array} \qquad 601$$

As in the preceding exercise, write the multiplier to one side rather than under the multiplicand. Multiply by the hundred's figure, 6, and write the result as shown.

3. Multiply 537 by 157.

$$\begin{array}{l} 537 \\ 2685 \\ 3759 \\ \hline 84309 \end{array} \qquad 157$$

Write the multiplier to one side as before. Multiply first by 5, the ten's figure, writing the result one place to right as indicated. Next multiply by the unit's figure, 7, and write the result one place further to the right, as illustrated.

Multiply, using the plan illustrated on the preceding page:

1. 234 by 21
2. 456 by 41
3. 728 by 81
4. 342 by 71
5. 376 by 61
6. 748 by 31
7. 839 by 91
8. 747 by 51
9. 856 by 61
10. 645 by 81
11. 746 by 701
12. 486 by 501
13. 839 by 401
14. 395 by 801
15. 692 by 901
16. 748 by 301
17. 689 by 601
18. 937 by 701
19. 437 by 6001
20. 748 by 8001
21. 834 by 341
22. 857 by 451
23. 968 by 561
24. 576 by 431
25. 832 by 741
26. 935 by 371
27. 347 by 521
28. 747 by 671
29. 873 by 771
30. 748 by 671
31. 746 by 146
32. 735 by 178
33. 647 by 149
34. 482 by 163
35. 965 by 158
36. 869 by 143
37. 456 by 145
38. 846 by 193
39. 695 by 148
40. 853 by 175

117. To multiply by numbers from 11 to 19; 91 to 99; 101 to 109, etc

Multiply 583 by 13; by 17.

$$\begin{array}{rl} 5830^{*} & 13 \\ 1749 & \\ \hline 7579 & \end{array} \qquad \begin{array}{rl} 583^{*} & 17 \\ 4081 & \\ \hline 9911 & \end{array}$$

Write the multiplier to the right. Multiply by the unit's figure and place the result as shown.

Multiply 719 by 93; by 99

$$\begin{array}{lrl} (100\times719) = & 71900^{*} & 93 \\ (7\times719) \quad = & 5033 & \\ \hline & 66867 & \end{array} \qquad \begin{array}{lrl} (100\times719) = & 719^{*} & 99 \\ (1\times719) \quad = & 719 & \\ \hline & 71181 & \end{array}$$

First, multiply by 100. (Annex two zeros). Then deduct 7×719, because 93 is 7 less than 100.

In the second case deduct 1×719, etc.

Multiply 847 by 103; by 107.

$$\begin{array}{rl} 84700^{*} & 103 \\ 2541 & \\ \hline 87241 & \end{array} \qquad \begin{array}{rl} 847^{*} & 107 \\ 5929 & \\ \hline 90629 & \end{array}$$

Write the multiplier to one side and multiply only by the unit's figure, placing the result as illustrated.

---

*The writing in of the zeros in these places is optional. The result is the same if the other figures are written in the correct position.

Multiply, using the plan illustrated on the preceding page:

1. 543 by 12
2. 467 by 17
3. 230 by 14
4. 876 by 17
5. 325 by 13
6. 634 by 16
7. 289 by 17
8. 980 by 15
9. 847 by 16
10. 455 by 13
11. 867 by 94
12. 389 by 98
13. 430 by 93
14. 245 by 95
15. 442 by 91
16. 331 by 92
17. 346 by 99
18. 239 by 98
19. 456 by 91
20. 782 by 93
21. 457 by 105
22. 987 by 102
23. 745 by 106
24. 923 by 101
25. 654 by 103
26. 432 by 102
27. 231 by 104
28. 346 by 107
29. 289 by 109
30. 319 by 101

118. To multiply by a number composed of factors.

1. Multiply 369 by 497.

```
   369
   497
  ----
  2583
18081
------
183393
```

First multiply by 7 in the usual way. The other part of the multiplier, 49, is 7 times 7; hence instead of multiplying 369 by 49, multiply 2583 by 7, placing the result as shown in the illustration, since the multiplier is 49 *tens*. Then add the partial products.

2. Multiply 657 by 321.

```
   657
   321
  ----
1971
  13797
------
210897
```

First multiply by 3, placing the result as shown. 21 is 7 times 3. Multiply the result already found by 7, writing the product two places to the right of the first partial product, since the 3 represents *hundreds* and the 21, *units*.

119. To multiply any two numbers of two figures.

Multiply 46 by 92. 46×92 = 4232.

First, 2 × 6 = 12; write 2, carry 1. Next, 2 × 4 + 1 (carried) = 9; also 9 × 6 = 54. Add 54 and 9, giving 63; write 3, carry 6. Next, 9 × 4 + 6 (carried) = 42, which makes a completed product of 4232.

CHECKING THE WORK

120. The following checks will be helpful in proving multiplication.

121. 1. Divide the product by one of the factors: the quotient will be the other factor. If there are more than two factors, the product divided by one of them will give a quotient equal to the product of all the other factors.

(*a*) $18 \times 25 = 450$. PROOF. $450 \div 18 = 25$.
(*b*) $4 \times 8 \times 9 \times 5 = 1440$. PROOF. $1440 \div 4 = 360 = 8 \times 9 \times 5$.

122. Casting out the nines.

$$\begin{array}{rl} 41 = 5 & \\ \underline{22} = \underline{4} & \\ 82 & \underline{}\;\; 20 = 2 \\ \underline{82} & \\ 902 = 2 & \end{array}$$

The excess of nines in the multiplicand is 5, and in the multiplier is 4. The product of the excesses 4 and 5 is 20, and the excess of nines in 20 is 2. The excess of nines in the product, 902, is 2. Hence the multiplication is probably correct.

The excess of nines in the product is equal to the excess of nines in the product of the excesses.

123. Casting out the elevens.

$$\begin{array}{rl} 248 = 6 & \\ \underline{37} = \underline{4} & \\ 1736 & 24 = 2 \\ \underline{744} & \\ 9176 = 2 & \end{array}$$

The excess of elevens in 248 is 6, $(8 - 4 + 2)$; the excess of elevens in 37 is 4, $(7 - 3)$. The product of the excesses is 4×6, or 24, and the excess of elevens in 24 is 2. The excess of elevens in 9176 is 2, $(6 + 1 + 11 - 7 - 9)$. Since the results agree, the multiplication is probably correct.

124. Multiply, and check the results. Use the check assigned by the teacher.

1. 43 by 29
2. 72 by 37
3. 48 by 27
4. 37 by 84
5. 43 by 56
6. 91 by 28
7. 96 by 47
8. 56 by 46
9. 83 by 34
10. 875 by 756
11. 1249 by 648
12. 3147 by 639
13. 2489 by 763
14. 3271 by 654
15. 5298 by 324
16. 6476 by 549
17. 8749 by 624
18. 9483 by 424
19. 64 by 36
20. 93 by 67
21. 94 by 69
22. 81 by 74
23. 82 by 29
24. 34 by 85
25. 35 by 63
26. 73 by 57
27. 79 by 67

DIVISION

125. Division is the process of finding how many times one number is contained in another.

SHORT METHODS

126. To divide by 10, 100, 1000, etc.

Divide 4370 by 10.

$$4370 \div 10 = 437$$

$$10\overline{)4370}\quad 437.0$$

The solution at the left shows that to divide by 10 it is necessary only to move the decimal point one place to the left.

In like manner, to divide by 100, 1000, etc., move the point 2 places, 3 places, etc., to the left.

127. Divide each of the following numbers by 10, 100, 1000, 10,000

1. 130000
2. 1250000
3. 40000
4. 750000
5. 1160000
6. 24750000
7. 400000
8. 1800000
9. 2000000
10. 14500000
11. 3489
12. 7248
13. 5673
14. 8470
15. 19872

128. To divide by 25, 50, 75, etc.

In many cases the work of division can be lessened by making the operation one of multiplication.

Divide 2800 by 25. $2800 \div 25 = 28 \times 4 = 112$.

25 is $\frac{1}{4}$ of 100. Divide 2800 by 100 by dropping the two zeros. But, in dividing by 100, you have divided by a number 4 times too large, consequently the quotient is only $\frac{1}{4}$ as large as it should be (Principle 17, page 117). To correct the error, multiply the 28 by 4.

NOTE. Further application of the principle illustrated in the last example is made in the chapter on Aliquot Parts, page 29.

129. Divide:

1. 1400 by 25
2. 1600 by 25
3. 1800 by 50
4. 2400 by 50
5. 142 by 25
6. 153000 by 250
7. 72000 by 500
8. 16000 by 500
9. 27000 by 75
10. 36000 by 250
11. 48000 by 500
12. 54000 by 750

130. To divide by a number composed of factors.

1. Divide 12726 by 63.

$9 \times 7 = 63.$

9)12726
7) 1414
202

The factors of 63 are 9 and 7. Instead of making a long division, divide by the factors, 9 and 7, using short division.

2. Divide 38971 by 64.

$8 \times 8 = 64.$

8)38971
8) 4871 3 rem.
608 7 rem.
$7 \times 8 + 3$ 59 rem.

The factors of the divisor are 8 and 8. Divide by the factors as before, reserving the remainders as shown. To find the remainder, multiply the first divisor, 8, by the second remainder, 7, and add the first remainder, 3. Thus, $7 \times 8 + 3 = 59$, remainder. Hence the result is 608, with a remainder of 59.

3. Divide 174684 by 504.

$7 \times 8 \times 9 = 504.$

7)174684
8) 24954 6 rem.
9) 3119 2 rem.
346 5 rem.

$5 \times 8 \times 7 = 280$
$2 \times 7 = 14$
$6 = 6$
300
300, remainder.

The factors of 504 are 7, 8, and 9. Divide by the factors as before. The quotient is 346. To find the remainder, multiply the third remainder by the first and second divisors, 7, and 8, ($5 \times 8 \times 7 = 280$). Next multiply the second remainder by the first divisor, 7, ($2 \times 7 = 14$). To the sum of these two products, add the first remainder, 6, giving 300 as the remainder.

4. Divide 58750 by 2400.

$4 \times 6 \times 100 = 2400.$
$4\overline{)587.50}$
$6\overline{)146}$ 3 rem.
24 2 rem.
$2 \times 4 + 3 = 11,$
1150, remainder.

The factors are 4, 6, and 100. Divide by 100 by "cutting off" the right-hand two figures of the dividend. Then divide by the factors 4 and 6. The quotient is 24. To find the remainder first treat the divisors and remainders as though the divisor, 100, and the remainder, 50, were not present. $2 \times 4 + 3 = 11$. To complete the remainder, annex the remainder, 50, to the 11, giving a remainder of 1150.

CHECKING THE WORK

131. The following methods will be found helpful in checking the work.

1. Multiply the divisor by the quotient and add the remainder (if any); the result should equal the dividend.

2. Cast out the nines.

$8879 \div 247 = 35$, with a remainder of 234.
Therefore, $8879 = 247 \times 35 + 234$
The excess of 9's in 8879 is 5.
The excess of 9's in 247 is 4.
The excess of 9's in 35 is 8.
The excess of 9's in 234 is 0.
$4 \times 8 + 0 = 32.$
The excess of 9's in 32 is 5.

Since the excess of 9's in the dividend is equal to the excess of 9's in the product of the excesses in the divisor and quotient plus the excess in the remainder, the division is probably correct.

3. Cast out the elevens.

$3504 \div 54 = 64$, with a remainder of 48.
Therefore, $3504 = 54 \times 64 + 48$
The excess of 11's in 3504 is 6.
The excess of 11's in 54 is 10.
The excess of 11's in 64 is 9.
The excess of 11's in 48 is 4.
$10 \times 9 + 4 = 94.$
The excess of 11's in 94 is 6.

Since the excesses agree, the division is probably correct.

132. Divide and check the work. Use the check assigned by the teacher.

1. 4872 by 36
2. 7349 by 42
3. 5987 by 56
4. 7879 by 72
5. 9478 by 84
6. 9764 by 96
7. 11492 by 99
8. 24763 by 108
9. 45972 by 54
10. 79489 by 88
11. 48900 by 3500
12. 147260 by 3600
13. 248750 by 4200
14. 329764 by 4800
15. 247689 by 4900
16. 324798 by 5600
17. 498749 by 5400
18. 897200 by 3600
19. 948720 by 48000
20. 672000 by 24000
21. 486000 by 18000

DIVISION BY CONTINUED SUBTRACTION

133. This method of division is often required in civil service examinations.

134. By the method of continued subtraction divide 784 by 149, and prove the work.

$$\begin{array}{rr} & 784 \\ 1. & \underline{149} \\ & 635 \\ 2. & \underline{149} \\ & 486 \\ 3. & \underline{149} \\ & 337 \\ 4. & \underline{149} \\ & 188 \\ 5. & \underline{149} \\ & 39 \end{array}$$

Subtract the divisor from the dividend, and from the successive remainders, until a remainder less than the divisor is obtained. The number of subtractions is the quotient.

PROOF.

$5 \times 149 + 39 = 784$; or, $784 \div 149 = 5$, with a remainder of 39.

135. Divide by continued subtraction and prove:

1. 489764 by 42875
2. 447859 by 48727
3. 128976 by 54738
4. 249768 by 55987
5. 473964 by 75428
6. 963874 by 85925
7. 389726 by 98762
8. 598370 by 87820
9. 397645 by 98435
10. 947680 by 94768
11. 247682 by 31463
12. 437263 by 48721

FACTORING

136. 6 is the product of what numbers?

Because $2 \times 3 = 6$, 2 and 3 are called the factors of 6.

137. What are the factors of 10? 15? 18? 21?

138. A prime number is one not exactly divisible by any number except itself and 1.

139. Prime factors are prime numbers.

140. **Factoring** is the process of separating a number or quantity into its factors.

REMARK. Factoring is important, not merely for the purpose of resolving a given number into its factors, but also for its assistance in the solution of problems. In many problems in fractions, practical measurements, and percentage, in fact, in all problems in which cancellation is used, the work involved in the solution is very materially lessened by the ability to see factors quickly.

The ability quickly to factor a given number depends upon a knowledge of the tests of divisibility, with which the student should become thoroughly familiar.

141. Tests of Divisibility.

1. All even numbers are divisible by 2.

Thus, 4, 8, 18, 38, 96, 144, are divisible by 2.

2. If the sum of the digits of any number is divisible by 3, the number is divisible by 3.

Thus, 144957 is divisible by 3 because $1 + 4 + 4 + 9 + 5 + 7 = 30$, and 30 is divisible by 3.

3. All numbers whose right-hand two figures are zeros, or express a number divisible by 4, are divisible by 4.

Thus, 100, 144, and 3288, are divisible by 4.

4. All numbers whose units' figure is either a zero or 5 are divisible by 5.

Thus, 60 and 135 are divisible by 5.

5. All even numbers, the sum of whose digits is divisible by 3, are divisible by 6.

Thus, 126, 918, and 45252 are divisible by 6.

In practice, notice first whether the number is even. If so, proceed as in test for 3.

6. 7, 11, and 13 will divide 1001 and any of its multiples; as, 3003, 8008, 12012, etc.

7. All numbers whose right-hand three digits are zeros, or express a number divisible by 8, are divisible by 8.

Thus, 3000, 6624, and 18232 are divisible by 8.

8. All numbers, the sum of whose digits is divisible by 9, are divisible by 9.

Thus, 45621 and 234819 are divisible by 9.

9. All numbers whose right-hand figure is 0 are divisible by 10.

Thus, 60, 140, 1190, are divisible by 10.

10. All numbers whose right-hand two figures are zeros, or express a number divisible by 25, are divisible by 25.

Thus, 9800, 4125, and 875 are divisible by 25.

11. All numbers whose right-hand three figures are zeros, or express a number divisible by 125, are divisible by 125.

Thus, 3000, 4625, and 3375 are divisible by 125.

142. Applying the tests given above, find the prime factors of 12,464.

$$\begin{array}{r@{)}r} 8 & 12464 \\ \hline 2 & 1558 \\ \hline 19 & 779 \\ \hline & 41 \end{array} \quad \text{Therefore, } 2\times2\times2\times2\times19\times41=12,464.$$

The "test" for 8 shows that 8 is a factor. The quotient 1558 is even; hence 2 is a factor. By trial 19 is found to be a factor, resulting in a quotient of 41, a prime number.

8 was used as a divisor first to save time. Its prime factors are known to be $2 \times 2 \times 2$. Therefore, the prime factors of 12,464 are $2 \times 2 \times 2 \times 2 \times 19 \times 41$.

143. There is no regular method of determining large factors. Unless one has access to factor tables, nothing remains but to try the prime numbers until the right one is found. There are, however, two fundamental principles of factoring that should be borne in mind:

1. *A factor of a number is also a factor of all the multiples of that number.*

Thus, 7 is a factor of 21. 7 is also a factor of 42, 63, 105, 189, 420, 1071, all of which are multiples of 21.

2. *A common factor of two numbers is a factor also of the sum or of the difference of those two numbers.*

Thus 4 is a common factor of 20 and 36. It is a factor also of 56 (20 + 36), and of 16 (36 − 20).

144. Find the prime factors of the following numbers:

1. 48
2. 64
3. 80
4. 105
5. 144
6. 160
7. 729
8. 343
9. 1125
10. 3375
11. 3654
12. 3248
13. 2664
14. 4064
15. 4512
16. 3824
17. 9664
18. 9750
19. 16296
20. 25728
21. 6875
22. 5375
23. 4004
24. 3224 906
25. 5872
26. 9536 317
27. 3856
28. 7328
29. 15309
30. 27783

GREATEST COMMON DIVISOR

145. The **greatest common divisor** of two or more numbers is the largest number that will exactly divide each of them.

146. The only practical application (so far as this work is concerned) of the principles involved in finding the G. C. D. is in reducing common fractions to their lowest terms.

GREATEST COMMON DIVISOR BY INSPECTION

147. In many cases the G. C. D. of two or more numbers can be determined by inspection by observing the following:

Principles. 1. *The G. C. D. of two or more numbers cannot be greater than the smallest number in the list.*

2. *The G. C. D. of two or more numbers cannot be greater than the smallest difference between any two numbers in the list.*

3. *The G. C. D. of two or more numbers is either* (*a*) *the smallest number,* (*b*) *the smallest difference, of* (*c*) *a factor of one of them.*

148. 1. Find the G. C. D. of 24, 72, 120, and 168.

By inspection it is seen that the smallest number, 24, will divide each of the several numbers, hence it is the G. C. D.

2. Find the G. C. D. of 70, 168, 224, 252.

The smallest number is 70. The smallest difference is 28 (252 − 224). By inspection 28 is seen not to be a divisor of 70. 28 = 2 × 14. Since 28 is not a common divisor, either 2 or 14 must be eliminated. Since the greatest common divisor is sought, eliminate the 2. 14 is seen to be a divisor of each of the numbers. Hence 14 is the G. C. D.

149. If the G. C. D. cannot be determined by inspection proceed as follows:

1. Find the G. C. D. of 188, 470, 705, and 940.

188 – 470 – 705 – 940
188 – 94 – 141 – 0
0 – 94 – 47 – 0
47 is the G. C. D.

Divide all the other numbers by the smallest number, writing the remainders as shown in the solution. The "divisor" must also be "brought down." Divide again by the smallest number. By inspection, it is now seen that 47 is the G.C.D.

150. Find, by inspection when possible, the G. C. D. of the following numbers.

1. 912 and 1121 19
2. 296 and 407
3. 792 and 1016 8
4. 629 and 1147
5. 387 and 559
6. 319 and 551
7. 85 and 119
8. 145 and 206
9. 2813 and 3589
10. 52, 91, and 143
11. 961, 1271, and 372
12. 68, 102, and 187
13. 2167, 2561, and 985
14. 3749, 8313, and 1141
15. 48, 72, 120, and 288
16. 1681, 2501, and 410
17. 6859, 11,191, and 3610
18. 90, 108, 144, and 111

LEAST COMMON MULTIPLE

151. A multiple of any given number is a number that will contain the given number without a remainder. Thus, 24 is a multiple of 4.

152. 24 is a **common multiple** of 4 and 6, because it contains 4 and 6 each without a remainder.

153. 24 is the **least common multiple** of 4, 6, 8, and 12, because it is the smallest number that will contain them without remainders.

LEAST COMMON MULTIPLE BY INSPECTION

154. In the great majority of practical cases the L. C. M. can be found by inspection by observing the following:

Principles. 1. *The L. C. M. of two or more numbers cannot be smaller than the largest number in the list.*

2. *The L. C. M. of two or more numbers is either* (*a*) *the largest number, or* (*b*) *a multiple of the largest number.*

1. Find the L. C. M. of 9, 16, and 24.

24, the largest number, is not the L. C. M. Hence, try 48, 72, 96, etc., until a number is found that is a common multiple. By trial it is found to be 144.

If the L. C. M. is not readily discovered by inspection, proceed as follows:

Find the L. C. M. of 6, 8, 12, 15, 20, 24, 30, and 36.

$$36 \,\big|\, \underline{\cancel{6}\text{-}\cancel{8}\text{-}\cancel{12}\text{-}\cancel{15}\text{-}20\text{-}24\text{-}30\text{-}36.}$$

$$\big|\, \underline{\cancel{5}\text{ - }2\text{ - }5\text{ - }1.}$$

$36 \times 2 \times 5 = 360.$

Write all the numbers in one line. Cancel all the numbers that are contained in any of the other numbers. Divide the remaining numbers by the largest number in the list having a factor common to one or more of the other numbers. Use 36. The G. C. D. of 36 and 20 is 4. 20 divided by 4 is 5. Write 5. The G. C. D. of 36 and 24 is 12. 24 divided by 12 is 2. Write 2. And so on through the list of numbers. A new line of numbers is now obtained. Proceed as at first. Cancel all numbers contained in some other number in the line. Divide as before if necessary, and continue the process until the remaining numbers have no common factor except unity. The **L. C. M. is the product of all the divisors and the remaining numbers.**

155. Find by inspection, when possible, the L. C. M. of:

1. 2, 4, 6, 12, 15.
2. 3, 6, 9, 18, 30.
3. 5, 12, 15, 30, 40.
4. 8, 12, 24, 60, 84.
5. 15, 25, 36, 75, 100.
6. 13, 15, 25, 65, 75.
7. 24, 36, 48, 42, 120.
8. 6, 20, 48, 100, 144.
9. 9, 16, 64, 80, 128.
10. 36, 45, 72, 105, 150.

CANCELLATION

156. Problems frequently arise in which the solution is best made like the following:

Divide the product of 36×18×21 by the product of 9×28×12.

$$\frac{\overset{3}{\cancel{36}}\times\overset{2}{\cancel{18}}\times\overset{3}{\cancel{21}}}{\cancel{9}\times\underset{\underset{2}{\cancel{4}}}{\cancel{28}}\times\cancel{12}}=\frac{9}{2}=4\tfrac{1}{2}.$$

Write the factors of the dividend above a horizontal line, and the factors of the divisor below the line, as shown in the solution. 9, below the line, is contained in 18, above the line. Both numbers are canceled, and the quotient, 2, is written near the 18. In the same way 12 is contained in 36, 3 times. Since 28 is not divisible by 21, both numbers are divided by their common factor, 7. The numbers 21 and 28 are both canceled, and the quotients, 3 and 4, are written near the numbers, respectively. 2 of the dividend now divides the 4 in the divisor; that is, the common factor, 2, is canceled in both dividend and divisor. (See **Principles of Division, No. 21, page 117.**) The product of the remaining factors above the line is now divided by the product of the factors left below the line, thus, $\frac{3\times3}{2}=\frac{9}{2}=4\frac{1}{2}$, the desired quotient.

Using cancellation, divide:

1. 8×6×12×15 by 3×4×5×18.
2. 15×12×24×39 by 10×13×4×16.
3. 125×80×96×216 by 50×60×32×540.
4. 48×56×63×72 by 36×78×42×108.
5. 30×45×9×12 by 5×50×18×60.
6. 6×12×80×20 by 4×9×75×120.
7. 8×24×36×160 by 12×18×30×60.
8. 36×45×54×75×68 by 144×15×36×51×65.
9. 57×39×76×91×96 by 114×65×95×160.
10. 38×64×68×69×217 by 133×12×17×460.

11. How many tubs of butter averaging 30 lb. each, at 56¢ per pound, will pay for 8 pieces of muslin, each piece containing 49 yd. at 30¢ per yard?

The cost of the muslin is represented by the product of 8 × 49 × $.30; the value of 1 tub of butter, by the product of 30 × $.56. The number of tubs of butter required is found by dividing the cost of the muslin by the value of 1 tub of butter. Thus:

$$\frac{\cancel{8}\times\overset{7}{\cancel{49}}\times\cancel{.30}}{\cancel{30}\times\underset{7}{\cancel{.56}}}=7\text{, number of tubs of butter.}$$

20×41

12. How many pounds of sugar at 15¢ a pound can be had in exchange for 20 doz. eggs at 48¢ per dozen?

13. At $1.20 a bushel, how many bushels of potatoes will pay for 3 chests of tea, weighing 36 lb. each, at 40¢ a pound?

14. How many boys earning 20¢ an hour, 8 hour a day, in 15 days, will earn as much as 4 men at 60¢ an hour, 10 hour a day, in 18 days?

15. If $560 earns $84 interest in 2 yr. 6 mo., how much interest will $640 earn in 3 yr. 6 mo., at the same rate?

16. At $3.60 per barrel, how many loads of apples, 21 bbl. to the load, will be required to pay for 18 bbl. of sugar, averaging 350 lb. to the barrel, at 6¢ per pound?

17. If 17 tons of coal cost $178.50, how much will 51 tons cost at the same rate?

18. Find the cost of 40 tubs of butter, each weighing 56 lb., if 28 tubs of the same quality, weighing 48 lb. each, cost $698.88.

19. How many yards of cloth worth 70¢ a yard, would be given in exchange for 4 loads of oats, each load containing 40 bags of 2 bu. each, if oats are worth 84¢ a bushel?

20. How many cases of eggs, each containing 30 doz., at 42¢ a dozen, will pay for 6 bbl. of sugar averaging 350 lb. to the barrel, at 6¢ a pound?

POSTAL RATES

157. First-Class Matter includes letters, postal cards, post-cards, and all matter wholly or partly in writing, whether sealed or unsealed (except manuscript copy) and all other matter sealed or closed against inspection.

The limit of weight of first class matter is 70 pounds.

Rates of Postage. Postal cards and post cards 1 cent each. All other first-class matter 2 cents for each ounce or fraction thereof.

Second-Class Matter includes newspapers and periodicals bearing notice of entry as second-class matter.

No limit of weight is prescribed.

Rate of Postage. One cent for each 4 ounces or fraction thereof.

Third-Class Matter includes circulars, newspapers and periodicals not admitted to second-class, manuscript copy, proof sheets, photographs, and other miscellaneous printed matter.

The limit of weight is 4 pounds.

Rate of Postage. One cent for each 2 ounces or fraction thereof.

Fourth-Class Matter, or Domestic Parcel Post mail includes merchandise, farm and factory products, seeds, plants, etc., books and catalogues, miscellaneous printed matter weighing more than 4 pounds, and all other mailable matter not embraced in the first, second, or third classes.

Parcel post matter must not be of a kind likely to injure the person of any postal employee or to damage the mail equipment or other mail matter, nor must it be of a character perishable within a period reasonably required for transportation and delivery.

Rates of postage on domestic parcel post matter are as follows:

(*a*) Parcels weighing 4 oz. or less, except books, seeds, plants, etc., 1 ¢ for each ounce or fraction thereof, for any distance.

(*b*) Parcels weighing 8 oz. or less containing books, seeds, plants, etc., 1 ¢ for each 2 oz. or fraction thereof, for any distance.

(*c*) Parcels weighing more than 8 oz. containing books, seeds, plants, etc., parcels of miscellaneous printed matter weighing more than 4 lb., and all other parcels of domestic parcel post, or fourth class matter weighing more than 4 oz. are chargeable according to distance or zone, at the pound rates, a fraction of a pound being considered a full pound.

Rates of Postage in the Different Zones

Local rate, 1st pound, 5 ¢; each additional 2 pounds, 1 ¢.
1st and 2d zones, 1 to 150 miles, 1st pound, 5 ¢; each additional pound, 1 ¢.
3d zone, 150 to 300 miles, 1st pound, 6 ¢; each additional pound, 2 ¢.
4th zone, 300 to 600 miles, 1st pound, 7 ¢; each additional pound, 4 ¢.
5th zone, 600 to 1000 miles, 1st pound, 8 ¢; each additional pound, 6 ¢.
6th zone, 1000 to 1400 miles, 1st pound, 9 ¢; each additional pound, 8 ¢.
7th zone, 1400 to 1800 miles, 1st pound, 11 ¢; each additional pound, 10 ¢.
8th zone, over 1800 miles, 1st pound, 12 ¢; each additional pound, 12 ¢.

The limit of weight for local delivery and for the first three zones is 70 pounds; for all other zones, 50 pounds. Parcel post matter may not exceed 84 inches in length and girth combined.

Insurance may be secured by a payment of 3 ¢ extra for an amount up to $5; 5 ¢ up to $25; 10 ¢ up to $50; 25 ¢ up to $100.

Parcels valued at not more than $100 may be sent "C. O. D." upon the payment of a fee of 10 cents, if the amount to be remitted does not exceed $50; or a fee of 25 ¢ for greater amounts up to $100. This fee includes insurance up to $50 and $100 respectively. The fee for the return, by postal money order, of the amount collected is paid by the addressee.

A parcel post guide and map are used to determine in which zone a post office is located from the office of mailing.

PROBLEMS

How much postage is required on the following?

1. A letter weighing $1\frac{1}{4}$ ounces.
2. A sealed package weighing $9\frac{3}{4}$ ounces.
3. A package of newspapers weighing 1 lb. 6 oz.

4. A photograph weighing 5½ ounces.

5. A mail order house sends 18 packages each weighing 9 lb. 4 oz. a distance of 900 miles.

6. A boy went to a post office to mail 3 letters, one of which weighed 1½ ounces, and the other two ½ ounce each, 4 post cards, a magazine weighing 8½ ounces, and a package of circulars weighing 7 ounces.

7. A parcel of merchandise weighing 18 pounds to a post office in the fifth zone.

8. A 40-pound parcel of books to the second zone.

9. An insured parcel of dry goods weighing 14 pounds 6 ounces, valued at $32.75, to a post office in the sixth zone.

10. A "C. O. D." insured parcel weighing 12½ pounds, valued at $18.50, to a post office in the seventh zone.

11. A 7-ounce book to a post office in the fifth zone.

12. A 10-ounce parcel of dry goods to the third zone.

13. A "C. O. D." parcel valued at $75 and weighing 20 lb. is mailed to the 8th zone. Find the cost of mailing it; also the cost to the addressee, including money order fee.

14. A man sends a 30-pound pail of butter to a post office in the second zone. How much is the postage?

15. A merchant in Toledo, O., telegraphed a jobber in Chicago, "Send C.O.D. by parcel post 100 dozen # 546 buttons." The buttons came billed at 22 cents a dozen. How much did the merchant have to pay for the buttons, including the money order fee?

16. A jeweler in Spokane, Wash., ordered from a wholesaler in San Francisco, Cal. (fifth zone), one dozen watches sent C.O.D. by parcel post. If the watches were valued at $7.50 each, and the parcel weighed 3½ pounds, (*a*) how much was the total cost of mailing the watches? (*b*) how much did they cost the jeweler in Spokane, including the money order fee? (*c*) if the watches were broken en route, how much insurance could be collected?

FRACTIONS

158. What part of a dollar is 50 ¢? 25 ¢? 75 ¢? $12\frac{1}{2}$ ¢? 10 ¢? 5 ¢? 1 ¢?

Each of these amounts is less than one dollar. They are fractional parts of a dollar.

159. A fraction is a part of a unit.

JACK DUDECK

50 ¢ = $\frac{1}{2}$ of a dollar = .50 of a dollar.
25 ¢ = $\frac{1}{4}$ of a dollar = .25 of a dollar.
75 ¢ = $\frac{3}{4}$ of a dollar = .75 of a dollar.
5 ¢ = $\frac{1}{20}$ of a dollar = .05 of a dollar.
1 ¢ = $\frac{1}{100}$ of a dollar = .01 of a dollar.

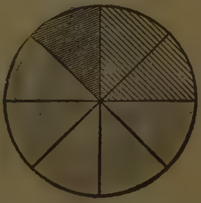
How many ways are there of writing a fraction?

Are $\frac{1}{2}$, $\frac{5}{10}$, and .5 equal in value?
Are $\frac{3}{4}$, $\frac{75}{100}$, and .75 equal in value?
Are $\frac{1}{20}$, $\frac{5}{100}$, and .05 equal in value?
Are $\frac{1}{100}$ and .01 equal in value?

160. There are, then, two ways of writing a fraction. In the common fraction form any number may be used as a denominator of the fraction. In the fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{20}$, and $\frac{1}{100}$, 2, 4, 20, and 100, respectively, are the denominators.

161. In the decimal fraction form, the denominator must be 10 or some power* of 10. In the fractions .5, .75, .05, and .125, the denominators are 10, 100, 100, and 1000, respectively.

162. Into how many parts is this circle divided?



What part of the circle is heavily shaded?

How much of it is lightly shaded?

How does the lightly shaded part compare in size with the heavily shaded part? $\frac{1}{4}=2\times\frac{1}{8}$

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*By "power of 10" is meant the product resulting from using 10 a given mumber of times as a factor. Thus, 10 × 10, or 100, is the second power of 10; 10 × 10 × 10, or 1000, is the 3d power of 10, etc.

This illustration emphasizes an important principle:

The larger the denominator, the smaller the fraction. Compare with the Principles of Division (No. 17, page 117.)

What part of the circle is not shaded?

Which part of the fraction, $\frac{5}{8}$, shows the number of parts into which the circle is divided?

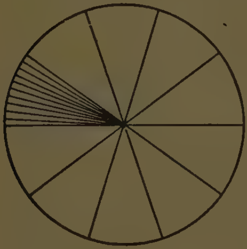
Which part shows the number of eighths in the fraction?

In the fraction, $\frac{5}{8}$, 8 is the **denominator**, and shows that the unit (the unit, in this case, is the circle) has been divided into 8 equal parts. 5 is the **numerator**, and shows that 5 of the 8 parts have been taken to form the fraction, $\frac{5}{8}$.

Important. For all practical purposes, a common fraction represents a division. $5 \div 8 = \frac{5}{8}$. These expressions are the same except in form. Each may be read "5 divided by 8." It is clear, therefore, that 5, the numerator of the fraction, is a dividend, and that 8, the denominator, is a divisor.

163. Decimal and common fractions are so closely associated in business problems, that it has been deemed wise to treat them together.

There are three things to learn about decimals: (1) The use of decimal point. (2) Notation and numeration. (3) The common fraction equivalent, and *vice versa.*



Into how many equal parts has this circle been divided?

One of these equal parts is called one tenth, written $\frac{1}{10}$ or .1.

Into how many equal parts has one of the tenths been divided?

If each of the tenths were divided into ten equal parts, into how many smaller parts would the entire circle be divided?

One of these small parts is what part of the circle? $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$ or .1 of .1 = .01. (Principles 2 and 4, page 116).

164. .1 and .01 are called **decimal fractions**, or, more often, **decimals.** (From the Latin word *decem,* meaning ten.) They are decimals because their denominators are 10 or some power of 10.

165. The **denominator** of a decimal is not written, but is indicated by the number of figures used to write the decimal.

166. In reading decimals, the denominator is named the same as though it were written. Thus, the decimal .015 is read "fifteen thousandths," the same as if written $\frac{15}{1000}$. The denominator is determined by the fact that the decimal contains 3 figures ("places"). For each place, one cipher is written to the right of the figure 1, making, in this case, 1000.

ORAL EXERCISE

167. Read the following decimals:

1. .4
2. .14
3. .05
4. .125
5. .001
6. .101
7. .427
8. .821
9. .025
10. .0025
11. 15.0125[*]
12. 28.02345
13. 75.00001
14. .00125
15. .00025
16. 112.01875
17. .125484
18. 236.324562
19. .000125
20. 10000.0001
21. .10101
22. .4755
23. 125.0125
24. .0000001

REDUCTION OF FRACTIONS

168. How many eighths equal $\frac{1}{4}$? How many quarters equal $\frac{1}{2}$? How many eighths equal $\frac{1}{2}$?

$$\frac{4}{8}=\frac{2}{4}=\frac{1}{2}, \text{ and } \frac{1}{2}=\frac{2}{4}=\frac{4}{8}$$

These equations illustrate what is meant by reduction of fractions. The form of the fraction, $\frac{4}{8}$, has been changed so as to read "$\frac{1}{2}$," but its value is not changed.

In the first equation, the fraction, $\frac{4}{8}$, has been reduced to its lowest terms.

In the second equation, the fraction, $\frac{1}{2}$, has been reduced to higher terms.

169. The terms of a fraction are its numerator and denominator.

170. A fraction is reduced to its lowest terms when its numerator and denominator cannot both be exactly divided by any whole number except 1.

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[*] Read "fifteen, *and* one hundred twenty-five ten-thousandths."

171. **1.** Reduce $\frac{126}{189}$ to its lowest terms.

$$3\Big|\frac{126}{189}=3\Big|\frac{42}{63}=7\Big|\frac{14}{21}=\frac{2}{3}$$

Divide both terms of the fraction by any common divisor, say 3. The resulting fraction is $\frac{42}{63}$. Divide again by any common divisor, as 3, obtaining $\frac{14}{21}$. Now divide both terms by 7, and the result is $\frac{2}{3}$, the lowest terms, because the numbers 2 and 3 cannot be divided by the same number.

2. Reduce $\frac{493}{899}$ to its lowest terms.

$$29\Big|\frac{493}{899}=\frac{17}{31}$$

In this fraction the common divisor cannot be readily determined by inspection. Find the greatest common divisor by the method explained on page 71. 29 is the G. C. D. Dividing both terms of the fraction by 29 gives $\frac{17}{31}$, the lowest terms.

Compare these solutions with the principles of division (No. 21, page 117.)

State the principle involved in the last two examples.

3. Reduce $\frac{301}{688}$ to its lowest terms.

$$\begin{array}{r} 688\\ 2\times301=\underline{602}\\ 86\end{array}$$

$$86=2\times43.$$

$$43\Big|\frac{301}{688}=\frac{7}{16}$$

From the denominator, 688, take 2 times the numerator, 301. The remainder, 86, is factored. 86 = 2 × 43. By inspection, 2 is not a common factor, but 43 is found, on trial, to be a common factor. Therefore, the fraction reduces to $\frac{7}{16}$, its lowest terms.

4. Reduce $\frac{7}{8}$ to fortieths.

$$40\div8=\ 5$$

$$\begin{array}{c}\underline{7}\times5=\underline{35}\\ 8\times5=40\end{array}$$

The denominator of the fraction desired is 40, which is 5 times as large as the denominator of the given fraction. If $\frac{7}{8}$ is to be changed to a fraction having a denominator 5 times as large the numerator also must be 5 times as large. Hence, multiplying both numerator and denominator by 5 gives the desired results, $\frac{35}{40}$.

Compare with the principles of division (No. 21, page 117).

5. Reduce .13 to ten-thousandths.

$$10{,}000\div100=100$$

$$13\times100=1300.\quad \text{Therefore, } .1300$$

The required denominator is 100 times as large as the denominator of the given fraction Therefore the numerator should be 100 times as large. Hence, the required decimal is .1300.

Observe that the process of reduction consists in annexing ciphers Show the application of the principles of division.

worke

14

EXERCISE

172. Reduce the following fractions to their lowest terms (mentally, when possible):

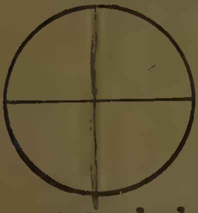
1. $\frac{72}{80}$, $\frac{84}{120}$, $\frac{96}{116}$
2. $\frac{125}{150}$, $\frac{48}{144}$, $\frac{56}{168}$
3. $\frac{111}{117}$, $\frac{156}{207}$, $\frac{165}{220}$
4. $\frac{45}{60}$, $\frac{45}{180}$, $\frac{55}{495}$
5. $\frac{1240}{1648}$, $\frac{1640}{2050}$, $\frac{3088}{4632}$
6. $\frac{1998}{2997}$, $\frac{5022}{6732}$, $\frac{2448}{7272}$
7. $\frac{437}{551}$, $\frac{609}{783}$, $\frac{341}{465}$
8. $\frac{648}{756}$, $\frac{244}{427}$ $\frac{201}{335}$,
9. $\frac{355}{568}$, $\frac{207}{552}$, $\frac{152}{171}$
10. $\frac{153}{187}$, $\frac{91}{143}$, $\frac{209}{231}$
11. $\frac{253}{299}$, $\frac{435}{464}$, $\frac{555}{740}$
12. $\frac{970}{1455}$, $\frac{410}{615}$, $\frac{684}{1026}$

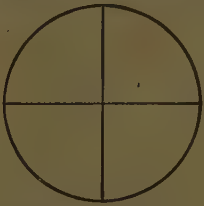
Change to higher terms:

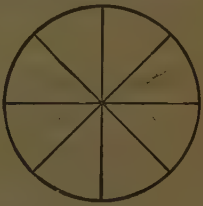
13. $\frac{3}{4}$ to 24ths.
14. $\frac{5}{8}$ to 48ths.
15. $\frac{7}{9}$ to 63ds.
16. .05 to thousandths.
17. .04 to hundred-thousandths.
18. $\frac{5}{6}$ to 36ths.
19. $\frac{8}{11}$ to 55ths.
20. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, to 12ths.
21. $\frac{3}{5}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{9}{10}$ to 40ths.
22. $\frac{3}{4}$, $\frac{13}{20}$, .5, .8 to 100ths.
23. $\frac{5}{8}$, $\frac{5}{6}$, .75, .125 to 24ths.
24. $\frac{3}{16}$, $\frac{2}{3}$, .33$\frac{1}{3}$, .16$\frac{2}{3}$ to 48ths

173. To reduce an improper fraction to a whole or a mixed number.

How many halves are there in 1 circle? in 2 circles? in 3 circles?
How many quarters are there 1 circle? in 2 circles? in 3 circles?
How many eighths are there in 1 circle? in 2 circles? in 3 circles?







1 circle $=\frac{?}{2}=\frac{?}{4}=\frac{?}{8}$. 2 circles $=\frac{?}{2}=\frac{?}{4}=\frac{?}{8}$. 3 circles $=\frac{?}{2}=\frac{?}{4}=\frac{?}{8}$.

$\frac{8}{2}$ circles = how many circles?
$\frac{5}{2}$ circles = how many circles?
$\frac{4}{2}$ circles = how many circles?
$\frac{6}{4}$ circles = how many circles?
$\frac{9}{4}$ circles = how many circles?
$\frac{12}{8}$ circles = how many circles?
$\frac{16}{8}$ circles = how many circles?
$\frac{20}{8}$ circles = how many circles?

All the fractions in this paragraph are improper fractions, because their numerators are equal to, or greater than, their denominators.

M. D.

174. A fraction having a numerator smaller than its denominator is a **proper fraction.**

175. Improper fractions may be reduced to whole or mixed numbers.

176. A **mixed number** is the sum of a whole number and a fraction. Thus, $2\frac{1}{4}$ is a mixed number; it is the sum of 2 and $\frac{1}{4}$.

177. Reduce $\frac{37}{5}$ to a mixed number.

$$37 \div 5 = 7\tfrac{2}{5}.$$

There are $\frac{5}{5}$ in one unit. In $\frac{37}{5}$ there are as many units as 5 is contained times in 37, or 7 times and $\frac{2}{5}$ over.

Therefore, $\frac{37}{5} = 7\frac{2}{5}$.

EXERCISE

178. Reduce the following improper fractions to whole or mixed numbers:

1. $\frac{43}{6}$ $7\frac{1}{6}$
2. $\frac{72}{10}$ $7\frac{1}{5}$
3. $\frac{31}{4}$ $7\frac{3}{4}$
4. $\frac{48}{8}$ 6
5. $\frac{53}{7}$ $7\frac{4}{7}$
6. $\frac{91}{8}$ $11\frac{3}{4}$
7. $\frac{88}{9}$ $9\frac{7}{9}$
8. $\frac{124}{7}$ $17\frac{5}{7}$
9. $\frac{263}{15}$ 111
10. $\frac{328}{12}$ 2
11. $\frac{264}{12}$ 22
12. $\frac{366}{15}$ 24 $\frac{\ }{5}$
13. $\frac{472}{4}$
14. $\frac{263}{16}$
15. $\frac{224}{128}$
16. $\frac{163}{10}$
17. $\frac{248}{100}$
18. $\frac{387}{100}$
19. $\frac{4897}{100}$
20. $\frac{2431}{1000}$
21. $\frac{6987}{100}$
22. $\frac{3480}{100}$
23. $\frac{5625}{1000}$
24. $\frac{4375}{1000}$
25. $\frac{675}{200}$
26. $\frac{990}{300}$
27. $\frac{4897}{25}$
28. $\frac{8000}{400}$
29. $\frac{4725}{25}$
30. $\frac{3125}{125}$

179. To reduce a mixed number to an improper fraction.

Reduce $4\frac{5}{8}$ to an improper fraction.

$$\begin{array}{r} 4\frac{5}{8} \\ \hline 32 \\ 5 \\ \hline \frac{37}{8} \end{array}$$

There are $\frac{8}{8}$ in one unit. In 4 units, or 4, there are 4 times $\frac{8}{8}$, or $\frac{32}{8}$; $\frac{32}{8} + \frac{5}{8} = \frac{37}{8}$. Therefore, $4\frac{5}{8} = \frac{37}{8}$.

In practice, the solution is as given in the margin, viz., multiply the whole number by the denominator of the fraction, and to the product add the numerator. Under the sum write the denominator of the fraction.

EXERCISE

180. Reduce the following mixed numbers to improper fractions (mentally, when possible):

1. $3\frac{1}{2}$
2. $4\frac{2}{5}$
3. $8\frac{3}{4}$
4. $9\frac{3}{8}$
5. $6\frac{3}{7}$
6. $9\frac{8}{9}$
7. $10\frac{3}{4}$
8. $5\frac{9}{10}$
9. $6\frac{7}{8}$
10. $9\frac{3}{7}$
11. $17\frac{3}{4}$
12. $19\frac{3}{8}$
13. $72\frac{3}{4}$
14. $96\frac{5}{9}$
15. $82\frac{3}{7}$

16. $85\frac{5}{7}$
17. $45\frac{5}{8}$
18. $19\frac{9}{11}$
19. $38\frac{8}{11}$
20. $87\frac{1}{2}$
21. $128\frac{3}{4}$
22. $236\frac{3}{7}$
23. $328\frac{5}{6}$
24. $482\frac{5}{9}$
25. $437\frac{3}{5}$
26. $927\frac{3}{8}$
27. $726\frac{6}{7}$
28. $322\frac{5}{8}$
29. $972\frac{3}{16}$
30. $1846\frac{3}{7}$

In the following pages both ways of writing fractions will be used. The student must be able to handle either form alone or both together.

As illustrated in the chapter on Aliquot Parts, the common fraction is the more convenient form to use in many instances. In other cases, the decimal form is better.

INTERCHANGE OF FRACTIONAL FORMS

181. The student already knows that $\frac{1}{2}$ of a dollar = 50 cents = .50 of a dollar. If this were not known, it could be determined thus: One dollar is written \$1.00. To find $\frac{1}{2}$ of any number, divide by 2.

$$\begin{array}{r} \$.50 \\ 2\overline{)\$1.00} \end{array}$$

This simple example illustrates the method of changing any common fraction to its equivalent decimal form, viz.,

Annex zeros to the numerator and divide by the denominator.

EXERCISE

182. Change the following fractions to their equivalent decimal form. (Find results mentally, when possible.)

1. $\frac{3}{20}$
2. $\frac{5}{8}$
3. $\frac{9}{32}$
4. $\frac{7}{25}$
5. $\frac{4}{15}$*
6. $\frac{5}{18}$
7. $\frac{9}{27}$
8. $\frac{4}{9}$
9. $\frac{4}{11}$
10. $\frac{3}{14}$
11. $\frac{1}{80}$
12. $\frac{3}{40}$
13. $\frac{1}{200}$
14. $\frac{13}{14}$
15. $\frac{15}{44}$
16. $\frac{4}{125}$
17. $\frac{18}{25}$
18. $\frac{21}{500}$
19. $\frac{4}{3125}$
20. $\frac{39}{75}$

*A common fraction in its lowest terms having a denominator that contains any factor other than 2 or 5 cannot be changed to a pure decimal. In such cases the division should be carried out two places and the fraction retained.

Thus,

$$\begin{array}{r} .26\frac{2}{3} \\ 15\overline{)4.00} \\ 30 \\ \hline 100 \\ 00 \\ \hline \frac{10}{15}=\frac{2}{3} \end{array}$$

Hence, $\frac{4}{15} = .26\frac{2}{3}$

If it were not known that 25¢ = .25 of a dollar = $\frac{1}{4}$ of a dollar, that fact could be determined by comparing 25¢ with 100¢ (100¢ being equal to one dollar). 25¢ is $\frac{25}{100}$ of 100¢.

$$5\,|\,\tfrac{25}{100} = 5\,|\,\tfrac{5}{20} = \tfrac{1}{4}. \quad \text{Hence, } .25 = \tfrac{1}{4}.$$

Observe that the decimal is written in the common fraction form by supplying its denominator, and reducing the resulting fraction to its lowest terms.

183. The denominator of the decimal is 1, with as many ciphers after it as there are places in the decimal.

Change $.46\frac{2}{3}$ to its common fraction form in its lowest terms.

$$.46\tfrac{2}{3} = \frac{46\frac{2}{3}}{100}$$

$$\underline{.46\tfrac{2}{3}}$$

$$\frac{46\frac{2}{3}\times 3}{100\times 3} = 10\,\Big|\,\frac{140}{300} = 2\,\Big|\,\frac{14}{30} = \frac{7}{15}, \text{ or } \frac{140}{300} = \frac{7}{15}.$$

$46\frac{2}{3}$ is a complex decimal. Its denominator is 100, giving a complex common fraction of $\frac{46\frac{2}{3}}{100}$. By the Principles of Division (No. 21, page 117), multiplying both dividend and divisor (numerator and denominator) by the same number does not change the value of the quotient. Hence, multiply both numerator and denominator by 3 (the denominator of the fraction $\frac{2}{3}$). The result is $\frac{140}{300}$, which equals $\frac{7}{15}$, the desired result.

EXERCISE

184. Change the following decimals to their equivalent common fraction form in their lowest terms. Determine results mentally, when possible.

1. .8
2. .24
3. .56
4. .625
5. $.71\frac{3}{7}$
6. .888
7. .032
8. .00125
9. .0005
10. $.012\frac{1}{2}$
11. $.006\frac{1}{4}$
12. $.002\frac{1}{2}$
13. $.003\frac{1}{3}$
14. .1875
15. $.09\frac{3}{8}$
16. $.09\frac{1}{11}$
17. $.07\frac{1}{7}$
18. .0015
19. .056
20. .104
21. $.0001\frac{1}{4}$
22. $.0003\frac{1}{3}$
23. $.0142\frac{6}{7}$
24. $.006\frac{2}{3}$

LEAST COMMON DENOMINATOR

185. What is the denominator of each of the following fractions: $\frac{5}{16}$, $\frac{11}{16}$, $\frac{9}{16}$, $\frac{13}{16}$, $\frac{1}{16}$, $\frac{3}{16}$, $\frac{7}{16}$, $\frac{15}{16}$?

Because their denominators are all alike, the fractions are said to have a **common denominator.** They are also called **similar fractions.**

M.X.D.

186. Compare these denominators with each other: $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{9}{10}$, $\frac{11}{16}$, $\frac{8}{9}$, .18, .232.

They are unlike or **dissimilar.**

What are dissimilar fractions?

187. Fractions must be made similar before they can be added (See Principles of Addition, No. 5, page 116.)

188. What is the least common multiple of 2, 4, 8, and 16?

189. **1.** Change $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{9}{16}$ to similar fractions, having the smallest denominator possible.

$$\frac{1}{2}=\frac{8}{16} \qquad \frac{3}{4}=\frac{12}{16} \qquad \frac{5}{8}=\frac{10}{16} \qquad \frac{9}{16}=\frac{9}{16}$$

If the fractions are to be similar, the denominators must be alike. The least common denominator is the same as the least common multiple of the denominators. The least common denominator can be determined mentally. (See page 73 for finding L. C. M.) It is 16.

As explained on page 82, reduce each of the fractions to 16ths. (State the principle of division that applies here.)

2. Reduce $\frac{5}{8}$, $\frac{11}{24}$, $.12\frac{1}{2}$, $.33\frac{1}{3}$, and .25 to similar fractions, having their least common denominator.

$$\frac{5}{8}=\frac{15}{24}$$
$$\frac{11}{24}=\frac{11}{24}$$
$$.12\tfrac{1}{2}=\frac{1}{8}=\frac{3}{24}$$
$$.33\tfrac{1}{3}=\frac{1}{3}=\frac{8}{24}$$
$$.25=\frac{1}{4}=\frac{6}{24}$$

Change the fractions in decimal form to their equivalent common fraction form, in their lowest terms. By inspection, it is seen that 24 is the least common denominator. Each fraction is now changed to a fraction having 24 for its denominator.

EXERCISE

190. Reduce the following fractions to similar fractions having their least common denominator. (Do the work mentally when possible, using pencil only to tabulate results.)

1. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$
2. $\frac{4}{5}$, $\frac{9}{10}$, $\frac{1}{2}$
3. $\frac{3}{4}$, $\frac{1}{8}$, .25
4. $\frac{5}{6}$, $\frac{2}{3}$, .1
5. $\frac{3}{4}$, $.06\frac{1}{4}$, $.12\frac{1}{2}$
6. $\frac{7}{8}$, .9, .375
7. $\frac{5}{8}$, $\frac{2}{3}$, $\frac{1}{12}$ 24
8. $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$ 12
9. $\frac{5}{16}$, $\frac{7}{24}$, $.62\frac{1}{2}$
10. $\frac{8}{9}$, $\frac{2}{3}$, $\frac{5}{18}$
11. $\frac{5}{9}$, $\frac{3}{4}$, $\frac{7}{18}$, $\frac{2}{3}$, $\frac{5}{36}$
12. $\frac{5}{16}$, .125, $\frac{3}{4}$, $.18\frac{3}{4}$
13. $\frac{3}{4}$, $87\frac{1}{2}$, $\frac{1}{2}$, .5, .25
14. $\frac{17}{24}$, $\frac{5}{6}$, $.33\frac{1}{3}$, $.16\frac{2}{3}$
15. $\frac{5}{12}$, $\frac{4}{9}$, $.08\frac{1}{3}$, $.33\frac{1}{3}$
16. $\frac{7}{8}$, $\frac{9}{16}$, $\frac{3}{4}$, $\frac{1}{6}$, $\frac{5}{12}$
17. .25, .625, $.06\frac{1}{4}$, .5
18. $\frac{3}{7}$, $\frac{5}{21}$, $\frac{9}{28}$, $\frac{3}{4}$, $\frac{1}{2}$ 54
19. $\frac{3}{8}$, $.31\frac{1}{4}$, $\frac{2}{3}$, $.16\frac{2}{3}$, $\frac{5}{12}$
20. $\frac{3}{11}$, $\frac{5}{9}$, $\frac{7}{33}$, $\frac{18}{99}$, $\frac{2}{3}$ 99

Find common D.

L. C. D. [illegible] X [illegible]

ADDITION OF FRACTIONS

191. 1 qt.+2 qt.+3 qt.=how many quarts?

What kind of numbers can be added?

1 fourth+3 fourths+2 fourths+3 fourths=how many fourths?

$\frac{1}{4}+\frac{3}{4}+\frac{2}{4}+\frac{3}{4}=?$

What kind of fractions can be added?

192. **1.** Add $\frac{5}{8}$, $\frac{7}{8}$, $\frac{3}{8}$, $\frac{1}{8}$, $\frac{2}{8}$.

$\frac{5}{8}+\frac{7}{8}+\frac{3}{8}+\frac{1}{8}+\frac{2}{8}=\frac{18}{8}=2\frac{1}{4}$.

Since these are similar fractions, they may be added by taking the sum of their numerators, which is 18, and writing it over the common denominator, 8. $\frac{18}{8}$, reduced to a mixed number, $=2\frac{1}{4}$.

2. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{11}{12}$.

$\frac{2}{3}=16$
$\frac{3}{4}=18$
$\frac{5}{8}=15$
$\frac{11}{12}=22$
$\frac{71}{24}=2\frac{23}{24}$

Before these fractions can be added they must be changed to similar fractions. By inspection, the least common denominator is seen to be 24. Instead of writing the 24 for each fraction, write it where it is needed, under the sum of the new numerators, as shown in the solution. Now find the numerators, writing each one after the fraction it represents. This arrangement makes the addition of the numerators very easy and quick, their sum being written, as shown, over the common denominator, 24; $\frac{71}{24}=2\frac{23}{24}$, the desired result.

3. Find the sum of $18\frac{2}{3}$, $33\frac{4}{5}$, and $68\frac{1}{2}$.

$18\ \frac{2}{3}=20$
$33\ \frac{4}{5}=24$
$68\ \frac{1}{2}=15$
$120\frac{29}{30}\quad \frac{59}{30}=1\frac{29}{30}$

Write the integers and fractions, slightly separated, as shown in the solution. Add the fractions first by the method already explained. Their sum is $1\frac{29}{30}$. Add the integers, carrying the 1 from the sum of the fractions. The total result is $120\frac{29}{30}$.

4. Find the sum of $13\frac{1}{3}$, 18.25, 64.345, and $56\frac{1}{7}$.

$13.333\frac{1}{3}=\ 7$
18.25
64.345
$56.142\frac{6}{7}=18$
$152.071\frac{4}{21}\quad \frac{25}{21}=1\frac{4}{21}$

These fractions are best added in their decimal form. The first thing to do is to change each common fraction to its decimal form. Next write the numbers as shown, with the decimal points under one another. In changing $\frac{1}{3}$ and $\frac{1}{7}$ to their decimal form, the student should observe that the decimals are carried out as many places as there are places in the longest decimal in any of the other numbers. Thus, the longest decimal given is .345, having 3 places. Therefore, $\frac{1}{3}$ and $\frac{1}{7}$ are changed to 3-place decimals. This being done, the remaining common fractions, $\frac{1}{3}$ and $\frac{6}{7}$, are parts of like orders and can be added. Add the common fractions as before, and carry the integral part as in the preceding example.

5. How many yards are there in 5 pieces of silk as follows: 41^2, 43^1, 44, 42^3, 47^1?

41^2
43^1
44
42^3
47^1
218^3

The small numbers written a little above and to the right of the numbers 41, 43, etc., indicate quarter yards. That is, 41^2 yd. is $41\frac{2}{4}$ yd.; 43^1 yd. is $43\frac{1}{4}$ yd., etc. This method of writing quarters is much used in the dry-goods business. These being similar fractions, the addition is made as already explained.

FRI.

EXERCISE

193. Add:

18
12
16
15
61
24

1. $\frac{3}{4}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{8}$
2. $\frac{5}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$
3. $\frac{5}{7}$, $\frac{1}{2}$, $\frac{3}{14}$, $\frac{3}{8}$
4. $\frac{3}{11}$, $\frac{1}{2}$, $\frac{5}{22}$, $\frac{1}{8}$
5. $18\frac{1}{2}$, $16\frac{2}{3}$, $17\frac{3}{8}$
6. $24\frac{5}{6}$, $31\frac{3}{4}$, $47\frac{1}{8}$
7. $14\frac{1}{4}$, 18.125, 27.5
8. $126\frac{1}{3}$, 162.5, 204.475
9 $\frac{5}{8}$, $2\frac{1}{2}$, 13.324
10. $\frac{3}{5}$, .4, $\frac{5}{8}$, .875
11. $\frac{4}{5}$, $\frac{7}{8}$
12. $\frac{5}{7}$, $\frac{9}{10}$
13. 18^1, 17^2, 16^3, 19^2, 22^1
14. 48^1, 72^3, 64^2, 89^2, 94^1
15. $75\frac{2}{3}$, $81\frac{1}{2}$, 63.427
16. $41\frac{5}{8}$, 72.325, $47.18\frac{2}{3}$
17. $32\frac{7}{8}$, $91\frac{3}{4}$, 52.135
18. 46^1, 72^2, 84^1, 92, 87^3
19. $19\frac{1}{2}$, $18\frac{1}{4}$, $82\frac{3}{8}$, $76\frac{3}{4}$
20. $21\frac{2}{3}$, $76\frac{4}{5}$, $91\frac{2}{3}$, 4.847

$\frac{3}{4}$

2 16
3
5 1
8

MENTAL ADDITION OF FRACTIONS

194. 1. Find the sum of $\frac{1}{3}$ and $\frac{1}{4}$

$$\frac{1}{3}=\frac{4}{12}$$
$$\frac{1}{4}=\frac{3}{12}$$
$$\frac{7}{12}$$

Observe that, in changing the fractions to similar fractions, the numerators of the similar fractions are the same numbers as the denominators of the given fractions, and that the common denominator is the product of the same numbers, 3 and 4. Therefore, to find the sum of two fractions having 1 for a numerator, take the sum of the denominators for the numerator, and the product of the denominators for the denominator.

2. Add $\frac{1}{3}$, $\frac{1}{8}$, $\frac{5}{24}$.

$$\frac{1}{3}+\frac{1}{8}=\frac{11}{24}$$
$$\frac{11}{24}+\frac{5}{24}=\frac{16}{24}=\frac{2}{3}$$

Notice that the sum of the first two fractions, $\frac{1}{3}$ and $\frac{1}{8}$, is a fraction ($\frac{11}{24}$) similar to the third fraction, $\frac{5}{24}$, which may then be added mentally.

The student should be always on the alert, looking for short methods.

195. Add the following:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{7}$	$\frac{1}{12}$	$\frac{1}{11}$
$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{7}$	$\frac{1}{9}$	$\frac{1}{8}$	$\frac{1}{7}$	$\frac{1}{2}$

11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
$\frac{1}{10}$	$\frac{1}{9}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{11}$	$\frac{1}{8}$	$\frac{1}{7}$	$\frac{1}{6}$	$\frac{1}{7}$
$\frac{1}{11}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{12}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{5}$

21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{1}{9}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{9}$	$\frac{1}{7}$
$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{3}$
$\frac{5}{6}$	$\frac{9}{20}$	$\frac{5}{6}$	$\frac{3}{14}$	$\frac{1}{6}$	$\frac{11}{12}$	$\frac{5}{27}$	$\frac{4}{15}$	$\frac{8}{45}$	$\frac{5}{21}$

165

196. Find the sum of $\frac{2}{5}$ and $\frac{2}{7}$.

$\frac{3}{5}+\frac{2}{7}=\frac{24}{35}$

$(7+5)\times 2=24$

$\frac{2}{5}$ is 2 times $\frac{1}{5}$, and $\frac{2}{7}$ is 2 times $\frac{1}{7}$. The sum of $\frac{1}{5}$ and $\frac{1}{7}$ is $\frac{12}{35}$. Therefore, the sum of $\frac{2}{5}$ and $\frac{2}{7}$ is 2 times $\frac{12}{35}$, or $\frac{24}{35}$. That is, take 2 times the sum of the denominators for the numerator of the sum. The denominator is the product of the denominators, the same as before.

197. Add the following:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{3}$	$\frac{2}{8}$	$\frac{2}{7}$	$\frac{2}{5}$	$\frac{2}{3}$	$\frac{2}{7}$	$\frac{2}{5}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{7}$
$\frac{2}{6}$	$\frac{2}{8}$	$\frac{2}{7}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{8}$	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{3}{5}$	$\frac{3}{6}$	$\frac{3}{8}$

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
$\frac{4}{5}$	$\frac{4}{9}$	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{6}{7}$	$\frac{6}{7}$	$\frac{2}{3}$	$\frac{2}{5}$ 18	$\frac{2}{7}$ 18	$\frac{2}{3}$ 10	$\frac{3}{4}$	$\frac{4}{5}$
$\frac{4}{7}$	$\frac{4}{11}$	$\frac{5}{9}$	$\frac{5}{7}$	$\frac{6}{8}$	$\frac{6}{10}$	$\frac{2}{7}$ 6	$\frac{2}{9}$ 10	$\frac{2}{9}$ 14	$\frac{2}{5}$ 6	$\frac{3}{5}$	$\frac{4}{7}$
						$\frac{8}{21}$ $\frac{8}{21}$	$\frac{4}{15}$ $\frac{40}{45}$	$\frac{1}{21}$ $\frac{3}{63}$	$\frac{11}{15}$ $\frac{27}{15}$	$\frac{7}{20}$ $=1\frac{9}{5}$	$\frac{8}{35}$

198. The student should drill carefully and thoroughly on additions like the following, till he can add simple fractions as readily as whole numbers.

$\frac{35}{63}$

Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{9}{16}$.

To add these fractions, the student should "*think*" all operations, naming only the results ("Think" the part in italics). *The least common denominator is* 16; *the first numerator is* 8; *the second is* 12, 20; *the third is* 10; 30, 39, $\frac{39}{16}=2\frac{7}{16}$.

$\frac{4}{5}$ $1\frac{1}{3}$ $\frac{8}{9}$ $\frac{114}{20}$

1 4
10
7
5
1
70

199. Add the following: (mentally)

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
$\frac{1}{3}$	$\frac{1}{2}$ 3	$\frac{1}{4}$ 4	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{8}$	$\frac{7}{9}$	$\frac{9}{12}$	$\frac{7}{12}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{1}{3}$ 2	$\frac{3}{8}$ 6	$\frac{7}{10}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{7}{16}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{5}{16}$
$\frac{1}{6}$	$\frac{5}{6}$ 5	$\frac{1}{2}$ 8	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{7}{12}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{7}{8}$
$\frac{5}{12}$	$\frac{2}{3}$ 4	$\frac{9}{16}$ 9	$\frac{1}{10}$	$\frac{7}{20}$ 7	$\frac{7}{12}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{5}{18}$	$\frac{3}{8}$	$\frac{2}{3}$	$\frac{3}{4}$
	14 6	27 2 $\frac{\cancel{1}\cancel{7}6}{3}$ = $1\frac{11}{16}$		20		$\frac{1}{2}$	$\frac{5}{8}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{4}$

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
$\frac{1}{3}$	$\frac{5}{9}$	$\frac{5}{8}$	$\frac{5}{16}$	$\frac{9}{10}$	$\frac{15}{32}$	$\frac{5}{18}$	$\frac{9}{16}$	$\frac{11}{15}$	$\frac{11}{24}$	$\frac{2}{3}$	$\frac{5}{9}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{16}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{3}$
$\frac{5}{6}$	$\frac{5}{6}$	$\frac{3}{4}$	$\frac{9}{16}$	$\frac{2}{3}$	$\frac{7}{8}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{3}{4}$
$\frac{3}{8}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{9}{32}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{11}{12}$	$\frac{11}{12}$
$\frac{1}{6}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{10}$	$\frac{3}{4}$	$\frac{5}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{5}{24}$	$\frac{1}{2}$	$\frac{1}{2}$

200. Add (mentally, when possible):

1. $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}+\frac{7}{8}$
2. $5\frac{4}{5}+\frac{1}{3}+\frac{1}{2}+\frac{7}{10}$
3. $\frac{5}{8}+\frac{9}{16}+.25+.37\frac{1}{2}$
4. $16\frac{3}{4}+17\frac{2}{3}+24\frac{5}{8}+56.5$
5. $47\frac{3}{5}+96.2+143\frac{9}{10}+54.25$
6. $122.125\frac{1}{3}+63\frac{1}{2}+88.0625+15\frac{1}{8}$
7. $49\frac{3}{4}+72\frac{5}{8}+99\frac{11}{16}+\frac{1}{2}$
8. $51^1+64^2+48^3+54+50^3$
9. $32\frac{3}{16}+29\frac{3}{8}+19\frac{3}{4}+54$
10. $78.125+31.5+83.333\frac{1}{3}+.66\frac{2}{3}$

11-30. For further drill add the fractions on page 87.

SUBTRACTION OF FRACTIONS

201. 1. 5 yd. minus 3 yd. =?

2. 5 eighths minus 3 eighths =?
$\frac{5}{8}-\frac{3}{8}=?$

3. What kind of fractions can be subtracted?

4. From $\frac{9}{11}$ take $\frac{3}{11}$.
$\frac{9}{11}-\frac{3}{11}=\frac{6}{11}$.

Since the fractions are similar, it is necessary only to take the difference between the numerators, 9 and 3, which = 6, and under the 6 to write the common denominator, thus, $\frac{6}{11}$.

2 8
20
8
35

4
9
2
5

5 ←
2

56
35

121

5
5

202. Find the difference between.

1. $\frac{5}{8}$ and $\frac{3}{8}$
2. $\frac{9}{11}$ and $\frac{7}{11}$
3. $\frac{5}{6}$ and $\frac{1}{6}$
4. $\frac{3}{7}$ and $\frac{2}{7}$
5. $\frac{9}{14}$ and $\frac{7}{14}$
6. $\frac{7}{12}$ and $\frac{5}{12}$
7. $\frac{7}{8}$ and $\frac{6}{8}$
8. $\frac{9}{16}$ and $\frac{7}{16}$
9. $\frac{4}{3}$ and $\frac{2}{3}$
10. .8 and .5
11. .11 and .09
12. .50 and .25

203. **1.** From $\frac{5}{8}$ take $\frac{1}{3}$.

$$\begin{array}{rr} \frac{5}{8}= & 15 \\ \frac{1}{3}= & 8 \\ \cline{2-2} & \frac{7}{24} \end{array}$$

Reduce the fractions to similar fractions, as in addition. Instead of adding the new numerators, subtract the one from the other, which gives the result, $\frac{7}{24}$.

2. From $15\frac{1}{3}$ take $9\frac{3}{4}$.

$$\begin{array}{rlr} 14 & ,\ \frac{4}{3}= & 16 \\ 9 & ,\ \frac{3}{4}= & 9 \\ \cline{1-1} \cline{3-3} 5\frac{7}{12} & & \frac{7}{12} \end{array}$$

$\frac{3}{4}$ is larger than $\frac{1}{3}$. First, therefore, "borrow" 1 from the 15, and add it to the $\frac{1}{3}$, expressing the result as an improper fraction, $\frac{4}{3}$. Now proceed as before: Change to similar fractions, and subtract, obtaining $5\frac{7}{12}$ as the result.

204. Find the value of:

non **1.** $\frac{7}{8}-\frac{3}{4}$
2. $\frac{9}{10}-\frac{5}{6}$
3. $\frac{3}{4}-\frac{2}{3}$
4. $\frac{9}{16}-\frac{1}{4}$
5. $1\frac{1}{2}-\frac{3}{4}$
6. $5\frac{2}{3}-3\frac{5}{6}$
7. $5\frac{1}{2}-2.75$
8. $18\frac{2}{3}-12.16\frac{2}{3}$
9. $24.83\frac{1}{3}-18\frac{1}{3}$
10. $63\frac{3}{8}-12.06\frac{1}{4}$
11. $14\frac{5}{9}-7.33\frac{1}{3}$
12. $38\frac{7}{8}-21\frac{11}{12}$
13. $56\frac{3}{7}-29.5$
14. $76\frac{3}{16}-67.18\frac{3}{4}$
15. $98.16\frac{2}{3}-21.135$
16. $88.124-57\frac{1}{3}$
17. $74\frac{7}{16}-28\frac{11}{12}$
18. $58\frac{8}{9}-27.9$
19. $73\frac{3}{7}-34\frac{9}{14}$
20. $84\frac{1}{4}-72.13$
21. $18.127-12.33\frac{1}{3}$
22. $26.872\frac{1}{3}-5\frac{1}{6}$
23. $39.458\frac{1}{3}-18.12\frac{1}{2}$
24. $79\frac{3}{11}-18\frac{3}{4}$
25. $71\frac{4}{5}-28\frac{7}{8}$
26. $82\frac{9}{10}-43\frac{1}{4}$
27. $67\frac{3}{16}-51\frac{3}{4}$
28. $54.128-27.063$
29. $57.45\frac{1}{3}-33.128$
30. $99\frac{5}{8}-74.25$

MENTAL SUBTRACTION OF FRACTIONS

205. **1.** From $\frac{1}{2}$ take $\frac{1}{3}$.

$$\begin{array}{r} \frac{1}{2}=\frac{3}{6} \\ \frac{1}{3}=\frac{2}{6} \\ \hline \frac{1}{6} \end{array}$$

In changing the fractions to similar fractions, notice that the numerators of the similar fractions are the same numbers as the denominators of the given fractions. The difference is $\frac{1}{6}$. Therefore, take the difference between the denominators for the required numerator, and the product of the denominators for the required denominator.

2. From $\frac{2}{3}$ take $\frac{2}{5}$.

$$\frac{2}{3}=\frac{10}{15}$$
$$\frac{2}{5}=\frac{6}{15}$$
$$\frac{4}{15}$$

The common denominator is 15. Observe that the new numerators are just twice the denominators of the given fractions, and that their difference, 4, is just twice the difference between the denominators 5 and 3. Therefore, take twice the difference between the denominators of the given fractions and write it over the common denominator.

206. Subtract (mentally):

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**	**12.**
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$
	$\frac{2}{15}$	$\frac{4}{21}$	$\frac{5}{24}$	6 [illegible]	$\frac{7}{30}$	$\frac{1}{20}$	$\frac{1}{12}$	$\frac{3}{28}$	$\frac{1}{8}$	$\frac{5}{36}$	$\frac{3}{20}$

13.	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**	**22.**	**23.**	**24**
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{1}{11}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{2}{7}$	$\frac{2}{5}$	$\frac{2}{7}$
$\frac{7}{44}$	$\frac{1}{6}$	$\frac{1}{30}$	$\frac{2}{35}$	$\frac{3}{40}$	$\frac{4}{45}$	$\frac{1}{10}$	$\frac{6}{55}$	$\frac{7}{60}$	$\frac{4}{35}$	4 [illegible]	$\frac{8}{21}$

207. Find (mentally, when possible) the value of:

1. $\frac{1}{2}+\frac{1}{3}-\frac{1}{6}$
2. $\frac{1}{3}+\frac{1}{4}-\frac{5}{12}$
3. $\frac{1}{3}+\frac{1}{5}-\frac{7}{15}$
4. $\frac{2}{3}+\frac{2}{7}-\frac{5}{21}$
5. $\frac{3}{4}+\frac{3}{5}-\frac{7}{20}$
6. $\frac{2}{7}+\frac{2}{9}-\frac{1}{3}$
7. $\frac{1}{3}+\frac{3}{8}-\frac{1}{12}$
8. $\frac{1}{4}+\frac{1}{5}-\frac{3}{10}$
9. $\frac{1}{8}+\frac{1}{9}-\frac{5}{24}$
10. $\frac{1}{4}+\frac{1}{7}-\frac{5}{14}$
11. $\frac{1}{2}+\frac{2}{3}-\frac{5}{6}$
12. $\frac{2}{3}+\frac{1}{4}-\frac{5}{12}$
13. $\frac{3}{4}+\frac{1}{3}-\frac{7}{12}$
14. $\frac{3}{5}+\frac{1}{2}-\frac{9}{10}$
15. $\frac{3}{4}+\frac{3}{8}-\frac{1}{2}$
16. $\frac{5}{6}+\frac{1}{2}-\frac{2}{3}$
17. $\frac{3}{8}+\frac{2}{3}-\frac{5}{12}$
18. $\frac{4}{5}+\frac{1}{3}-\frac{11}{15}$
19. $\frac{5}{8}+\frac{3}{4}-\frac{7}{8}$
20. $\frac{3}{5}+\frac{3}{7}-\frac{9}{35}$
21. $\frac{7}{8}-\frac{1}{2}+\frac{3}{4}-\frac{3}{8}+\frac{5}{16}$
22. $\frac{9}{16}-\frac{1}{2}+\frac{7}{8}-\frac{3}{16}+\frac{1}{4}$
23. $\frac{5}{6}+\frac{1}{3}-\frac{1}{2}-\frac{1}{6}+\frac{2}{3}$
24. $\frac{4}{9}+\frac{2}{3}-\frac{5}{6}+\frac{1}{2}-\frac{1}{3}$
25. $\frac{5}{8}+\frac{1}{4}+\frac{1}{2}-\frac{3}{8}-\frac{1}{2}$
26. $\frac{15}{16}-\frac{1}{2}-\frac{1}{4}+\frac{3}{8}+\frac{3}{4}$
27. $\frac{9}{10}-\frac{4}{5}+\frac{1}{2}+\frac{3}{4}-\frac{17}{20}$
28. $\frac{8}{9}-\frac{1}{6}-\frac{1}{3}+\frac{2}{18}-\frac{1}{4}$
29. $\frac{11}{12}-\frac{3}{4}+\frac{1}{6}+\frac{1}{3}-\frac{1}{2}$
30. $\frac{11}{16}+\frac{1}{4}-\frac{5}{8}+\frac{1}{2}-\frac{3}{4}$

MULTIPLICATION OF FRACTIONS

208. To multiply a whole number by a fraction or a fraction by a whole number.

1. 4 times 3 gallons = how many gallons?
2. 4 times 3 fifths = how many fifths? $4\times\frac{3}{5}=?$
3. Does $4\times\frac{3}{5}=\frac{3}{5}\times4$? (Principle 13, page 117.)

Change mixed fractions [illegible]

Cancel all factors

4. Compare these two solutions.

$$5\times\frac{3}{7}=\frac{15}{7}=2\tfrac{1}{7}. \qquad \frac{3}{7}\times5=\frac{15}{7}=2\tfrac{1}{7}.$$

(× may be read "times," "multiplied by," or "of." The first of these two equations may be read "5 times $\frac{3}{7}=\frac{15}{7}$," or "5 multiplied by $\frac{3}{7}=\frac{15}{7}$," the second may be read "$\frac{3}{7}$ of $5=\frac{15}{7}$," or "$\frac{3}{7}$ multiplied by $5=\frac{15}{7}$." It is not proper to say "5 of $\frac{3}{7}$," or "$\frac{3}{7}$ times 5.".)

State the principle of division that applies in these equations See page 117.

5. Compare these two solutions:

$$7\times\frac{5}{14}=\frac{35}{14}=\frac{5}{2}=2\tfrac{1}{2}.$$

$$7\times\frac{5}{\underset{2}{\cancel{14}}}=\frac{5}{2}=2\tfrac{1}{2}$$

Observe that, in the first solution, the numerator of the fraction is multiplied by 7, and that the resulting fraction is reduced to its lowest terms by *dividing both* terms of the fraction by 7.

In the second solution, notice that by dividing the denominator of the fraction, 14, by the multiplier, 7, the result, $\frac{5}{2}$, is obtained at once, without the intermediate step of multiplying by 7 (Principles 18, 19, page 117).

TO THE STUDENT. Study each problem carefully and determine the shortest solution. As far as possible, make all computations mentally, using pen or pencil to tabulate results.

209. Find the value of:

1. $5\times\frac{7}{8}$
2. $8\times\frac{3}{4}$
3. $9\times\frac{7}{9}$
4. $6\times\frac{3}{5}$
5. $5\times\frac{4}{7}$
6. $9\times\frac{8}{11}$
7. $3\times\frac{2}{7}$
8. $4\times\frac{8}{15}$
9. $7\times\frac{6}{11}$
10. $5\times\frac{9}{10}$
11. $6\times\frac{5}{6}$
12. $8\times\frac{9}{16}$
13. $\frac{5}{8}\times16$
14. $\frac{7}{10}\times15$
15. $\frac{7}{9}\times27$
16. $\frac{15}{16}\times48$
17. $\frac{3}{4}\times64$
18. $\frac{8}{11}\times55$
19. $\frac{11}{12}\times11$
20. $\frac{5}{9}\times15$
21. $\frac{8}{9}\times24$
22. $\frac{9}{8}\times24$
23. $\frac{7}{15}\times60$
24. $\frac{8}{15}\times40$
25. $\frac{5}{9}$ of 148
26. $\frac{3}{4}$ of 123
27. $\frac{8}{9}$ of 72
28. $\frac{5}{16}$ of 38
29. $\frac{11}{12}$ of 66
30. $\frac{7}{8}$ of 84
31. $\frac{15}{16}$ of 5
32. $\frac{19}{20}$ of 50
33. $\frac{21}{22}$ of 66
34. $\frac{12}{25}$ of 75
35. $\frac{13}{20}$ of 48
36. $\frac{15}{19}$ of 57

210. In all cases in which the fraction is in its decimal form, proceed in one of two ways: (1) If the decimal is one of the familiar aliquot or aliquant parts, use it in its common fraction form as illustrated in the chapter on Aliquot Parts. (2) In all other cases solve as follows:

Multiply 425 by .217.

$$\begin{array}{r} 425 \\ .217 \\ \hline 2975 \\ 8925 \\ \hline 92.225 \end{array}$$

Perform the multiplication in the same manner as for simple numbers, and point off as many places in the product as there are places in either or both the multiplier and multiplicand.

211. Multiply: Twice

1. 48 by $.16\frac{2}{3}$
2. 93 by .123
3. 133 by .416
4. 216 by .75
5. 874 by .613
6. 838 by .456
7. 971 by .375
8. 1240 by .625
9. 368 by .649
10. 487 by .1245
11. 696 by .875
12. 389 by .0625
13. 496 by .083
14. 979 by .005
15. 845 by .001
16. 859 by .0101
17. 59 by .101
18. 84 by .0125
19. 96 by .125
20. 88 by .09
21. 138 by .0001
22. 4585 by .01
23. 7345 by .003
24. 250 by .004

The student should observe carefully that on multiplying by such decimals as .1, .01, .001, .0001, and so on, the product contains the same figures as the multiplicand, with as many places pointed off as there are places in the decimal. Thus, multiply 4789 by .001. The product is 4.789, the same order of digits, with 3 places pointed off.

Write the answers to the following:

25. 8432×.0001
26. 9678×.1
26. 8439×.001
28. 9637×.00001
29. 45000×.01
30. 64873×.00001
31. 069478×.000001
32. 83902×.01
33. 84800×.001
34. 12489×.01
35. 34876×.0001
36. 48749×.01
37. 87634×.001
38. 85634×.1
39. 7964×.0001

212. To find the product of an integer and a mixed number.

1. At $\$\frac{1}{2}$ a yard, how much will 4 yd. of cloth cost?
2. At \$1 a yard, how much will 4 yd. of cloth cost?

Compare the cost of 4 yd. at $\$\frac{1}{2}$ a yard and 4 yd. at \$1 a yard, with the cost of 4 yd. at $\$1\frac{1}{2}$ a yard.

3. At \$3 a yard, how much will $\frac{1}{2}$ a yard of silk cost? How much will 6 yd. cost? How much will $6\frac{1}{2}$ yd. cost?

4. How many are $6\times4\frac{1}{2}$? $8\times3\frac{1}{4}$? $9\times5\frac{1}{3}$? $3\frac{1}{2}\times8$? $7\frac{1}{5}\times10$? $9\frac{1}{4}\times4$? $6\frac{1}{3}\times6$?

213. 1. Multiply 864 by $13\frac{3}{8}$.

$$\begin{array}{r} 864 \\ 13\frac{3}{8} \\ \hline 324 \\ 2592 \\ 864 \\ \hline 11556 \end{array}$$

First multiply mentally by the fraction $\frac{3}{8}$; thus, $\frac{1}{8}$ of 864 is 108; 3 × 108 = 324. Next, multiply by the whole number, 13, and add the results.

2. Multiply $231\frac{2}{3}$ by 43.

$$\begin{array}{r} 231\frac{2}{3} \\ 43\phantom{\frac{2}{3}} \\ \hline 3)86\phantom{\frac{2}{3}} \\ \hline 28\frac{2}{3} \\ 693\phantom{\frac{2}{3}} \\ 924\phantom{0\frac{2}{3}} \\ \hline 9961\frac{2}{3} \end{array}$$

First, multiply the fraction $\frac{2}{3}$ by 43. If the integer, 43, is not exactly divisible by the denominator of the fraction, multiply the numerator of the fraction by 43; 43 × 2 = 86. Divide 86 by 3, the denominator of the fraction, obtaining $28\frac{2}{3}$. Then multiply 231 by 43, and add results.

3. Multiply 583 by $31.64\frac{3}{4}$.

$$\begin{array}{r} 31.64\frac{3}{4} \\ 583\phantom{\frac{3}{4}} \\ \hline 4)1749\phantom{\frac{3}{4}} \\ \hline 437\frac{1}{4} \\ 9492\phantom{\frac{3}{4}} \\ 25312\phantom{0\frac{3}{4}} \\ 15820\phantom{00\frac{3}{4}} \\ \hline 18450.49\frac{1}{4} \end{array}$$

Note that the multiplicand, 583, is used as the multiplier for the purpose of shortening the work (Prin. 13, page 117). First multiply 583 by the numerator of the fraction, and divide the product by the denominator, as in the preceding example. Multiply 31.64 by 583 the same as though 31.64 were a whole number, and add the results. In the product point off 2 places, for the 2 places in the number $31.64\frac{3}{4}$.

214. Find the value of:

#

1. $10\times8\frac{1}{2}$
2. $12\times4\frac{2}{3}$
3. $16\times8\frac{3}{4}$
4. $24\times5\frac{3}{8}$
5. $33\times11\frac{5}{6}$
6. $76\times51\frac{3}{5}$
7. $21\times7\frac{3}{7}$
8. $67\times18\frac{4}{9}$
9. $49\times88\frac{3}{8}$
10. $83\times31\frac{2}{3}$
11. $59\times83\frac{3}{5}$
12. $38\times17\frac{5}{7}$

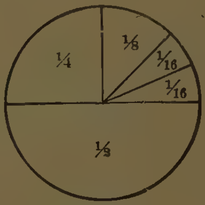
160

16,0

176

13. $16\frac{3}{5} \times 28$
14. $18\frac{3}{4} \times 115$
15. $14\frac{5}{12} \times 93$
16. $15\frac{6}{7} \times 99$
17. $24\frac{3}{14} \times 50$
18. $89\frac{11}{15} \times 160$
19. $39\frac{3}{11} \times 138$
20. $58\frac{4}{7} \times 44$
21. $91\frac{15}{16} \times 110$
22. $38\frac{3}{16} \times 92$
23. $47\frac{7}{9} \times 86$
24. $69\frac{3}{17} \times 85$
25. $67 \times 4.13\frac{3}{8}$
26. $87 \times .312\frac{1}{3}$
27. $34 \times 4.37\frac{5}{6}$
28. $39 \times 3.82\frac{3}{4}$
29. $98 \times 74.66\frac{2}{3}$
30. $38.13\frac{3}{7} \times 45$
31. $56.42\frac{3}{11} \times 66$
32. 3.123×86
33. $71 \times 16.10\frac{2}{3}$
34. $5.91\frac{2}{3} \times 84$
35. $8.31\frac{1}{4} \times 34$
36. $19.36\frac{3}{16} \times 59$

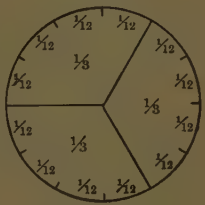
215. To multiply a fraction by a fraction.



1. How much is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{8}$?

2. If a yard of cloth costs $\frac{2}{3}$ of a dollar, how much will $\frac{1}{2}$ of a yard cost?

216. 1. What is $\frac{3}{4}$ of $\frac{2}{3}$?



Let the circle represent a unit divided into thirds. If each third is divided into four equal parts, the circle will be divided into 12 equal parts, or twelfths. $\frac{2}{3} = \frac{8}{12}$. $\frac{3}{4}$ of $\frac{2}{3}$, then, is the same as $\frac{3}{4}$ or $\frac{8}{12}$, which is equivalent to $\frac{3}{4}$ of 8 parts. $\frac{1}{4}$ of 8 parts is 2 parts; 3 × 2 parts is 6 parts, or $\frac{6}{12} = \frac{1}{2}$. Therefore, $\frac{3}{4}$ of $\frac{2}{3} = \frac{1}{2}$.

Instead of all the work indicated in this explanation, which has been given to make clear the principles involved, the solution may be made very short. Observe the following.

$$\frac{3}{4} \text{ of } \frac{2}{3} = \frac{\cancel{3}\times\cancel{2}}{\underset{2}{\cancel{4}}\times\cancel{3}} = \frac{1}{2}.$$

By applying cancellation, the work of the solution is reduced to a minimum.

Some problems do not admit of cancellation. For instance, in finding the value of $\frac{5}{6}$ of $\frac{7}{8}$, cancellation cannot be applied. The solution consists in multiplying together the numerators for the numerator of the result, and multiplying together the denominators for the denominator of the result.

The expression is equivalent to $\frac{5\times 7}{6\times 8} = \frac{35}{48}$.

2. Multiply .43 by .032.

$$\begin{array}{r} .43 \\ .032 \\ \hline 86 \\ 129 \\ \hline .01376 \end{array}$$

Multiply as in whole numbers, pointing off as many places in the product as there are places in both multiplicand and multiplier.

217. Find the products:

1. $\frac{2}{3}$ of $\frac{9}{10}$
2. $\frac{5}{8}$ of $\frac{4}{5}$
3. $\frac{7}{9}$ of $\frac{9}{14}$
4. $\frac{5}{6}$ of $\frac{11}{15}$
5. $\frac{3}{4}$ of $\frac{7}{8}$
6. $\frac{11}{12}$ of $\frac{3}{4}$
7. $\frac{7}{15}$ of $\frac{9}{11}$
8. $\frac{8}{9}$ of $\frac{11}{12}$
9. $\frac{7}{10}$ of $\frac{15}{28}$
10. $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{2}{3}$
11. $\frac{9}{10}$ of $\frac{5}{6}$ of $\frac{6}{7}$
12. $\frac{7}{3}$ of $\frac{8}{9}$ of $\frac{9}{14}$
13. $\frac{15}{16}$ of $\frac{32}{45}$ of $\frac{90}{128}$
14. $\frac{11}{12}$ of $\frac{6}{7}$ of $\frac{1}{2}$
15. $\frac{3}{5}$ of $\frac{10}{21}$ of $\frac{11}{15}$
16. $\frac{5}{6}$ of $\frac{3}{8}$ of $\frac{9}{16}$
17. $\frac{9}{10}$ of $\frac{8}{15}$ of $\frac{45}{56}$
18. $\frac{7}{12}$ of $\frac{24}{35}$ of $\frac{15}{16}$
19. .4123 × .814
20. .963 × .0001
21. 8.24 × .001
22. .4237 × 1.001
23. .954 × 10.11
24. 76.98 × 11.11
25. 8.329 × 8.96
26. 4.73 × 100.10
27. 89.48 × 3.824

218. To multiply a mixed number by a fraction.

1. What is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{2}$ of 4? Add the two answers.
2. What is $\frac{1}{2}$ of $4\frac{1}{2}$? $\frac{1}{3}$ of $6\frac{1}{2}$? $\frac{1}{4}$ of $8\frac{1}{4}$?
3. What is $\frac{2}{3}$ of $6\frac{1}{2}$? $\frac{3}{4}$ of $12\frac{1}{2}$? $\frac{4}{5}$ of $20\frac{1}{2}$?

219. Find the value of $\frac{7}{8}$ of $313\frac{4}{5}$.

$$\frac{7}{\underset{2}{\cancel{8}}} \text{ of } \frac{\cancel{4}}{5} = \frac{7}{10}$$

$$\begin{array}{l} \frac{7}{8} \text{ of } 313 = \frac{2191}{8} = 273\frac{7}{8} \\ \quad 273\ \frac{7}{8} = \frac{35}{} \\ \quad\quad\ \ \frac{7}{10} = \frac{28}{} \\ \hline \quad 274\frac{23}{40} \quad \frac{63}{40} = 1\frac{23}{40} \end{array}$$

First, multiply the fractions together, obtaining $\frac{7}{10}$.

Next, take $\frac{7}{8}$ of 313, which gives $273\frac{7}{8}$. (For explanation, see page 96.)

Now add $\frac{7}{10}$ and $273\frac{7}{8}$, and the desired result is obtained.

220. Find the products:

1. $\frac{3}{5} \times 18\frac{8}{9}$
2. $\frac{4}{7} \times 33\frac{3}{8}$
3. $\frac{9}{11} \times 16\frac{4}{7}$
4. $\frac{8}{9} \times 39\frac{4}{5}$
5. $\frac{5}{8} \times 91\frac{3}{7}$
6. $\frac{7}{9} \times 88\frac{4}{7}$
7. $\frac{9}{14} \times 84\frac{7}{10}$
8. $\frac{7}{12} \times 127\frac{5}{8}$
9. $\frac{6}{7} \times 148\frac{7}{8}$
10. $\frac{3}{4} \times 131\frac{3}{5}$
11. $48\frac{7}{8} \times \frac{9}{10}$
12. $68\frac{7}{9} \times \frac{3}{4}$

13. $137\frac{3}{4} \times \frac{6}{7}$
14. $246\frac{7}{8} \times \frac{5}{6}$
15. $337\frac{4}{5} \times \frac{3}{4}$
16. $498\frac{3}{7} \times \frac{2}{3}$
17. $468\frac{15}{16} \times \frac{3}{4}$
18. $483\frac{3}{11} \times \frac{4}{5}$
19. $591\frac{4}{5} \times \frac{4}{5}$
20. $563\frac{2}{3} \times \frac{3}{4}$
21. $\frac{3}{16} \times 416\frac{3}{8}$
22. $\frac{5}{9} \times 382\frac{3}{4}$
23. $\frac{7}{11} \times 438\frac{2}{3}$
24. $\frac{5}{6} \times 938\frac{3}{4}$
25. $\frac{3}{4} \times 414\frac{5}{8}$
26. $\frac{7}{8} \times 214\frac{2}{3}$
27. $\frac{3}{8} \times 328\frac{2}{5}$
28. $\frac{4}{5} \times 564\frac{5}{8}$
29. $\frac{7}{12} \times 366\frac{6}{7}$
30. $\frac{9}{10} \times 455\frac{5}{9}$

221. To multiply a mixed number by a mixed number.

Find the value of $31\frac{3}{4} \times 17\frac{5}{8}$.

$31\frac{3}{4}$
$17\frac{5}{8}$
$\frac{15}{32}$, 15
$19\frac{3}{8}$, 12
$12\frac{3}{4}$, 24
527
$559\frac{19}{32}$ $\quad \frac{51}{32} = 1\frac{19}{32}$

1. $\frac{5}{8}$ of $\frac{3}{4} = \frac{15}{32}$
2. $\frac{5}{8}$ of $31 = 19\frac{3}{8}$
3. $17 \times \frac{3}{4} = 12\frac{3}{4}$
4. $17 \times 31 = 527$

This solution is what is known as the "four-step process" of multiplying mixed numbers. For clearness, the four steps are written out in full. The student should observe these four steps very carefully: (1) $\frac{5}{8}$ of $\frac{3}{4} = \frac{15}{32}$, which is written as a part of the product. (2) $\frac{5}{8}$ of $31 = 19\frac{3}{8}$, which is written as in the solution. (3) $17 \times \frac{3}{4} = 12\frac{3}{4}$, which is written under the $19\frac{3}{8}$. (4) $17 \times 31 = 527$. Add the several results. The final result is $559\frac{19}{32}$.

Multiply $29\frac{1}{4}$ by $15\frac{1}{4}$.

$29\frac{1}{4}$
$15\frac{1}{4}$
$11\frac{1}{16}$
435
$446\frac{1}{16}$

1. $\frac{1}{4}$ of $\frac{1}{4} = \frac{1}{16}$
2. $\left\{\begin{matrix} \frac{1}{4} \text{ of } 29 \\ \frac{1}{4} \text{ of } 15 \end{matrix}\right\}$ $\frac{1}{4}$ of $44 = 11$
3. $15 \times 29 = 435$

It is frequently possible to shorten the "four-step process" as illustrated in this exercise. In all cases in which the fractions are alike, the integral part of the multiplier and the multiplicand may be added and their sum multiplied by *one* of the fractions. "Steps" 2 and 3 are combined into one "step."

222. Multiply:

1. $15\frac{3}{8} \times 16\frac{1}{4}$
2. $18\frac{3}{4} \times 26\frac{1}{2}$
3. $27\frac{5}{6} \times 34\frac{3}{5}$
4. $37\frac{3}{7} \times 29\frac{4}{7}$
5. $38\frac{3}{4} \times 34\frac{1}{4}$
6. $56\frac{4}{9} \times 9\frac{3}{4}$
7. $4\frac{1}{2} \times 9\frac{3}{8}$
8. $76\frac{3}{4} \times 12\frac{1}{2}$
9. $89\frac{4}{5} \times 17\frac{1}{2}$
10. $93\frac{3}{5} \times 27\frac{3}{4}$
11. $87\frac{3}{8} \times 29\frac{7}{9}$
12. $91\frac{1}{2} \times 41\frac{1}{3}$
13. $47\frac{2}{3} \times 38\frac{5}{6}$
14. $93\frac{3}{4} \times 86\frac{2}{3}$
15. $74\frac{3}{4} \times 19\frac{1}{5}$

16. $88\frac{4}{5}\times 55\frac{5}{8}$
17. $1.34\frac{3}{8}\times .19\frac{4}{7}$
18. $2.28\frac{4}{5}\times .67\frac{3}{4}$
19. $2.14\frac{5}{16}\times .48\frac{1}{4}$
20. $.246\frac{4}{9}\times .96\frac{3}{4}$
21. $3.24\frac{5}{8}\times .56\frac{4}{5}$
22. $6.32\frac{2}{3}\times .16\frac{2}{3}$
23. $4.56\frac{3}{5}\times .59\frac{3}{5}$
24. $5.98\frac{3}{4}\times 1.22\frac{3}{4}$
25. $8.46\frac{5}{8}\times 1.38\frac{5}{8}$
26. $9.66\frac{2}{3}\times 1.44\frac{2}{7}$
27. $8.62\frac{1}{2}\times 2.33\frac{1}{3}$

Find the total value of each of the following:

Fractions, when not expressed in full, are to be considered as fourths Thus, $\$.07^2=\$.07\frac{2}{4}$.

28.

27^3 yd. at $\$.07^2$.
31^1 yd. at $.08^3$.
45^2 yd. at $.09^1$.
56^3 yd. at $.07^3$.
38^2 yd. at $.06^1$.
51^2 yd. at $.07^3$.

29.

35^1 yd. at $\$.07^1$.
43^3 yd. at $.08^1$.
72^1 yd. at $.05^2$.
53^3 yd. at $.06^2$.
58^2 yd. at $.04^3$.
62^1 yd. at $.07^1$.

30.

64^1 yd. at $\$.08^2$.
76^2 yd. at $.07^3$.
48^3 yd. at $.08^2$.
39^2 yd. at $.04^3$.
17^2 yd. at $.12^3$.
19^3 yd. at $.13^3$.

31. total

72^3 yd. at $\$.10^1$.
42^2 yd. at $.09^2$.
63^1 yd. at $.07^2$.
16^3 yd. at $.15^2$.
22^2 yd. at $.18^3$.
31^3 yd. at $.23^2$.

32.

21^3 yd. at $\$.07^2$.
34^3 yd. at $.06^3$.
56^2 yd. at $.08^2$.
47^1 yd. at $.09^1$.
53^3 yd. at $.07^3$.
29^1 yd. at $.07^1$.

33.

44^1 yd. at $\$.08^1$.
29^3 yd. at $.07^1$.
64^2 yd. at $.07^3$.
29^3 yd. at $.05^2$.
12^2 yd. at $.26^1$.
14^3 yd. at $.35^2$.

34.

59^2 yd. at $\$.05^2$.
39^3 yd. at $.09^3$.
45^1 yd. at $.06^2$.
73^2 yd. at $.08^2$.
67^3 yd. at $.07^2$.
83^1 yd at 09^1.

35.

26^1 yd. at $\$.07^2$.
32^3 yd. at $.07^2$.
49^2 yd. at $.08^3$.
38^2 yd. at $.22^2$.
59^3 yd. at $.33^2$.
47^2 yd at $.17^2$.

36.

32^2 yd. at $\$.06^2$.
54^1 yd. at $.07^1$.
69^3 yd. at $.09^3$.
57^1 yd. at $.07^1$.
48^2 yd. at 06^2.
27^3 yd. at $.05^3$.

MENTAL MULTIPLICATION OF MIXED NUMBERS

223. When each fraction is $\frac{1}{2}$.

1. Multiply $6\frac{1}{2}$ by $6\frac{1}{2}$.

$$(a)\quad \begin{array}{r} 6\frac{1}{2} \\ \underline{6\frac{1}{2}} \\ 3\frac{1}{4} \\ 3 \\ \underline{36} \\ 42\frac{1}{4} \end{array} \qquad (b)\quad \begin{array}{r} 6\frac{1}{2} \\ \underline{6\frac{1}{2}} \\ 42\frac{1}{4} \end{array}$$

In (*a*) the complete "four-step" solution (page 99) is given. Note that the sum of the two cross products (3 and 3) is 6. Hence, instead of taking 6 × 6, and adding 6, add 1 to the multiplicand, and take 6 × 7, as in (*b*).

2. Multiply 9.5 by 9.5.

$$\begin{array}{r} 9.5 \\ \underline{9.5} \\ 90.25 \end{array}$$

As in (*b*), example 1, .5 × .5 = .25. Add 1 to one of the 9's and multiply by the other 9, and obtain 90. Hence, 9.5 × 9.5 = 90.25.

224. Multiply (mentally):

1. $8\frac{1}{2}$ by $8\frac{1}{2}$
2. $7\frac{1}{2}$ by $7\frac{1}{2}$
3. 3.5 by 3.5
4. 4.5 by 4.5
5. $2\frac{1}{2}$ by $2\frac{1}{2}$
6. $1\frac{1}{2}$ by $1\frac{1}{2}$
7. $10\frac{1}{2}$ by $10\frac{1}{2}$
8. 11.5 by 11.5
9. $13\frac{1}{2}$ by $13\frac{1}{2}$
10. 12.5 by 12.5
11. $5\frac{1}{2}$ by $5\frac{1}{2}$
12. 6.5 by 6.5
13. $9\frac{1}{2}$ by $9\frac{1}{2}$
14. $15\frac{1}{2}$ by $15\frac{1}{2}$
15. $20\frac{1}{2}$ by $20\frac{1}{2}$

225. 1. Multiply $7\frac{1}{2}$ by $5\frac{1}{2}$.

$$\begin{array}{r} 7\frac{1}{2} \\ \underline{5\frac{1}{2}} \\ 41\frac{1}{4} \end{array}$$

$\frac{1}{2}$ of 7 + $\frac{1}{2}$ of 5 = $\frac{1}{2}$ of (7 + 5) = 6. Take 5 × 7, and add the 6, which gives $41\frac{1}{4}$ as the product.

2. Multiply $9\frac{1}{2}$ by $4\frac{1}{2}$.

$$\begin{array}{r} 9\frac{1}{2} \\ \underline{4\frac{1}{2}} \\ 42\frac{3}{4} \end{array}$$

The sum of the integers (4 + 9 = 13) being an odd number, $\frac{1}{2}$ of the sum gives a mixed number, $6\frac{1}{2}$. $\frac{1}{2}$ of $\frac{1}{2}$ = $\frac{1}{4}$, to which is added the $\frac{1}{2}$ of the mixed number ($6\frac{1}{2}$), giving $\frac{3}{4}$, the fractional part of the product. Now take 4 times 9, and add the 6, obtaining $42\frac{3}{4}$, the desired result.

In problems like the last two examples, observe first whether the sum of the integers is even or odd. If even, write as the first part of the product $\frac{1}{4}$, .25, or 25, as the case may require; if odd, write $\frac{3}{4}$, .75, or 75. For the remainder of the product multiply the integers together, and add the integral part of half their sum. (Compare with Art. 107.)

When the fractions are alike.

3. Multiply $8\frac{2}{3}$ by $5\frac{2}{3}$.

$$\begin{array}{r} 8\frac{2}{3} \\ \underline{5\frac{2}{3}} \\ \frac{4}{9} \\ 8\frac{2}{3} \\ \underline{40} \\ 49\frac{1}{9} \end{array}$$

First multiply $\frac{2}{3}$ by $\frac{2}{3}$. Next find $\frac{2}{3}$ of 13, (8 + 5), which is $8\frac{2}{3}$. Then find 5 × 8, which is 40. It should be observed that although $\frac{2}{3}$ of 13 gives a mixed number, $8\frac{2}{3}$, the solution is shorter than by the other method. Now there are only two fractions to be added, instead of three, as there would be if 8 and 5 were multiplied separately.

The student will be surprised at the rapidity with which he will be able to make multiplications like the above examples, after a little practice.

226. Multiply (mentally):

1. $6\frac{1}{2}\times10\frac{1}{2}$
2. $8\frac{1}{3}\times7\frac{1}{3}$
3. $5\frac{3}{4}\times11\frac{3}{4}$
4. 8.25×4.25
5. $6\frac{1}{3}\times9\frac{1}{3}$
6. $7\frac{1}{2}\times9\frac{1}{2}$
7. 11.25×9.25
8. $7\frac{2}{3}\times8\frac{2}{3}$
9. $10\frac{1}{5}\times15\frac{1}{5}$
10. 11.5×8.5
11. $14\frac{1}{4}\times8\frac{1}{4}$
12. $5\frac{3}{5}\times7\frac{3}{5}$
13. $12\frac{3}{4}\times9\frac{3}{4}$
14. $14\frac{1}{3}\times7\frac{1}{3}$
15. $15\frac{3}{7}\times6\frac{3}{7}$
16. $18\frac{2}{5}\times7\frac{2}{5}$
17. $16\frac{2}{3}\times8\frac{2}{3}$
18. $11\frac{3}{4}\times7\frac{3}{4}$
19. $13\frac{2}{3}\times2\frac{2}{3}$
20. 15.75×1.75
21. $14\frac{4}{7}\times4\frac{4}{7}$

227. Find the total value of each of the following groups:

1.

65 crates at $6.50.
85 crates at 7.50.
4½ bu. at .75.
9½ bu. at .85.

2.

10½ lb. at $.75.
5½ lb. at .65.
7½ lb. at .07½.
3½ lb. at .15½

3.

12½ A. at $125.
15½ A. at 155.
16½ A. at 85.
20½ A. at 75.

4.

5¼ yd. at $.07¼.
9¼ yd. at .18¼.
7¼ yd. at .09¼.
13¼ yd. at .07¼.

5.

8¾ lb. at $.08¾.
9¾ lb. at .11¾.
5¾ lb. at 3.75.
12¾ lb. at 4.75.

6.

15½ bu. at $1.50.
350 lb. at 1.50 per cwt.
2500 ft. at 18.50 per M.
4500 ft. at 6.50 per M.

228. The 44 hour week (8 hr. daily from Monday to Friday inclusive, and 4 hr. on Saturday) is fast becoming the standard week of labor.

The scale of wages is based on the hourly or daily rate. The day is thought of as beginning at 8 A.M. and ending at 5 P.M., with 1 hr. for luncheon, except Saturday, when the day ends at 12 M.

Hours of labor between 8 A.M. and 5 P.M. (or between any other regularly prescribed hours of beginning and ending the day of labor, as is done in certain kinds of business) is *regular time.* Hours of labor after 5 P.M. (or other prescribed closing hour) is generally considered *overtime.*

Overtime is usually paid for at a higher rate than regular time. The extra wage may be a quarter, or a half greater than, or even double, the regular wage. It is expressed as *time and a quarter*, *time and a half*, or *double time.* In terms of figures that means that a man who receives 60¢ an hour regular time would receive for 1 hour over time, 75¢ if he was allowed time and a quarter ($1\frac{1}{4}\times 60\text{¢}=75\text{¢}$); 90¢ if he was allowed time and a half ($1\frac{1}{2}\times 60\text{¢}=90\text{¢}$), etc.

The following illustration will show how an employee's weekly wages are determined.

	M	T	W	Th	F	S	Hours		Rate	Wages		
							Reg. Time	Over Time		Reg. Time	Over Time	Total
Anson, John	8	9	10	5	7	4	40	3	60c	24.00	2.70	26.70

Regular time and overtime are reckoned separately. The regular time for Tuesday and Wednesday is 8 hr. for each day. Hence the total regular time is 8 hr.+8 hr.+8 hr.+5 hr.+7 hr +4 hr.=40 hr. The overtime is 1 hr. on Tuesday and 2 hr. on Wednesday, or a total of 3 hr. The rate (wage per hour) being 60¢, the regular wages amount to 40×60¢ or $24.00. It is assumed, in this problem, that time and a half is allowed for overtime. Since the regular rate is 60¢, the overtime rate is 90¢ per hour. The overtime wages amount to 3×90¢, or $2.70. The total wages are $26.70.

In many kinds of business the money due an employee for wages is put into a "pay envelope" and handed to him at the close of the week's work. The number and kinds of bills and coins required are determined by means of a "currency break-up" or change sheet. For the wages in the above problem, together with the wages of several other employees, the currency break-up would appear as follows:

No.	Amt.	$20	$10	$5	$2	$1	50c	25c	10c	5c	1c
1.	$26.70	1		1		1	1		2		
2.	27.18	1		1	1				1	1	3
3.	31.23	1	1			1			2		3
4.	29.87	1		1	2		1	1	1		2
5.	34.69	1	1		2		1		1	1	4
	149.67	5	2	3	5	2	3	1	7	2	12
		100	20	15	10	2	1.50	.25	.70	.10	.12

Having thus determined the correct amount of change required, the result is copied on a *currency memorandum*, and sent to the bank with a pay-roll check. In cashing the check the paying teller provides the exact amount of change called for. The payroll clerk is then able to fill the pay envelopes without delay.

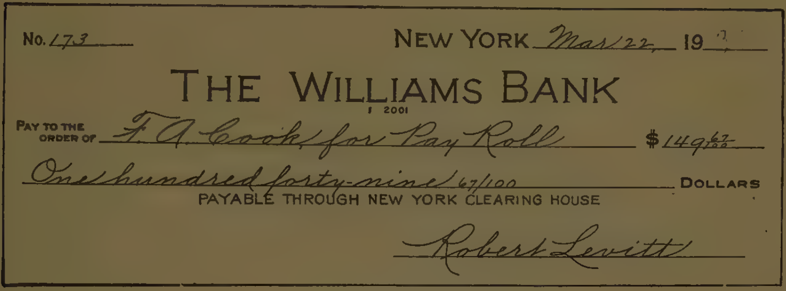
THE COLONIAL BANK

CURRENCY MEMORANDUM

NEW YORK, *Mar. 22, 1924*

DEPOSITOR *Robert Levitt*

	DOLLARS	CTS
Bills $ 1	*2*	
" 2	*10*	
" 5	*15*	
" 10	*20*	
" 20	*100*	
" 50		
" 100		
Coin: Pennies		*12*
" Nickels		*10*
" Dimes		*70*
" Quarters		*25*
" Halves	*1*	*50*
" Dollars		
" Gold		
Total	*149*	*67*



No. 173 NEW YORK Mar 22, 19

THE WILLIAMS BANK

1 2001

PAY TO THE ORDER OF F. A. Cook, for Pay Roll $149 67/100

One hundred forty-nine 67/100 DOLLARS

PAYABLE THROUGH NEW YORK CLEARING HOUSE

Robert Levitt

. Pay-Roll Check

229. **1.** The following time sheet is based on a 44-hour week. Allow time and a half for overtime. Using the form shown on page 103 find the total wages for each employee. Prepare the currency break-up, a currency memorandum, and write the pay-roll check.

No.	M	T	W	Th	F	S	Hours		Rate	Wages		
							Reg. Time	Over Time		Reg. Time	Over Time	Total
1.	8	7	7½	6	9	4			.60			
2.	7	8½	6	9	9	5			.60			
3.	9	10	7	6	7½	6½			.64			
4.	8	8	7	8	8	6			.62			
5.	9	7	6	4½	8	5			.50			
6.	10	8	7	9	8	4			.60			
7.	9	7	8	6	9	4			.50			
8.	8½	7	6	9½	8½	7½			.64			

2. In the following time sheet the rate is for an 8-hour day, (44-hour week) with time and a quarter for overtime. Prepare the payroll, currency memorandum and check, using the above forms.

No.	Names of Employees	M	T	W	Th	F	S	Rate
1.	Addison, James	8	10	9	11	2	4	6.40
2.	Buell, Henry	9	6	9	8½	9	4	6.00
3.	Crane, John	10	9½	8	7	8	3	6.00
4.	Crawford, Wm.	9	9	8	6	10	4	6.00
5.	Jones, Albert	10	8	9½	9	0	4	6.00
6.	Landis, Frank	8	4	9	8	10	4	5.00

3. Find the total time and the wages for the week. Make no allowance for overtime.

Form No. 3

No.........18........

Name........*William MacCormack*................................

Total Time.........................Hr. Rate, 55¢.

Total Wages.......................................$.......

Week Ending..........*January 7*...........................19.....

In	Out	In	Out	In	Out
7.50	11.30	12.20	5.30		
		12.30	5.30	6.45	11.30
7.45	11.30	12.45	5.45		
7.55	12.15			6.10	11.15
		12.00	5.40	6.30	11.20
8.00	12.20	12.50	6.10		
8.30	12.15	1.30	6.10		

4. A gang of carpenters working under the 8-hour law, with a maximum day of 10 hours, and one-half extra pay for overtime, worked one week as follows.

Time Sheet. Week Ending March 22, 1924.

Names	M.	T.	W.	Th.	F.	S.	Total Time	Rate per Day
Bigelow, A. E. (foreman)	8	8½	8	9	8	10		$7.50
Arnold, A. W.	8	8	8	8	8	9		7.00
Babcock, C. D.	8	8	8	9	8	8		7.00
Comstock, L. O. . . .	8	8	8	8½	8	9		7.00
Evans, M. R.	8	8	8	8½	8	8		7.00
Geraghty, P. O. . . .	8	8½	8	8½	8	9		7.00
Morrow, G. W.	8	8¼	8	8	8	8		7.00
Peters, A. B.	8	8½	8	8¼	8	8		6.00
Rogers, H. F.	8	8¼	8	8	8	9		6.00
Simler, H.	8	8	8	8	8	8		6.00

Prepare a pay roll for the gang.

230. Instead of working by the day or the hour, some employees are paid by the piece. In that case their wages vary in proportion to the amount of work done. To find the amount of daily or weekly wages is simply to multiply the price per piece by the number of pieces of work completed.

In some factories the piecework plan is modified so that a rapid or skilled worker receives a higher price per piece than a slower or unskilled worker receives. It may be that it has been found from experience that the completion of from 32 to 35 pieces is a standard or average day's work, and that the manufacturer can afford to pay 15¢ apiece for the labor. To provide a slow worker with tools and machinery, heat, light, and factory space in which to work, costs just as much as it does to provide for a fast worker. Assume that the overhead expense averages $1 a day for each employee. Then if one worker completes 25 articles per day, the overhead expense per article is 4¢; if he completes 33 articles per day, the overhead is 3¢ each; and if he completes 40 articles per day, the overhead is but 2½¢ each.

Therefore, to equalize cost of production per article, a "differential rate" is paid. A slow worker receives less, and a fast worker receives more, per piece.

1. Using the following schedule of rates, find the daily wages and the total wages for the week, for each employee:

PIECEWORK PAY ROLL

NAMES	NUMBER OF PIECES					
	M.	T.	W.	TH.	F.	S.*
Brown, John	33	31	34	28	36	20
Carey, Leon	28	30	27	32	33	18
Dunne, Theo.	24	26	29	27	30	15
Evers, Mark	34	36	40	38	39	21

SCHEDULE OF RATES

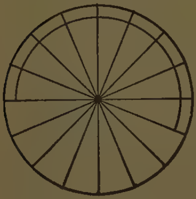
PIECES PER DAY	PRICE PER PIECE
20 to 23	13½¢
24 to 27	14¢
28 to 31	14½¢
32 to 35 (Standard)	15¢
36 to 39	15½¢
40 to 43	16¢
44 to 47	16½¢

*Four hours only. Use the rate for twice the number of pieces finished.

DIVISION OF FRACTIONS

231. To divide a fraction by an integer.

Into how many parts is this circle divided? What part of the circle is marked off by the curved line? If these $\frac{9}{16}$ are divided into 3 equal parts, how many sixteenths will there be in each part?



$$\tfrac{9}{16} \div 3 = \tfrac{3}{16}.$$

How does the fraction $\frac{3}{16}$ compare in value with the fraction $\frac{9}{16}$? Formulate a second way of dividing a fraction by an integer. (Prin. 17, page 117.)

232. **1.** Divide $\frac{5}{8}$ by 3. $\frac{5}{8} \div 3 = \frac{5}{24}$.

Multiply the denominator of the fraction by the integer, 3, and write the numerator, 5, over the product. This divides the fraction, because multiplying the divisor (denominator) divides the quotient (which, in this case is the fraction). (Prin. 17, page 117.)

2. Divide $\frac{6}{11}$ by 2. $\frac{6}{11} \div 2 = \frac{3}{11}$.

Divide the numerator of the fraction by the integer, 2, and write the result over the denominator, 11. This divides the fraction, because dividing the dividend (numerator) divides the quotient (the fraction). (Prin. 20, page 117.)

This solution should always be used when the integer is an exact divisor of the numerator of the fraction.

3. Find the value of $.0344 \div 8$.

$$\begin{array}{r} .0043 \\ 8\overline{).0344} \end{array}$$

The first thing to do when the divisor is a whole number, as in this case, is to place the point for the quotient. Place it directly above the point in the dividend, as shown in the example. By dividing thus, "8 is contained in 0, 0 times; 8 in 3, 0 times; 8 in 34, 4 times, etc."; placing the figures of the quotient above the figures of the dividend, as here shown, the position of the decimal point in the quotient is always right.

233. Find the value of:

1. $\frac{6}{7} \div 3$
2. $\frac{8}{9} \div 4$
3. $\frac{5}{8} \div 3$
4. $\frac{9}{10} \div 6$
5. $.0125 \div 5$
6. $.464 \div 8$
7. $\frac{7}{8} \div 3$
8. $\frac{9}{16} \div 3$
9. $\frac{11}{15} \div 3$
10. $.968 \div 4$
11. $.8464 \div 16$
12. $.0035 \div 5$
13. $\frac{7}{20} \div 5$
14. $\frac{10}{17} \div 8$
15. $\frac{11}{18} \div 9$
16. $\frac{24}{31} \div 6$

17. $.3125 \div 25$
18. $.738 \div 9$
19. $.001 \div 100$
20. $.00128 \div 16$
21. $\frac{19}{21} \div 4$
22. $\frac{15}{26} \div 5$
23. $\frac{11}{4} \div 5$
24. $\frac{25}{6} \div 5$
25. $\frac{19}{8} \div 3$
26. $\frac{31}{32} \div 6$
27. $\frac{43}{8} \div 7$
28. $.125 \div 50$
29. $.0125 \div 500$
30. $.8448 \div 8$
31. $\frac{159}{160} \div 3$
32. $\frac{249}{328} \div 83$
33. $\frac{896}{1024} \div 128$
34. $\frac{1089}{10} \div 121$
35. $\frac{1144}{12} \div 104$
36. $.00126 \div 63$
37. $.216 \div 24$
38. $.4888 \div 8$
39. $.03036 \div 11$
40. $.3069 \div 11$

234. To divide a mixed number by an integer.

If \$$1\frac{1}{2}$ is divided equally among 3 boys, how much will each receive? If 9 lb. of coffee cost \$$2\frac{1}{4}$, what part of a dollar does 1 lb. cost?

$1\frac{1}{2} \div 3 = ?$ $2\frac{1}{4} \div 9 = ?$ $2\frac{2}{3} \div 8 = ?$ $1\frac{3}{5} \div 4 = ?$

235. 1. Divide $5\frac{5}{8}$ by 7.

$$5\tfrac{5}{8} = \tfrac{45}{8}$$
$$\tfrac{45}{8} \div 7 = \tfrac{45}{56}$$

When the mixed number is small, change it to an improper fraction, and divide as in dividing a fraction by an integer.

2. Divide $3478\frac{2}{9}$ by 8.

$$\begin{array}{r} 434\frac{7}{9} \\ 8\overline{)3478\frac{2}{9}} \end{array}$$
$$6\tfrac{2}{9} = \tfrac{56}{9}$$
$$\tfrac{56}{9} \div 8 = \tfrac{7}{9}$$

In problems like this, in which the mixed number is much larger than the divisor, *do not* change the mixed number to an improper fraction. Divide the integral part of the dividend by the divisor as if there were no fraction in the dividend. 8 is contained in 3478, 434 times, with a remainder of 6, which, with the fraction $\frac{2}{9}$, makes a total remainder of $6\frac{2}{9}$. Dividing $6\frac{2}{9}$ by 8 as explained in the preceding example gives the completed quotient, $434\frac{7}{9}$.

The student should divide $6\frac{2}{9}$ by 8, and all similar expressions, mentally. It is written out in full here for the sake of clearness.

3. Divide 42.3 by 25.

$$\begin{array}{r} 1.692 \\ 25\overline{)42.300} \\ 25 \\ \hline 17\,3 \\ 15\,0 \\ \hline 2\,30 \\ 2\,25 \\ \hline 50 \\ 50 \\ \hline \end{array}$$

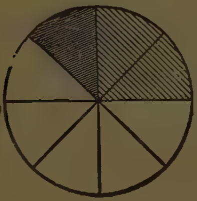
Place the point for the quotient directly above the point in the dividend. The important part, now, is to write the first figure of the quotient in the right place. Observe that 1, the first figure of the quotient, is written over the 2, the figure farthest to the right in the dividend, that is used to contain the divisor, 25. Continue the division, annexing ciphers in the dividend as needed, and placing the figures of the quotient over each succeeding figure in the dividend, respectively.

NOTE. If the figures of the quotient are placed correctly, the last, or right-hand figure, of the quotient will stand directly above the last figure of the dividend.

236. Find the value of:

1. $3\frac{3}{4} \div 5$
2. $6\frac{2}{3} \div 10$
3. $8\frac{3}{4} \div 7$
4. $3\frac{2}{3} \div 7$
5. $9\frac{4}{5} \div 14$
6. $18\frac{3}{4} \div 8$
7. $26\frac{7}{8} \div 4$
8. $47\frac{5}{9} \div 2$
9. $167\frac{8}{9} \div 21$
10. $568\frac{3}{7} \div 43$
11. $828\frac{3}{11} \div 69$
12. $939\frac{4}{15} \div 22$
13. $896\frac{4}{5} \div 15$
14. $948\frac{5}{8} \div 12$
15. $776\frac{7}{9} \div 9$
16. $3924\frac{5}{8} \div 12$
17. $4964\frac{3}{4} \div 11$
18. $6847\frac{5}{9} \div 96$
19. $3478\frac{3}{8} \div 84$
20. $9674\frac{5}{9} \div 99$
21. $8496\frac{7}{11} \div 64$
22. $4589\frac{7}{10} \div 5$
23. $896\frac{3}{11} \div 17$
24. $974\frac{3}{16} \div 11$
25. $897\frac{4}{5} \div 15$
26. $969\frac{3}{14} \div 7$
27. $896\frac{3}{20} \div 8$
28. $487\frac{5}{7} \div 10$
29. $389\frac{7}{12} \div 6$
30. $967\frac{3}{4} \div 5$
31. $478.336 \div 16$
32. $394.128 \div 8$
33. $48.3744 \div 24$
34. $9674.88 \div 48$
35. $364.64 \div 160$
36. $511.375 \div 125$
37. $895.4 \div 110$
38. $6.4872 \div 4$
39. $9.674\frac{2}{3} \div 8$
40. $4.592 \div 16$
41. $9.34\frac{1}{2} \div 12$
42. $6.43\frac{2}{3} \div 5$
43. $6.38\frac{7}{8} \div 9$
44. $9.72\frac{5}{9} \div 11$
45. $106.88\frac{8}{9} \div 15$

237. To divide an integer, a fraction, or a mixed number by a fraction.



How many times does the whole circle contain $\frac{1}{2}$ of the circle?

How many times does it contain $\frac{1}{4}$ of the circle? $\frac{1}{8}$ of the circle?

$$1 \div \tfrac{1}{2} = ? \quad 1 \div \tfrac{1}{4} = ? \quad 1 \div \tfrac{1}{8} = ?$$

If $\frac{1}{2}$ is contained in 1, 2 times, how many times is $\frac{1}{2}$ contained in 2? in 3? in 4? in 6? in 10?

$$1 \div \tfrac{1}{2} = 2 = \tfrac{2}{1}$$
$$3 \div \tfrac{1}{2} = 6 = \tfrac{6}{1}$$

Observe, in the first equation, that the answer, 2, or its equivalent, $\frac{2}{1}$, is the fraction $\frac{1}{2}$ inverted; that is, the denominator is written for the numerator, and the numerator for the denominator.

In the second equation, notice that the answer 6, or $\frac{6}{1}$, is 3 times as large as the answer in the first equation. That is, to divide by $\frac{1}{2}$ is to multiply by its reciprocal, 2.

In the same circle count off 3 eighths. How many times will these $\frac{3}{8}$ be contained in the circle?

Call the "eighths" "parts," and divide 8 parts (that is, the circle) by 3 parts. 8 parts ÷ 3 parts $= 2\frac{2}{3}$ times. That is, $1 \div \frac{3}{8} = 2\frac{2}{3}$. But $2\frac{2}{3} = \frac{8}{3}$, and $\frac{8}{3}$ is the fraction $\frac{3}{8}$ inverted.

The result of 1 divided by any fraction is that fraction inverted.

238. The reciprocal of a fraction is the fraction inverted.

239. The reciprocal of any number is 1 divided by that number. The reciprocal of 4 is $\frac{1}{4}$; of 8 is $\frac{1}{8}$; etc.

What is the reciprocal of $\frac{3}{4}$? of $\frac{5}{8}$? of $\frac{2}{3}$? of 10? of 12?

How may an integer be divided by a fraction?

To divide by a fraction, multiply by its reciprocal.

1. Divide $6\frac{2}{3}$ by $\frac{4}{5}$.

$$6\tfrac{2}{3} \div \tfrac{4}{5} = 6\tfrac{2}{3} \times \tfrac{5}{4} = \frac{\overset{5}{\cancel{20}}}{3} \times \frac{5}{\cancel{4}} = \frac{25}{3} = 8\tfrac{1}{3}.$$

To divide by $\frac{4}{5}$ is to multiply by its reciprocal, $\frac{5}{4}$. $6\frac{2}{3}$ multiplied by $\frac{5}{4}$ gives $\frac{25}{3} = 8\frac{1}{3}$.

2. Divide $\frac{4}{5}$ of $\frac{5}{8}$ of $\frac{2}{3}$ by $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{9}{10}$.

$\frac{4}{5}$ of $\frac{5}{8}$ of $\frac{2}{3} \div \frac{3}{4}$ of $\frac{2}{3}$ of $\frac{9}{10} =$

$$\frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{5}}{\underset{2}{\cancel{8}}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\overset{2}{\cancel{4}}}{3} \times \frac{\cancel{3}}{\cancel{2}} \times \frac{10}{9} = \frac{20}{27}.$$

In dividing by a compound fraction, invert all the separate fractions composing the divisor, and treat the problem as an exercise in cancellation.

NOTE. A compound fraction is a fraction of a fraction: thus $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{4}{5}$ of $\frac{8}{9}$ of $\frac{7}{8}$, etc.

3. Divide 4.8325 by .025.

$$
\begin{array}{rr}
 & 193.3 \\
{}_{\wedge}025.) & \overline{4_{\wedge}832.5} \\
 & \underline{25} \\
 & 233 \\
 & \underline{225} \\
 & 82 \\
 & \underline{75} \\
 & 75 \\
 & \underline{75}
\end{array}
$$

When the divisor is a decimal, change it to an integer by moving its point to the right-hand side of the decimal. In this example the point is moved three places, which is equivalent to multiplying by 1000. Since the divisor is multiplied by 1000, the dividend must likewise be multiplied by 1000 (Prin. 21, page 117). Therefore, the point in the dividend is moved 3 places to the right. Before dividing, place the point for the quotient directly above the point in the dividend after it is moved.

The division is now performed in the same manner as in dividing by integers.

4. Divide 5000 by .0005.

$$\begin{array}{r} 1000\,0000. \\ {}_{x}0005.\overline{)5000_{\wedge}0000.} \end{array}$$

As in the preceding example, change the divisor to an integer by moving the point to the right-hand side of the decimal, that is, by multiplying by 10,000. The dividend also must be multiplied by 10,000; that is, the point must be moved 4 places to the right. In this example it is necessary to make the places by annexing ciphers.

Remember, *the point in the dividend* MUST *be moved in the same direction as many places as the point in the divisor is moved to make the divisor a whole number. Annex ciphers when necessary.*

Remember, also, to place the point for the quotient *before* dividing.

The process of division is then the same as in simple numbers.

240. Find the value of:

1. $\frac{2}{3} \div \frac{3}{4}$
2. $\frac{4}{5} \div \frac{9}{10}$
3. $\frac{5}{11} \div \frac{7}{8}$
4. $\frac{6}{7} \div \frac{5}{9}$
5. $1\frac{1}{2} \div \frac{2}{3}$
6. $2\frac{1}{4} \div \frac{4}{5}$
7. $9\frac{1}{3} \div \frac{7}{8}$
8. $15\frac{1}{4} \div \frac{3}{4}$
9. $\frac{2}{3}$ of $\frac{4}{5} \div \frac{5}{8}$ of $\frac{6}{7}$
10. $\frac{5}{8}$ of $\frac{3}{4} \div \frac{9}{10}$ of $\frac{5}{4}$
11. $\frac{8}{9}$ of $\frac{3}{4} \div \frac{11}{15}$ of $\frac{15}{22}$
12. $28\frac{1}{2} \times \frac{3}{19} \div \frac{5}{16}$ of $\frac{4}{5}$
13. $\frac{7}{15}$ of $22\frac{1}{2} \div \frac{3}{13}$ of $\frac{26}{33}$
14. $28\frac{2}{3} \div \frac{4}{15}$
15. $18\frac{3}{4} \div \frac{5}{8}$
16. $.2862 \div .009$
17. $34.74 \div .018$
18. $49.52 \div .16$
19. $.004 \div .0002$
20. $.400 \div .004$
21. $.800 \div .0001$
22. $.008 \div .0001$
23. $.12\frac{1}{2} \div .62\frac{1}{2}$
24. $4.875 \div .0625$
25. $8.5 \div .3125$
26. $40{,}000 \div .002$
27. $404.04 \div .20$
28. $96.848 \div 1.60$
29. $74.64 \div 1.25$
30. $395.20 \div .0125$
31. $16 \div \frac{5}{8}$
32. $28 \div \frac{4}{7}$
33. $36 \div \frac{12}{17}$
34. $89 \div \frac{2}{3}$
35. $56 \div \frac{6}{7}$
36. $38 \div \frac{19}{20}$
37. $42 \div \frac{6}{11}$
38 $32 \div \frac{16}{17}$
39. $48 \div \frac{9}{10}$

241. To divide an integer or a mixed number by a mixed number.

1. Divide 24 by $7\frac{1}{2}$.

Write the dividend and divisor as for division of simple numbers. The divisor being a mixed number, it is first necessary to "clear of fractions." This is done by multiplying the divisor by 2, the denominator of the fraction in the divisor. Multiplying the divisor by 2 gives 15. If the divisor is multiplied by 2, the dividend must also

$$\begin{array}{r} 7\frac{1}{2})24 \\ \underline{2\quad 2} \\ 3\frac{1}{5} \\ 15\overline{)48} \\ \underline{45} \\ 3 \end{array} \qquad \frac{3}{15} = \frac{1}{5}$$

be multiplied by 2 (why?), which gives 48. The divisor and the dividend are now both integral, and the division is easily completed.

2. Divide $19.44\frac{4}{9}$ by $4.16\frac{2}{3}$.

$$
\begin{array}{rr}
 6 & \\
4.16\frac{2}{3}) & 19.44\frac{4}{9} \\
9 & 9 \\ \hline
\end{array}
$$

$$
\begin{array}{rr}
 & 4.66\frac{2}{3} \\
37_x5.0) & \overline{175_x0.00} \\
 & 150\ 0 \\ \hline
 & 25\ 0\ 0 \\
 & 22\ 5\ 0 \\ \hline
 & 2\ 5\ 00 \\
 & 2\ 2\ 50 \\ \hline
 & \frac{2\ 50}{3\ 75} = \frac{2}{3}
\end{array}
$$

Multiply both dividend and divisor by 9, the least common multiple of the denominators of the fractions $\frac{2}{3}$ and $\frac{4}{9}$, retaining the decimal points in the products. Move the points each one place to the right, and divide as already explained. The quotient is $4.66\frac{2}{3}$.

242. **Find the value of:**

1. $472\frac{3}{4} \div 24\frac{2}{3}$
2. $867 \div 15\frac{1}{6}$
3. $978\frac{5}{8} \div 19\frac{1}{2}$
4. $5.26\frac{2}{3} \div .46\frac{2}{3}$
5. $8.49\frac{1}{2} \div 2.12\frac{3}{8}$
6. $349\frac{3}{5} \div 16\frac{1}{4}$
7. $8749 \div 22\frac{1}{4}$
8. $8396 \div 78\frac{2}{3}$
9. $7469\frac{3}{5} \div 152\frac{1}{2}$
10. $9119\frac{2}{3} \div 428\frac{1}{2}$
11. $16.31\frac{1}{2} \div 1.21\frac{1}{4}$
12. $329\frac{3}{5} \div 26\frac{1}{4}$
13. $897\frac{2}{3} \div 31\frac{1}{4}$
14. $598\frac{4}{7} \div 24\frac{1}{2}$
15. $939\frac{3}{8} \div 42\frac{2}{8}$
16. $978 \div 85\frac{1}{8}$
17. $125 \div 1\frac{2}{3}$
18. $250 \div 1.66\frac{2}{3}$
19. $375 \div 3.33\frac{1}{3}$
20. $964\frac{5}{8} \div 16\frac{1}{4}$
21. $493\frac{2}{3} \div 42\frac{3}{4}$
22. $967\frac{5}{6} \div 91\frac{1}{3}$
23. $83.92 \div 5.125$
24. $49.87\frac{1}{2} \div 2.49\frac{3}{8}$
25. $39.37\frac{1}{2} \div 1.31\frac{1}{4}$
26. $478\frac{3}{7} \div 21\frac{2}{3}$
27. $674\frac{5}{6} \div 91\frac{1}{2}$
28. $845\frac{3}{8} \div 14\frac{1}{4}$
29. $7689 \div 18\frac{5}{6}$
30. $598\frac{3}{5} \div 14\frac{1}{2}$
31. $963\frac{8}{9} \div 8\frac{2}{3}$
32. $248\frac{3}{4} \div 19\frac{1}{2}$
33. $732\frac{3}{8} \div 19\frac{3}{4}$
34. $864\frac{5}{8} \div 21\frac{1}{2}$
35. $438\frac{6}{7}\ \ 219\frac{3}{7}$
36. $644\frac{3}{8} \div 16\frac{1}{2}$
37. $849\frac{5}{9} \div 13\frac{2}{3}$
38. $674\frac{5}{9} \div 11\frac{1}{3}$
39. $84.96\frac{1}{2} \div 15\frac{1}{4}$
40. $60.83\frac{1}{3} \div 12\frac{1}{6}$
41. $.0482\ \ .0002$
42. $59.68\frac{1}{2} \div .0005$
43. $11.25 \div .003\frac{3}{4}$
44. $131 \div .008\frac{1}{3}$
45. $45.75 \div .001\frac{2}{3}$

243. Fraction review drill.

Add at sight, (a) vertically, (b) horizontally:

1.

$\frac{1}{2}, \frac{1}{4}$
$\frac{3}{4}, \frac{1}{8}$
$\frac{3}{8}, \frac{1}{2}$
$\frac{1}{4}, \frac{7}{8}$
$\frac{3}{4}, \frac{7}{8}$
$\frac{5}{8}, \frac{1}{2}$
$\frac{7}{8}, \frac{3}{4}$
$\frac{1}{2}, \frac{7}{8}$

2.

$\frac{1}{3}, \frac{5}{6}$
$\frac{1}{2}, \frac{1}{6}$
$\frac{1}{4}, \frac{1}{6}$
$\frac{3}{4}, \frac{1}{3}$
$\frac{1}{6}, \frac{2}{3}$
$\frac{1}{2}, \frac{5}{6}$
$\frac{2}{3}, \frac{1}{2}$
$\frac{1}{4}, \frac{5}{6}$

$45\frac{1}{2}$ $21\frac{1}{12}$
38 $83\frac{1}{12}$
$3\frac{5}{12}$ $3\frac{1}{12}$ $3\frac{1}{2}$

3.

4 $2\frac{1}{2}, 3\frac{1}{3}$ 8
2 $4\frac{1}{4}, 2\frac{1}{6}$ 2
1 $3\frac{1}{8}, 5\frac{1}{3}$ 9
6 $7\frac{3}{4}, 4\frac{2}{3}$ 8
3 $6\frac{3}{8}, 5\frac{1}{4}$ 2
4 $7\frac{1}{2}, 5\frac{5}{6}$ 10
4 $8\frac{1}{2}, 7\frac{5}{6}$ 10
4 $5\frac{1}{2}, 3\frac{2}{3}$ 8
28 $\frac{46}{12}$ 3

4.

$\frac{1}{4}, \frac{1}{5}, \frac{1}{2}$
$\frac{3}{4}, \frac{1}{2}, \frac{1}{3}$
$\frac{3}{8}, \frac{1}{4}, \frac{1}{6}$
$\frac{1}{2}, \frac{3}{10}, \frac{1}{4}$
$\frac{5}{8}, \frac{7}{10}, \frac{1}{2}$
$\frac{3}{4}, \frac{1}{4}, \frac{1}{3}$
$\frac{3}{4}, \frac{2}{5}, \frac{1}{2}$
$\frac{1}{4}, \frac{9}{10}, \frac{1}{2}$

$\frac{5}{6}$ $3\frac{1}{2}$ $\frac{3}{12}$ 11

State results at sight:

5.

$\frac{2}{3}-\frac{1}{4}=?$
$\frac{3}{4}-\frac{1}{3}=?$
$\frac{1}{2}-\frac{1}{3}=?$
$\frac{5}{6}-\frac{1}{2}=?$
$\frac{5}{6}-\frac{2}{3}=?$
$\frac{5}{8}-\frac{1}{2}=?$

6.

$\frac{1}{3}+?=\frac{7}{12}$
$\frac{3}{4}+?=\frac{7}{8}$
$\frac{1}{2}+?=\frac{5}{6}$
$\frac{1}{4}+?=\frac{11}{12}$
$\frac{1}{3}+?=\frac{1}{2}$
$\frac{1}{2}+?=\frac{5}{8}$

$45\frac{1}{2}$ 3
$46\frac{5}{6}$ 5
$2\frac{1}{3}$

7.

$\frac{1}{2}+\frac{1}{3}-\frac{1}{4}=?$
$\frac{1}{3}+\frac{1}{5}-\frac{2}{15}=?$
$\frac{2}{3}-\frac{1}{2}+\frac{5}{6}=?$
$\frac{3}{4}+\frac{3}{8}-\frac{1}{2}=?$
$\frac{5}{6}-\frac{1}{3}+\frac{3}{4}=?$
$1-\frac{3}{4}+\frac{1}{3}=?$

$37\frac{5}{}$
$83\frac{1}{3}$

8.

$\frac{3}{4}+?=1\frac{1}{2}$
$\frac{2}{3}+?=1\frac{1}{2}$
$\frac{3}{8}+?=1\frac{1}{4}$
$\frac{1}{4}+?=2\frac{1}{8}$
$\frac{1}{2}+?=5\frac{1}{4}$
$\frac{1}{3}+?=3\frac{1}{6}$

Multiply at sight:

9.

$\frac{1}{3}$ of $12=?$
$\frac{1}{4}$ of $24=?$
$\frac{3}{4}$ of $16=?$
$\frac{2}{3}$ of $27=?$
$\frac{3}{8}$ of $32=?$
$\frac{5}{6}$ of $36=?$

10.

$8\times\frac{3}{8}=?$
$9\times\frac{2}{3}=?$
$7\times\frac{3}{4}=?$
$6\times\frac{4}{5}=?$
$5\times\frac{5}{8}=?$
$4\times\frac{3}{8}=?$

11.

$6\times3\frac{1}{2}=?$
$8\times4\frac{3}{4}=?$
$9\times6\frac{1}{3}=?$
$12\times4\frac{1}{2}=?$
$10\times6\frac{1}{4}=?$
$6\times7\frac{2}{3}=?$

12.

$\frac{2}{3}$ of $\frac{3}{4}=?$
$\frac{3}{4}$ of $\frac{5}{8}=?$
$\frac{1}{2}$ of $\frac{3}{7}=?$
$\frac{1}{3}$ of $\frac{3}{4}=?$
$\frac{5}{6}$ of $\frac{2}{3}=?$
$\frac{3}{8}$ of $\frac{1}{2}=?$

13.	14.	15.	16.	17.	18.	19.	20.
5	6	8	$2\frac{1}{2}$	$3\frac{1}{4}$	$6\frac{1}{2}$	9	7
$\frac{4}{5}$	$\frac{2}{3}$	$\frac{2}{3}$	6	3	3	$3\frac{1}{2}$	$4\frac{1}{3}$

Divide, stating answers at sight:

21.	**22.**	**23.**	**24.**
$\frac{6}{7} \div 3 = ?$	$\frac{9}{16} \div 3 = ?$	$8 \div \frac{2}{3} = ?$	$3\frac{1}{2} \div 2 = ?$
$\frac{8}{9} \div 2 = ?$	$\frac{11}{12} \div 2 = ?$	$9 \div \frac{3}{4} = ?$	$4\frac{1}{4} \div 5 = ?$
$\frac{5}{8} \div 4 = ?$	$\frac{5}{9} \div 3 = ?$	$16 \div \frac{4}{5} = ?$	$4\frac{1}{2} \div 3 = ?$
$\frac{2}{3} \div 6 = ?$	$\frac{5}{6} \div 2 = ?$	$7 \div \frac{1}{2} = ?$	$3\frac{3}{4} \div 5 = ?$
$\frac{3}{4} \div 4 = ?$	$\frac{7}{8} \div 7 = ?$	$8 \div \frac{3}{4} = ?$	$8\frac{1}{3} \div 5 = ?$
$\frac{5}{7} \div 5 = ?$	$\frac{1}{4} \div 4 = ?$	$12 \div \frac{6}{7} = ?$	$3\frac{1}{3} \div 4 = ?$

In the following exercises estimate results in advance of written work:

25. Add	**26.** Add	**27.** Subtract	**28.** Subtract
$23\frac{1}{2}$	$\frac{1}{16}$	$412\frac{3}{4}$	$15\frac{1}{16}$
4.032	11.9426	$87\frac{7}{8}$	$9.68\frac{3}{4}$
$17\frac{1}{2}$	5.9949		
$16\frac{1}{6}$	$7\frac{1}{3}$		

Multiply:

29.	**30.**	**31.**	**32.**
$41\frac{2}{3}$	$32\frac{3}{4}$	$27\frac{2}{3}$	$23\frac{2}{3}$
16	22	$16\frac{1}{2}$	$22\frac{2}{3}$

33.	**34.**	**35.**	**36.**
$28\frac{1}{2}$	$33\frac{3}{4}$	$17\frac{1}{4}$	$27\frac{1}{3}$
$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{4}$

Divide:

37.	**38.**	**39.**	**40.**
$5\overline{)62\frac{1}{2}}$	$9\overline{)82\frac{1}{3}}$	$7\frac{3}{4}\overline{)423}$	$17\frac{1}{2}\overline{)216\frac{3}{4}}$

41.	**42.**	**43.**	**44.**
$41\frac{2}{3} \div \frac{1}{2}$	$62\frac{3}{8} \div \frac{1}{3}$	$29\frac{1}{3} \div \frac{2}{3}$	$54\frac{1}{8} \div \frac{3}{4}$

State results at sight:

45.	**46.**
450 lb. @ $2.50 per C.	4500 lb. @ $4.50 per M.
650 lb. @ $3.50 per C.	6500 lb. @ $8.50 per M.
975 lb. @ $3.75 per C.	9250 lb. @ $7.25 per M.
1125 lb. @ $5.25 per C.	7250 lb. @ $5.25 per M.

SOME FUNDAMENTAL PRINCIPLES OF ARITHMETIC

244. Every problem in arithmetic is based on one or more fundamental principles or laws. A knowledge of these principles and of their application to the solution of problems is essential to success in arithmetical calculations.

Instead of stating the principles of notation, addition, subtraction, etc., in connection with these topics, it has been deemed advisable to group them in one chapter, in order that the proper emphasis may be placed upon them.

It is suggested that special attention be given to these principles, as frequent reference will be made to them.

PRINCIPLES OF NOTATION AND NUMERATION

245. **1.** *Each removal of any significant figure** *one place to the left toward or from the decimal point multiplies its value by ten.*

2. *Each removal of any significant figure one place to the right toward or from the decimal point divides its value by ten.*

3. *Each cipher annexed to an integer multiplies its value by ten.*

4. *Each cipher prefixed to a decimal divides its value by ten.*

PRINCIPLES OF ADDITION

5. *Only like numbers or parts of like numbers can be added.*

6. *The sum is composed of the same kind of units as are the addends.*

PRINCIPLES OF SUBTRACTION

7. *Only like numbers or parts of like numbers can be subtracted.*

8. *The remainder is composed of the same kind of units as are the minuend and subtrahend.*

9. *The minuend is equal to the sum of the subtrahend and the remainder.*

---

*A significant figure is a figure that possesses a value of its own. Thus, all the figures, 1, 2, 3, etc., except 0, are significant figures. 0 has no value of its own.

PRINCIPLES OF MULTIPLICATION

10. *The multiplicand may be either an abstract or a concrete number.*

11. *The multiplier is an abstract number.*

12. *The multiplicand and the product are like numbers.*

13. *The multiplier and the multiplicand may be interchanged without affecting the value of the product. (This principle is called the commutative law of multiplication.)*

14. *The product of two factors divided by either of them will give the other one. (If there are more than two factors in any product, the product divided by any one of the factors will give a quotient equal to the product of all the other factors.)*

PRINCIPLES OF DIVISION

15. *The dividend is equal to the product of the divisor and the quotient (plus the remainder, if any).*

16. *The dividend and the remainder are like numbers.*

17. *Multiplying the divisor divides the quotient by the same number.*

18. *Dividing the divisor multiplies the quotient by the same number.*

19. *Multiplying the dividend multiplies the quotient by the same number.*

20. *Dividing the dividend divides the quotient by the same number.*

21. *Multiplying or dividing both divisor and dividend by the same number does not change the value of the quotient.*

REMARK. Some of the above principles are already familiar to the student and are applied unconsciously to the solution of problems. Others (Nos. 11, 12, 14, 15, 16, 17, 18, 19, 20, and 21) are not so familiar, and need emphasis. In the number of its practical applications, Principle No. 14 is the most important.

APPLICATION OF THE FUNADAMENTL PRINCIPLES OF ARITHMETIC TO THE SOLUTION OF PROBLEMS

246. The solution of problems requires a knowledge of three things: (1) The student must know how to add, subtract, multiply, and divide; (2) he must know in what order these operations are to be performed; and (3) he must be able to make the right combinations of numbers in any given problem.

The order of performing the operations and the proper combining of numbers depend upon a knowledge of the fundamental principles of arithmetic.

The following illustrations show the practical application of the most important principles to the solution of problems.

Very many problems are reducible to one or more of three type forms. As type problems, the following may be taken as examples:

Type 1. $\frac{3}{4}$ of \$48 = ?
Type 2. $\frac{5}{8}$ of what number = \$45?
Type 3. What part of \$90 = \$75?

In the first example given, the operation is obviously one of multiplication, and the product is \$36 (Prin. 12).

In the second example one factor and a product are given. The product is \$45, and the given factor is $\frac{5}{8}$. The missing factor is indicated by the words "what number." By Principle 14, if \$45 is divided by $\frac{5}{8}$, the other factor will be obtained. $\$45 \div \frac{5}{8} = \$72$.

Many problems are reducible to this type form. It is worthy of the careful attention of the student.

In the third example there are also given a product, \$75, and a factor, \$90. The product divided by the given factor gives the other factor. $\$75 \div \$90 = \frac{75}{90} = \frac{5}{6}$. $\frac{5}{6}$ is the missing factor.

In the second example the missing factor was the multiplicand (Prin. 12). In the third example it was the multiplier (Prin. 11).

Practice in reducing problems to their appropriate type form will be found helpful. Following are a few illustrations and explanations:

1. A farmer had 48 cd. of wood and sold $\frac{3}{4}$ of it. How many cords did he sell?

First, after reading the problem, note the question, "How many cords did he sell?" The problem answers this question by saying that he "sold $\frac{3}{4}$ of it." What is "it"? "It" is 48 cd. Now put the number in the place of "it," and you have $\frac{3}{4}$ of 48 cd. = number of cords sold. The problem is now reduced to the form of the first type problem (Prin. 12).

2. A street is paved for a distance of 960 yd., which is $\frac{4}{5}$ of its length. What is the length of the street?

Read the problem. Note the question to be answered. Is the street all paved? What part of it is paved? How many yards are paved? How do

"$\frac{4}{5}$ of the length of the street" and "960 yd." compare? (They are alike.) Therefore write,

$\frac{4}{5}$ of the length of the street = 960 yd. or
$\frac{4}{5}$ of ? yards = 960 yd.

960 yd. is the product obtained by multiplying some number by $\frac{4}{5}$. Therefore,

960 yd. ÷ $\frac{4}{5}$ = 1200 yd., length of street. (Prin. 14, Type 2.)

3. A man's annual income is $2500, and his expenses for the year are $1500. What part of his income does he save?

Read the problem till you know what it means. Note the question carefully.

The question is, "What part of his income did he *save?*" He saved the difference between $2500 and $1500, which is $1000. He saved $1000. His income was $2500.

Now write the question in the shortest way, using numbers instead of words where possible.

? part of $2500 = $1000? (Prin. 14, Type 3.)

Reduce each of the following problems to its proper type form:

1. A merchant bought 480 yd. of muslin and sold $\frac{5}{8}$ of it. How many yards did he sell?

2. A boy's total expenses for one year were $1280, of which amount $400 was spent for board. His board bill was what part of all his expenses?

3. A manufacturing concern pays profits amounting to $\frac{3}{16}$ of its capital. If the profits are $15,000, what is the capital?

4. A man sold $\frac{3}{8}$ of his farm for $4612½. At the same rate how much was the whole farm worth?

5. A dealer bought goods for $12½, and sold them for $15. The gain was what part of the cost? The gain was what part of the selling price?

6. If a hatter buys a hat for $6, and has to sell it for $4½, he loses what part of the cost? of the selling price?

7. The first of these lines is 1½ in. long, the second 1¼ in. long.

1923.______________________________
1924.__________________________

If the first line represents a merchant's sales in 1923, and the second one, his sales in 1924, the amount of sales in 1924 was what part of the sales in 1923? The sales in 1924 were how much (fractional part) less than in 1923? The sales in 1923 were how much greater than in 1924?

PROBLEM ANALYSIS

247. The analysis of a problem consists in stating the reason for each successive step in the solution.

1. How long will it take a man earning \$15 a week to pay for a motor cycle costing \$200, if his expenses are \$7 a week?

Analysis. If he earns \$15 a week and spends \$7 a week, he can save \$8 a week. It will take as many weeks to pay for the motor cycle as \$8 is contained times in \$200, or 25 times. Therefore, it will take 25 weeks to pay for the motor cycle.

2. If $3\frac{1}{2}$ pounds of sugar cost 21 ¢, how much will 12 pounds cost?

Analysis. If $3\frac{1}{2}$ pounds cost 21 ¢, 1 pound will cost as many cents as $3\frac{1}{2}$ is contained times in 21, or 6 times. If 1 pound of sugar costs 6 ¢, 12 pounds will cost 12 × 6 ¢, or 72 ¢.

REVIEW PROBLEMS IN FRACTIONS

Mental

248. Find the results as indicated:

1. $\frac{1}{2} + \frac{1}{3}$; $\frac{1}{2} - \frac{1}{3}$; $\frac{1}{2} \times \frac{1}{3}$; $\frac{1}{2} \div \frac{1}{3}$.

2. $\frac{4}{5} + \frac{1}{2}$; $\frac{4}{5} - \frac{1}{2}$; $\frac{4}{5} \times \frac{1}{2}$; $\frac{4}{5} \div \frac{1}{2}$.

3. $\frac{3}{7} + \frac{3}{8}$; $\frac{3}{7} - \frac{3}{8}$; $\frac{3}{7} \times \frac{3}{8}$; $\frac{3}{7} \div \frac{3}{8}$.

4. $4\frac{1}{2} + 2\frac{1}{4}$; $4\frac{1}{2} - 2\frac{1}{4}$; $4\frac{1}{2} \times 2\frac{1}{4}$; $4\frac{1}{2} \div 2\frac{1}{4}$.

5. $7\frac{1}{3} + 3\frac{2}{3}$; $7\frac{1}{3} - 3\frac{2}{3}$; $7\frac{1}{3} \times 3\frac{2}{3}$; $7\frac{1}{3} \div 3\frac{2}{3}$.

6. If $\frac{3}{4}$ is subtracted from a given fraction, the remainder will be $\frac{1}{6}$. What is the fraction?

7. $\frac{5}{8}$ added to a certain fraction gives $1\frac{7}{24}$. What is the fraction?

Analyze as many of the following problems as the teacher directs:

8. If a man spends, on the average, $\$\frac{5}{8}$ a day for street car fare, in what time will he spend \$100?

9. A boy bought a bicycle for \$45, and sold it for $\frac{8}{9}$ of what he gave for it. How much less did he receive than he paid?

10. Two men bought a motor boat for \$450, each paying half. Later each sold one third of his share at cost. How much had each invested then, and what part of the boat did each own?

$\frac{2}{12}$

$\frac{5}{6}$ 12 10 $\frac{11}{12}$ 22

$\frac{3}{4}$ 9 $\frac{3}{4}$ 18

11. I bought $11\frac{3}{8}$ lb. of tea at $\$\frac{1}{2}$ a pound and 6 lb. of coffee at $\$\frac{3}{8}$ a pound. How much change should I receive from a ten-dollar bill?

12. A young lady bought $7\frac{1}{2}$ yd. muslin at $8\frac{1}{2}$ ¢ a yard. What change ought she to receive from a half dollar and a quarter?

13. A woman bought $10\frac{1}{2}$ yd. of silk at $\$2\frac{1}{2}$ a yard, and $3\frac{1}{2}$ yd. lining at $.25 a yard. She gave in payment three ten-dollar bills. What change should she receive?

14. I sold a horse for $75, which was $\frac{5}{6}$ of what it cost. How much did I lose?

15. A crate containing 10 doz. oranges cost $4.50. If the oranges sell at the rate of 2 for 11 ¢, what is the gain?

16. A bunch of 8 doz. bananas cost $1. If one dozen spoil and the rest are sold at the rate of 3 for 10 ¢, what is the gain?

17. Two railroads carry merchandise 450 mi., for $3.60. If the first road carries it 200 mi., how much should each receive?

18. Two men engage to do a piece of work for $157.50. The first works 18 da. and the other works 27 da. How shall the money be divided?

19. $3\frac{1}{2}$ doz. eggs at $.40 a dozen will pay for how many pounds of rice at $8\frac{3}{4}$ ¢ a pound?

20. I sold $\frac{3}{8}$ of my apples for $90. At that rate, how much were they all worth?

21. A merchant by selling cloth at 42 ¢ a yard gained $\frac{1}{6}$ of the cost. Find the cost.

22. I bought a quantity of cloth for $180. By selling it at $2 a yard, I gained $\frac{1}{3}$ of the cost. How many yards were there?

23. A buying agent charges $\frac{1}{16}$ of the net cost of goods as his commission. If his commission for one month amounts to $240, what is the value of the goods bought?

24. I sold $\frac{3}{7}$ of my land for $1500. If I sold 75 A., how much land have I left, and how much is it worth at the same rate?

25. A man picked $\frac{3}{4}$ of a bushel of pears and sold $\frac{1}{2}$ of them for $.45. At that rate, how much would a bushel cost?

26. A man had $\$37\frac{1}{2}$, and spent $\$12\frac{1}{2}$. What part of his money did he spend?

27. A clerk earns $87\frac{1}{2}$ a month, and spends $30 for board and $20 for other expenses. What part of his money does he save?

28. In a certain factory 56 men were employed. During a dull season 24 of them were laid off. What part of the men continued working?

29. In an orchard there are 120 trees; 96 of them are apple trees, 16 are pear trees, and the remainder are cherry trees. What part of all is each kind of trees?

30. Three railroads carry a piece of freight 540 mi. The first carries it 180 mi., the second 300 mi., and the third the remainder of the distance. What fractional part of the distance does each carry it?

PROBLEMS

(Solve mentally when possible.)

249. 1. $\frac{3}{8}$ of a pole 32 ft. long is decayed. How many feet are good?

2. A tree 76 ft. high breaks 19 ft. from the ground. What part is broken off?

3. A man has $\frac{1}{2}$ of his money in one bank, $\frac{1}{3}$ of it in another, and the remainder in a third bank. If the amount in the first bank is $660, how much is there in each of the other banks?

4. A man sold a horse for $150, which was $\frac{3}{4}$ of what he paid for it. Find the loss.

5. The wholesale price of Rio coffee at New York increased from $7\frac{1}{4}$¢ in August, one year, to $8\frac{3}{4}$¢ in August, the following year. What fractional part of the first price was the increase?

6. A farmer sold $\frac{3}{5}$ of his sheep for $360. At the same rate what is the value of all his sheep? What is the value of those he did not sell?

7. A boy spent $\frac{3}{5}$ of his money for a bicycle, and had $24 left. How much money had he at first?

8. A man had $\frac{1}{2}$ of his sheep in one pasture, $\frac{1}{4}$ of them in another pasture, and the rest of them, which was 33 sheep, in a third pasture. How many sheep had he?

Days	Class	Study	eat	[illegible]	sleep
	5 m.	0	1 m.	12:56 m.	12.

9. A gentleman's estate is so divided that $\frac{3}{8}$ of it is woodland, $\frac{1}{4}$ of it is water, and the remaining 600 A. are under cultivation. How many acres are there in the estate?

10. A house and lot are worth \$6300. If the lot is worth $\frac{4}{5}$ as much as the house, how much is each worth?

11. A gentleman left his son \$6500, which amount was $2\frac{1}{6}$ times as much as he left to his daughter. How much did the daughter receive?

12. A owned $\frac{4}{7}$ of a farm and B the remainder. If A had $24\frac{3}{4}$ A. more than B, how many acres had each?

13. A house and a barn are valued at \$14,805. Find the value of each if the barn is worth $\frac{2}{5}$ as much as the house.

14. Find the cost of each, if a house and lot together cost \$14,000, the lot costing .4 as much as the house.

15. A gentleman left his estate to his three sons. The eldest received .40 of it, the next son .35, and the youngest, \$7500. Find the value of the estate.

16. The cost of furnishing two rooms is \$480. If the cost of one is $\frac{7}{9}$ as much as the cost of the other, what is the cost of each?

17. The income from a certain business for two years was \$27,500. Find the income for each year if the second year's income was .20 greater than the first.

18. What number increased by $\frac{3}{4}$ of itself is 651?

19. If goods are sold for \$90, and a profit of $\frac{1}{5}$ of the cost is made, find the cost.

20. If the profit on eggs is 3 ¢ per dozen, and the eggs sell for 45 ¢ a dozen, what part of the cost is gained?

21. The profit on a pair of shoes is \$.75. If the profit is .20 of the cost, for how much per pair do the shoes sell?

22. What number decreased by $\frac{3}{7}$ of itself is equal to 56?

23. Find the cost of an article sold for \$36, if $\frac{1}{7}$ of the cost was lost.

24. A hatter marked all his straw hats down $\frac{1}{4}$. What was the marked price of a hat that he now sells for \$4.50? If he made a profit of $\frac{1}{2}$ of the cost *before* marking the hat down, what part of the cost does he make *after* marking it down?

25. If a man buys $868\frac{1}{2}$ yd. of cloth and sells $\frac{2}{3}$ of it at $14\frac{3}{4}$¢ a yard, and the remainder at $12\frac{3}{4}$¢ a yard, how much does he receive for all of it?

26. A field of $9\frac{1}{2}$ A. was bought for $380. The first year's wheat crop averaged 20 bu. to the acre. If wheat was worth 90¢ per bushel, the value of the crop was what part of the purchase price?

27. Find the total value of the following articles: $1\frac{3}{4}$ doz. buttons at 15¢; $2\frac{3}{4}$ yd. ribbon at $9\frac{1}{2}$¢; $3\frac{1}{2}$ yd. lace at 65¢; $\frac{3}{4}$ yd. velvet at $2.90.

28. I bought 360 sheep for $1200. I sold $\frac{2}{3}$ of them at $4 per head, and $\frac{1}{4}$ of the remainder died. At what price per head must the rest of them be sold to gain $\frac{1}{5}$ of the cost of all?

29. A fruit dealer buys 10 doz. oranges for $2.30. If 2 doz. spoil, at what price per dozen must he sell the good ones to gain $\frac{1}{4}$ of the cost?

30. From the sum of $36\frac{3}{4}+47\frac{1}{3}+18\frac{1}{8}$ take the sum of $31\frac{2}{3}$ and $24\frac{3}{8}$.

31. A coffee importer purchases green coffee at $16\frac{1}{2}$¢ a pound. Freight and other expenses amount to $3\frac{1}{2}$¢ a pound. If, in roasting, the coffee loses $\frac{1}{6}$ of its weight, find the cost of a pound of roasted coffee.

32. A young man buys $13\frac{3}{4}$ A. of land at $36.50 per acre. If he works for $27\frac{1}{2}$¢ an hour, 8 hr. a day, and spends $6.60 a week for board and other expenses, how long will it take him to pay for the land?

33. If eggs are bought at 42¢ a dozen and sold at the rate of 8 eggs for 30¢, find the gain on 500 doz. eggs.

34. In the following time sheet, 48 hours make a week's work, $\frac{1}{2}$ extra is allowed for over time. Find weekly wages of each man:

Names	M.	T.	W.	Th.	F.	S.	Weekly Wage
A. Anderson	8	9	9	8	9	10	$36
B. Brown	9	8	9	8	10	10	30
C. Custer	10	9	9	8	8	10	25
G. French	8	7	10	9	8	9	32

35. Out of 900 bushels of potatoes put in storage October 15, 45 bushels were found unsound April 1. What fractional part of the whole were sound?

36. If an investment of $22,500 produces an annual income of $1800, how much should an investment of $25,000 produce at the same rate?

37. Of a stock of cloth containing 1728 yd., 720 yd. were sold at one time, and 576 yd. at another time. What part of the cloth remained unsold?

38. To excavate a certain basement required the removal of 5648 cu. yd. of rock and earth. If $\frac{3}{8}$ was rock and cost $4.25 per cu. yd. and the remainder was earth costing 80¢ per cubic yd., what was the cost of digging the basement?

39. A man exchanges $8\frac{1}{2}$ bushels of potatoes at $1.20 per bu. for 24 pounds of coffee at $27\frac{1}{2}$¢ a pound, and $4\frac{1}{2}$ pounds of tea. How much was the tea worth per pound?

40. A lady paid for furs $157.10; for jewelry $219.10; for shoes $22.80; for millinery $35; and had expended but $\frac{7}{20}$ of her money. How many dollars had she at first?

41. By selling a house for $2400 I lost $600. What part of the cost did I lose?

42. A quantity of wheat contains 4250 bushels. $\frac{3}{10}$ of it is sold at $1.25 per bushel. One half of the remainder is sold at $1.15, and the rest at $1.20. Find the total selling price.

43. A merchant has three pieces of silk containing $13\frac{1}{8}$ yd., $21\frac{1}{4}$ yd. and $16\frac{1}{2}$ yd. respectively; at $1.75 a yard, how much must be paid for these three pieces?

44. The last reading of my gas meter was 67,300 cu. ft.; the previous reading was 64,900 cu. ft. At $1.35 per thousand cu. ft., find the amount of my gas bill.

45. At $64 per ton, find the cost of fertilizer for 600 young peach trees, allowing $1\frac{1}{4}$ lb. to a tree.

46. C sells a machine for $3217.50, and thereby loses $\frac{1}{4}$ of its cost. How much money did he lose, and how much did he pay for the machine?

47. B sells a machine for $719.10 at a gain of $\frac{1}{5}$ of the cost. How much did he gain and what was the cost price?

48. Find the total cost of 4325 pounds of coal at \$10.75 a ton; 6284 feet of lumber at \$42.50 a thousand; 3684 pounds of pork at \$19.50 a hundred.

49. A dealer bought 38 bags of potatoes each containing $2\frac{3}{4}$ bu. at \$.87$\frac{1}{2}$ a bushel. He sold them at a gain of $\frac{1}{5}$ of the cost. Find the gain and the selling price of all.

50. A merchant imported 1260 bags of coffee. He sold at one time 300 bags, and at another time $\frac{9}{16}$ of the remainder. What part of the coffee remained unsold?

51. A hatter marked his straw hats down $\frac{1}{3}$ to close out his stock. His profit was then \$.20 per hat, which was $\frac{1}{10}$ of the cost. At what price did he sell them before marking them down?

The following is a form used in making an annual statement of receipts and disbursements by the business manager of a high school paper:

Tenth Annual Financial Statement of the Business Manager of *The High School News* for the Year ended June, 1924.

	RECEIPTS								DISBURSEMENT						BALANCE	
	Sales		Ads		Mdse.		Total		Printing		Mdse.		Total			
Balance															275	90
September	98	60	15		12	50	126	10	110	60			110	60	15	50
October	102	30	25	75	60	40	188	45	112	70	42	25	154	95	33	50
November	127	40	35	50	62	30	225	20	121	50	35	20	156	70	68	50
etc.																

52. Prepare a form similar to the above and tabulate the receipts and disbursements for the football game between the Commerce High School and the Central Business College. There were sold to members of the athletic association 1528 tickets at 25 ¢; to other students 732 tickets at 35 ¢; and at the gate 2406 tickets at 50 ¢. The expenses were for printing and advertising, \$45; for the use of the athletic field, one third of the total receipts from ticket sales; miscellaneous expenses, \$28.75.

The same general form may be used for making a statement of receipts and disbursements in your school lunch room.

250. 1. A man sold 5 cows for $100, $127.50, $140, $165, and $170 respectively. Find the average selling price per cow.

$100+$127.50+$140 +$165+$170=$702.50, selling price of all.
$702.50÷5=$140.50, average price.

To find the average of several values, divide the sum of the values by the number of values.

2. In driving from New York to Chicago an automobilist's speedometer registered on successive days as follows: 215 mi., 175 mi., 130 mi., 190 mi., 121 mi., and 217 mi. How many miles a day did he average?

3. In one year a man received monthly checks as follows: $29.87, $98.63, $97.85 $111.54, $128.03, $101.48, $120.26, $142.38, $107.75, $116.11, $134.60, and $139.67. Find the average amount received per month.

4. In a class of 40 students, 16 were boys and 24 were girls, written in the school records, for convenience, thus, 16-24. The daily attendance for 1 week was as follows: 15-22, 14-23, 16-21, 13-24, 16-24. Find the average daily attendance for boys and for girls and the average for the class. Which group had the better average?

5. During a given term of school a pupil had seven tests in business arithmetic, with ratings as follows: 90, 86, 81, 95, 63, 72, 85. What was the average rating?

6. A tire costing $28.75 was put on an automobile when the speedometer registered 13,496 miles. At the time of its removal, owing to its being worn out, the speedometer registered 18,932 miles. Assuming that the other three tires wore equally well, find the average tire cost per mile of driving the car, correct to the nearest tenth of a mill.

7. On the first Saturday of each month of a given fiscal year the wholesale weekly price of rubber per pound, at New York City was $1.45, $1.87, $1.92, $2.03, $1.84, $1.75, $1.76, $1.84, $2.06, $2.80, $2.47, $2.28, respectively. Find (1) the average price for the year; (2) what fractional part the average price is greater than the first price given.

EXAMINATIONS

SPEED TEST

251. Minimum time, thirty minutes; maximum time, one hour. Suggestion for marking: Allow 100 credits if test is completed correctly in minimum time. Deduct one credit for each minute required beyond minimum time.

1. Add the following:

241693798	$693.05
473185459	7.847
391533768	78.98
427936857	1489.86
819348673	.75
473925165	974.98
274639827	6.
315987352	87.87
675431298	9748.098
897316984	7.96
931258369	75.899
687316984	17.

2.

From	Take	Difference
752361	582834	
1021314	987476	
7934.19	986.74	
784	95.9876	

3. Find the total value of the following:

222^1 yd. @ 3^2 ¢

166^2 yd. @ 4^1 ¢

265^1 yd. @ 7^1 ¢

163^2 yd. @ 9^2 ¢

254^3 yd. @ 8^1 ¢

212 yd. @ 4^3 ¢

4. Complete the following bill:

Chicago, Ill., July 1, 1924.

H. P. Jones

Bought of

D. C. Roy.

3 bbl. Sugar,	640#	@	$6\frac{1}{4}$¢		
5 chests Tea,	680#	@	75 ¢		
3 chests Tea,	498#	@	$66\frac{2}{3}$¢		
6 bbl. Flour,	1176#	@	4 ¢		
Total					

5. Find the total value of the following:

3290 lb. @ $2.75 per hundredweight.

4332 lb. @ 2.75 per hundredweight.

3570 lb. @ $1.87\frac{1}{2}$ per hundredweight

5792 lb. @ 22.50 per 1000 lb.

2396 lb. @ 16.50 per 1000 lb.

32690 lb. @ 12.80 per 1000 lb.

6. Add 873 to 1,301 continuously till the sum equals or exceeds 10,000. (Show all the work.)

7. Divide 23,236 by 2237 by continued subtraction. (Show all the work.)

8. Reduce to equivalent fractions having the least common denominator:

(*a*) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{12}$, $\frac{7}{8}$. (*c*) $\frac{9}{16}$, $\frac{5}{8}$, $\frac{5}{24}$, $\frac{3}{32}$.

(*b*) .75, $\frac{5}{6}$, $\frac{7}{9}$, $\frac{5}{18}$. (*d*) .80, $\frac{2}{3}$, $\frac{4}{15}$, .3.

9. Reduce to common fractions in their lowest terms:

.065 .0625 .225 .00125

10. Perform the operation indicated:

$15\frac{3}{5} \times 17\frac{1}{3}$ $426\frac{3}{4} \div 21\frac{1}{2}$ $48.72 \times .00125$ $54.7325 \div .001$

SPEED TEST

252. **1.** Add horizontally and vertically. Check by adding the totals:

$36,968	$ 7,634	$14,262	$——
22,635	28,329	37,257	——
9,564	82,378	6,295	——
49,877	69,764	34,472	——
Grand total			$——

2. Find the gain on each of the following items and the total gain:

Selling Price	Cost	Gain
$40,000.00	$37,624.37	$——
9,437.83	5,819.24	——
4,341.85	2,838.96	——
57,640.33	48,924.29	——
89,754.00	68,947.25	——
Total		$——

Perform the following indicated operations:

3. $425 \div \frac{5}{6} =$

$234 \div .33\frac{1}{3} =$

$.25 \times \frac{8}{9} =$

$9\frac{1}{8} - 4\frac{3}{16} =$

$\frac{1}{3} + \frac{1}{6} + \frac{1}{9} =$

4. $9 \times 12 \times 14 \div 42 =$

$95 \times 8 \times 3 \div 19 =$

$52 \times 8 \times 7 \div 91 =$

$20 \times 21 \times 22 \div 154 =$

$(48 \times 36 \times 24) \div (16 \times 12) =$

5. Find the value of each of the following:

$$\frac{\frac{3}{4}+\frac{5}{6}-\frac{1}{3}}{\frac{3}{4}}\times 12\tfrac{1}{2} = \qquad \frac{\frac{13}{15}}{\frac{7}{5}}\times\frac{15}{16}\times\frac{14}{65} =$$

6. Reduce each of the following groups to equivalent fractions having the least common denominators:

(*a*) $\frac{9}{24}$ $\frac{13}{18}$ $\frac{7}{12}$ $\frac{11}{36}$ =

(*b*) $\frac{3}{5}$ $\frac{4}{7}$ $\frac{11}{14}$ $\frac{9}{10}$ =

(*c*) $\frac{4}{15}$ $\frac{5}{21}$ $\frac{4}{27}$ $\frac{2}{63}$ =

(*d*) $\frac{5}{12}$ $\frac{8}{21}$ $\frac{11}{28}$ $\frac{13}{42}$ =

7. Multiply and prove:

76897 × 328
45723 × 126

8. Divide and prove:

$6806\frac{1}{4} \div 412\frac{1}{2}$

9. Find the total cost of the following:

250 bu. @ \$ .75 = \$ ——
340 bu. @ .80 = ——
1360 bu. @ $1.12\frac{1}{2}$ = ——
1840 bu. @ 1.25 = ——
1480 bu. @ $.62\frac{1}{2}$ = ——
Total . . \$——

10. Find the total interest at 6% on the following:

\$500.00 for 90 da. = \$ ——
750.00 for 80 da. = ——
1000.00 for 30 da. = ——
1200.00 for 45 da. = ——
895.50 for 20 da. = ——
Total . . \$ ——

WRITTEN TEST

253. **1.** A company employs 17 laborers at \$3.50 per day; 28 mechanics at \$6.60 per day; 9 teamsters at \$4.50 per day; 2 bookkeepers at \$25 a week; and a superintendent at \$3120 a year. What is the weekly pay roll of the company?

2. A man sold a farm of $248\frac{1}{2}$ A. at $\$45\frac{3}{4}$ per acre, and with the proceeds bought another farm of 125 A. What price per acre did he pay for the new farm?

3. A person owned $\frac{5}{8}$ of a mine and sold $\frac{3}{4}$ of his share for \$1710. What was the value of the mine?

4. Find the net cost of the following bill of hardware: 12 kegs cut nails, 8*d*., 1200 lb. @ 5 ¢ per pound; 5 doz. handsaws, 26 in., @ \$25 per dozen; 4 doz. carpet stretchers at \$5.50 per dozen; 20 doz. garden rakes @ \$6.25 per dozen; and 12 doz. try squares @ \$7.50 per dozen. Less 20% on the entire bill.

5. Find the total cost of the following: 8 pieces broadcloth, 56^1, 58^3, 57, 59^2, 55^1, 60^1, 61^2, 58^3 yd. at \$6.25; 5 pieces corduroy, 47^2, 44^2, 43^3, 46^2, 45^1 yd. at \$1.75; and 7 pieces storm serge, 43^1, 47^2, 48^3, 44, 45^2, 48^2, 47^3 yd. at \$2.25.

6. A man had on hand in the morning cash in the safe amounting to \$206.20, and in the bank \$1379.30. During the day he received cash in currency amounting to \$909.24 and checks amounting to \$489 36. Cash deposited in the bank, \$604.37. Checks drawn on the bank account, \$3.97, \$47.86, \$396.25, \$49.83, \$246.97. Cash paid out in bills and coin, \$49.86. Show the condition of the cash and bank accounts in the evening.

7. Find the total cost of 64 bags of sugar, each containing $12\frac{1}{2}$ lb. at $13\frac{1}{2}$ ¢ a pound; 19 bbl. of flour at \$13.50 a barrel; 1740 eggs at 47 ¢ a dozen; 375 pineapples at \$12.75 per hundred; 42 qt. of walnuts at $14\frac{1}{2}$ ¢ a quart.

8. A stenographer gets $\$116\frac{2}{3}$ per month. How long will it take him to pay for a house and lot costing \$2300, if his yearly expenses average \$1000? Reduce time to years, months, and days.

9. Find the cost of the following: 348 eggs at 43 ¢ per dozen; 643 lb. sugar at $13\frac{1}{2}$ ¢ per pound; and 7750 lb. coal at \$9.75 per ton.

10. In a hospital having 1048 inmates there were consumed the following articles: Meat, fresh, 287,138 lb., average price, \$.146; meat, smoked, 7614 lb., average price \$.23; meat, canned, 7239 lb., average price \$.126; poultry, 4128 lb., average price, \$.234. Find the total cost and the cost per capita.

PRACTICAL MEASUREMENTS

TIME

254. The time between two dates may be expressed in three ways; viz. compound time, exact time; and bankers' time.

255. Compound time is expressed in years, months, and days. It is determined by the method of compound subtraction. The time from Dec. 14, 1921 to Oct. 11, 1924 is reckoned thus:

1924	10	11
1921	12	14
2	9	27

The compound time is 2 yr. 9 mo. 27 da. Compound time is used in the majority of the ordinary business transactions involving time, especially if the period is more than a year.

256. The method is based on a 360-da. year—12 mo. of 30 da. each. "One half a month" is fifteen days; one quarter of a year is three calendar months; and one half of a year is six calendar months.

257. A month from a given day in any month is the same day in the next month except in those months in which there is no day to correspond with the given day in the month from which the time is reckoned. For instance, 1 mo. from Jan. 14 is Feb. 14, and 5 mo. from Feb. 1 is July 1. But 1 mo. from Jan. 29, 30, or 31 is Feb. 28 (29th in leap year), and 3 mo. from Aug. 31 is Nov. 30, etc.

258. There is a discrepancy of 5 da. per annum between compound time and exact time, but business men generally are willing to forego the difference on account of the facility of reckoning compound time.

259. **Exact time** is expressed in days, or years and days. It is determined by counting the exact number of days between two dates. The exact time from May 3 to Sept. 28 is reckoned thus:

May, 28 days remaining.
June, 30 days.
July, 31 days.
Aug., 31 days.
Sept., 28 days.
148 days.

The first date, May 3, is omitted; the last date, Sept. 28, is included. For exceptions, see Art. 445, note 1.

260. Exact time is used chiefly for periods of time less than one year, by the government in making interest calculations, by bankers in reckoning bank discount, and by some business men.

261. **Bankers' time** is expressed in days, or months and days. It is reckoned by counting months for the whole months and exact days for any remaining part of a month. To illustrate: From Jan. 20 to June 15, is 4 mo. and 26 da. 4 mo. from Jan. 20 is May 20, and from May 20 to June 15 is 26 da.

This method is used by bankers in reckoning interest on mortgages, notes, etc., for fractional periods of time.

PROBLEMS

262. Find the compound time between:

1. May 15, 1923 and Aug. 12, 1926.
2. Apr. 4, 1920 and Jan. 3, 1926.
3. Dec. 26, 1921 and Nov. 13, 1925.
4. Oct. 13, 1923 and June 1, 1926.
5. Sept. 1, 1921 and July 30, 1926.
6. Sept. 28, 1924 and Feb. 14, 1927.
7. Nov. 20, 1919 and Jan. 1, 1926.
8. Dec. 31, 1922 and May 1, 1924.
9. June 30, 1921 and May 15, 1924.
10. July 4, 1920 and Dec. 25, 1924.

Find the exact time and the bankers' time between:

11. May 4 and Oct. 9.
12. April 24 and Dec. 1.
13. Feb. 15 and Nov. 10.
14. June 30 and Nov. 20.
15. Jan. 15 and July 1.
16. Feb. 18 and June 12.
17. May 13 and Dec. 10.
18. June 28 and Nov. 4.
19. Apr. 16 and Sept. 5.
20. Mar. 30 and Sept. 8.

Find the compound time and the exact time between:

21. Apr. 1, 1921 and Jan. 30, 1923.
22. Mar. 30, 1922 and Feb. 28, 1923.
23. Dec. 26, 1918 and Jan. 1, 1923.
24. Nov. 20, 1922 and July 21, 1924.
25. Oct. 18, 1921 and July 8, 1924.
26. Sept. 17, 1922 and Aug. 12, 1923.

27. July 16, 1920 and June 1, 1923.

28. June 14, 1922 and May 5, 1924.

263. This is a condensed time-table of the Atchison, Topeka and Santa Fe Railway, extending from Chicago to San Francisco.

CHICAGO, KANSAS CITY AND CALIFORNIA

WEST—Via La Junta and Albuquerque

Mls	STATIONS		1 Daily	The Scout By Days	9 Daily	The Navajo By Days
			PM		AM	
0	Lv Chicago......	A.T.&S.F.	8.55	Su Mo Tu We Th Fr Sa	9.25	Su Mo Tu We Th Fr Sa
451	Ar Kansas City..	“	9.25	Mo Tu We Th Fr Sa Su	9.30	“ “ “ “ “ “ “
451	Lv Kansas City..	A.T.&S.F.	10.00	Mo Tu We Th Fr Sa Su	10.00	Su Mo Tu We Th Fr Sa
578	“ Emporia......	“	2.30	“ “ “ “ “ “ “	1.45	Mo Tu We Th Fr Sa Su
684	“ Hutchinson...	“	6.30	“ “ “ “ “ “ “	4.35	“ “ “ “ “ “ “
804	“ Dodge City...	“	10.07	“ “ “ “ “ “ “	7.25	“ “ “ “ “ “ “
1007	“ La Junta.....	“	4.55	Tu We Th Fr Sa Su Mo	12.55	“ “ “ “ “ “ “
1222	“ Las Vegas....	“	2.00	“ “ “ “ “ “ “	7.55	“ “ “ “ “ “ “
1287	Ar Lamy........	“	4.55	“ “ “ “ “ “ “	10.40	“ “ “ “ “ “ “
1354	Lv Albuquerque..	“	8.30	“ “ “ “ “ “ “	1.30	Tu We Th Fr Sa Su Mo
1515	“ Gallup.......	“	3.10	We Th Fr Sa Su Mo Tu	7.45	“ “ “ “ “ “ “
1702	“ Flagstaff......	“	9.45	“ “ “ “ “ “ “	1.40	“ “ “ “ “ “ “
1786	Ar Williams......	“	10.55	“ “ “ “ “ “ “	2.48	“ “ “ “ “ “ “
1736	Lv Williams......	G. C.	1.15	We Th Fr Sa Su Mo Tu	......	....
1800	Ar Grand Canyon	“	4.00	“ “ “ “ “ “ “	......	....
......	Lv Grand Canyon	G. C.	8.20	We Th Fr Sa Su Mo Tu	8.20	Tu We Th Fr Sa Su Mo
......	Ar Williams......	“	11.00	“ “ “ “ “ “ “	11.00	“ “ “ “ “ “ “
1736	Lv Williams......	A.T.&S.F.	11.05	We Th Fr Sa Su Mo Tu	2.55	Tu We Th Fr Sa Su Mo
1759	Ar Ash Fork.....	“	12.01	“ “ “ “ “ “ “	3.45	“ “ “ “ “ “ “
1936	“ Needles.......	“	5.50	“ “ “ “ “ “ “	8.35	“ “ “ “ “ “ “
2105	“ Barstow......	“	1.55	Th Fr Sa Su Mo Tu We	2.35	We Th Fr Sa Su Mo Tu
2186	“ San Bernardino	“	5.30	“ “ “ “ “ “ “	5.35	“ “ “ “ “ “ “
2246	“ Los Angeles...	“	8.15	Th Fr Sa Su Mo Tu We	8.10	“ “ “ “ “ “ “
2246	Lv Los Angeles...	“	9.45	“ “ “ “ “ “ “	9.45	“ “ “ “ “ “ “
2373	Ar San Diego....	“	1.20	“ “ “ “ “ “ “	1.20	“ “ “ “ “ “ “
2105	Lv Barstow.....	A.T.&S.F.	3.00	Th Fr Sa Su Mo Tu We	3.00	We Th Fr Sa Su Mo Tu
2176	“ Mojave.......	“	4.50	“ “ “ “ “ “ “	4.50	“ “ “ “ “ “ “
2245	Ar Bakersfield....	“	7.45	“ “ “ “ “ “ “	7.45	“ “ “ “ “ “ “
2414	“ Merced.......	“	12.55	“ “ “ “ “ “ “	12.55	“ “ “ “ “ “ “
2558	“ Oakland......	“	7.14	“ “ “ “ “ “ “	7.14	“ “ “ “ “ “ “
2564	Ar San Francisco.	“	7.20	“ “ “ “ “ “ “	7.20	“ “ “ “ “ “ “
			PM		PM	

264. Use Central Time from Chicago to Dodge City; Mountain Time from Dodge City to Borstow, and Pacific Time from Borstow to San Francisco or San Diego.

1. Find the average speed of train No. 1 from Chicago to Kansas City; from Kansas City to Albuquerque; from Chicago to Grand Canyon; from Kansas City to Los Angeles; from Albuquerque to Needles; from Needles to San Francisco; from Chicago to San Francisco.

2. Find the running time of No. 9 from Chicago to Dodge City; from Chicago to La Junta; from Chicago to San Diego; from Chicago to Oakland; from Kansas City to Ash Fork; from Needles to Los Angeles; from Kansas City to San Francisco.

265. Counting forward or backward from a given date.

1. Count forward 76 da. from June 23.

76 da
7 da. remaining in June.
69
31 da. in July
38
31 da. in August
7 September.
Therefore, September 7.

There are 7 da. left in June after June 23. Subtract 7 da. from 76 da., obtaining 69 da. to count forward after June 30. Subtract 31 da. for July, which leaves 38 da. to count forward after July 31. Subtract 31 da. for August, which leaves 7 da. to count into September, or September 7.

2. Count back 83 da. from May 10.

83 da.
10 da. back in May
73
30 da. in April
43
31 da. in March
12 da. back into Feb.
Feb. 28 – 12 da. =
Feb. 16.

In counting backward from May 10, note that 10 da. are subtracted for May, leaving 73 da. to count back from Apr. 30. Subtract 30 da. for April, and 31 da. for March. The 12 da. remaining are to be deducted from the 28 da. in February, which leaves February 16.

Note. Fractional parts of a day are not recognized in law. Hence, as soon as it is past midnight on May 9, it is May 10, and the whole day is considered, in law, to be gone. Therefore, in counting back from the 10th, there are 10 days to count back through to count back out of May.

PROBLEMS

266. Count both forward and backward:

1. 95 da. from July 10.
2. 110 da. from June 30.
3. 57 da. from Apr. 15.
4. 132 da. from June 1.
5. 160 da. from June 30.
6. 173 da. from Oct. 15, 1922.
7. 208 da. from Sept. 1, 1922.
8. 142 da. from Dec. 30, 1921.
9. 111 da. from Jan. 10, 1922.
10. 131 da. from Feb. 1, 1923.
11. 168 da. from Mar. 31, 1922.
12. 174 da. from Feb. 28, 1923.
13. 60 da. from July 1, 1923.
14. 90 da. from May 1, 1922.
15. 89 da. from May 31, 1922.
16. 101 da. from Nov. 1, 1923.
17. 201 da. from July 31, 1923.
18. 179 da. from Aug. 19, 1922.
19. 120 da. from Apr. 30, 1922.
20. 229 da. from Dec. 1, 1923.

PAINTING, PLASTERING, PAPERING, CARPETING

267. Painting and plastering are estimated by the square yard.

There is no definite rule regarding allowances for openings. It is a matter for special agreement, and should be mentioned in the contract when made.

268. Papering is estimated by the roll.

269. Most American wall papers are 18 in. wide, a single roll being 24 ft. long, and a double roll, 48 ft. long. Imported papers vary in width and length.

270. As there is always more or less waste in cutting and matching wall paper, it is better to use double rolls. The exact number of rolls cannot always be determined in advance.

271. In estimating the number of rolls for a room, paper hangers generally deduct the total width of openings from the perimeter of the room. Then they reckon the number of strips to cover the remaining surface. The number of rolls is found by dividing the total number of strips required by the number of whole strips that can be cut from 1 roll.

The spaces over and under the windows, over the doors, etc., are covered with the parts of strips left from each roll.

272. Carpet is estimated and sold by the linear yard. Linoleum is generally sold by the square yard.

273. Carpets vary in width, the more common widths being 1 yd. for Ingrain, and $\frac{3}{4}$ yd. for Axminster, Brussels, Moquette, Velvet, and Wilton.

274. Carpet may be bought in any *length* desired, but only in *whole* strips; hence, in order to determine the number of yards for a floor, the number of strips must be known.* The number of yards is then found by multiplying the yards in 1 strip by the number of strips.

275. A room 16 ft. long, 14 ft. wide, and $8\frac{1}{2}$ ft. high, has 3 windows each 3 ft. by 6 ft. and 1 door 3 ft. by 7 ft. Find the cost of plastering the walls and ceiling at $1.10 per square yard, allowing for the openings.

2 × (16 ft. + 14 ft.) = 60 ft., perimeter of room.
$8\frac{1}{2}$ × 60 sq. ft. = 510 sq. ft. in the walls.
16 × 14 sq. ft. = 224 sq. ft. in the ceiling.
510 sq. ft. + 224 sq. ft. = 734 sq. ft. in ceiling and walls.
3 × 3 × 6 sq. ft. = 54 sq. ft. in the windows.
3 × 7 sq. ft. = 21 sq. ft. in the door.
54 sq. ft. + 21 sq. ft. = 75 sq. ft. in all openings.
734 sq. ft. − 75 sq. ft. = 659 sq. ft., net area of ceilings and walls.

$$\frac{659 \times \$1.10}{9} = \$80.54, \text{ cost of plastering.}$$

First find the perimeter of the room and then multiply the result by the height of the room. The result, 510 sq. ft., is the area of the walls. The ceiling is a rectangle, 16 ft. by 14 ft., whose area is 224 sq. ft. The total area of ceiling and walls is the sum of 510 sq. ft. and 224 sq. ft., or 734 sq. ft. Next find the total area of windows and doors and deduct the result from 734 sq. ft. to show the net area to be plastered. That gives 659 sq. ft. Dividing by 9 sq. ft. to the square yard and multiplying $1.10 by the quotient gives $80.54, as the cost of plastering.

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* If any whole number of strips exactly covered a floor, and there were no waste in matching the pattern, the number of yards for a floor could be found by dividing the area of the floor by the area of a yard of carpet, but these conditions occur so rarely that the statement above may be taken as correct.

276. In writing the dimensions of rooms it is customary to write the length first, then the width, then the height. It is also usual to write 10 ft. thus, 10′, and 8 in., 8″. Hence a room 15 ft. long, 13 ft. wide, and 8 ft. 4 in. high, may be written "a room 15′ × 13′ × 8′ 4″."

1. Find the cost of plastering the walls and ceiling of a room 18′ × 16′ × 9′, allowing for one half the area of 2 windows 3′ × 6′ 6″, and 2 doors 3′ 6″ × 7′, at $1.20 per square yard.

2. A room is 16′ × 12′ × 8′ 6″. It has 3 windows 4′ × 5′ 6″, and 1 door 3′ × 6′ 6″. At 95 ¢ a square yard, find cost of plastering the walls and ceiling, making no allowance for openings.

3. At 80 ¢ a square yard find the cost of plastering the walls and ceiling of a room 26′ × 16′ × 10′ 6″, allowing for one half of the area of 4 windows 4′ × 7′ 6″, and 3 doors 4′ × 8′.

4. Find the cost of kalsomining the walls and ceiling of a room 22′ 6″ by 18′ by 9′ 6″, at $1.40 per square yard, making no allowance for openings.

5. Allowing 100 sq. ft. for openings, find the cost, at $1.05 per square yard, of plastering the walls and ceiling of a room 19′ 6″ × 15′ × 9′.

How many yards of Brussels carpet are required for a room 16 ft. long and 15 ft. wide?

15 ft. ÷ $2\frac{1}{4}$ ft. = $6\frac{2}{3}$ = 7, no. strips.

$\frac{7 \times 16}{3}$, = $37\frac{1}{3}$, no. yards.

Unless otherwise specified, carpet is generally considered to be laid with strips running lengthwise of the room. Hence, to find the number of strips, divide the width of the floor by the width of the carpet. 15 ft. ÷ $2\frac{1}{4}$ = $6\frac{2}{3}$, the number of strips, but as carpet has to be bought in whole strips, 7 strips are necessary. Each strip is 16 ft. long. The number of yards, therefore, is $\frac{1}{3}$ of 7 times 16, or $37\frac{1}{3}$.

6. How many yards of Wilton carpet are required for a room 18 ft. by 16 ft.?

7. A hall 40 ft. long and 30 ft. wide has an ingrain carpet on the floor. Find the cost of the carpet at $2.50 per yard.

8. A parlor is 16′ 6″ × 14′. How many yards of velvet carpet does it take for the floor?

9. A stairway has 12 steps each 10 inches wide. Each step has a riser of 8 inches. How many yards of carpet are required for the stairway?

Find the number of rolls of paper required for a room 14 ft. by 12 ft. 6 in. wide, and 8 ft. 6 in. high, allowance being made for 2 windows each $3\frac{1}{2}$ ft. wide, and for 2 doors each 3 ft. 9 in. wide.

$2 \times (14$ ft. $+ 12\frac{1}{2}$ ft.$) = 53$ ft., perimeter of room.
$(2 \times 3\frac{1}{2}$ ft.$) + 2 \times 3\frac{3}{4}$ ft.$) = 14\frac{1}{2}$ ft., total width of openings.
53 ft. $- 14\frac{1}{2}$ ft. $= 38\frac{1}{2}$ ft., net perimeter to be papered.
$38\frac{1}{2}$ ft. $\div 1\frac{1}{2}$ ft. $= 25\frac{2}{3} = 26$ strips required.
48 ft. $\div 8\frac{1}{2}$ ft. $= 5$ whole strips from a double roll.
26 strips $\div 5$ strips $= 5\frac{1}{5} = 6$, no. rolls required.

First find the perimeter of the room, and from it deduct the total width of all openings. That leaves $38\frac{1}{2}$ ft. as the net perimeter to be papered. The number of strips of paper required is determined by dividing the net perimeter by the width of a roll of paper, hence $38\frac{1}{2}$ ft. $\div 1\frac{1}{2}$ ft. gives $25\frac{2}{3}$, or 26 strips needed. Since paper is usually sold in double rolls, the number of strips that can be cut from 1 double roll is found by dividing 48 ft. by $8\frac{1}{2}$ ft. (the length of 1 strip). That gives 5 strips from the double roll. Since 26 strips are needed, it takes as many double rolls as 5 strips are contained times in 26 strips, or 5 and a fraction, which makes it necessary to buy 6 double rolls.

10. Find the cost of papering a room 20′ × 18′ × 10′6″ at $1.45 a double roll; allow for 2 windows 4′ wide; 1 door 3′6″ wide.

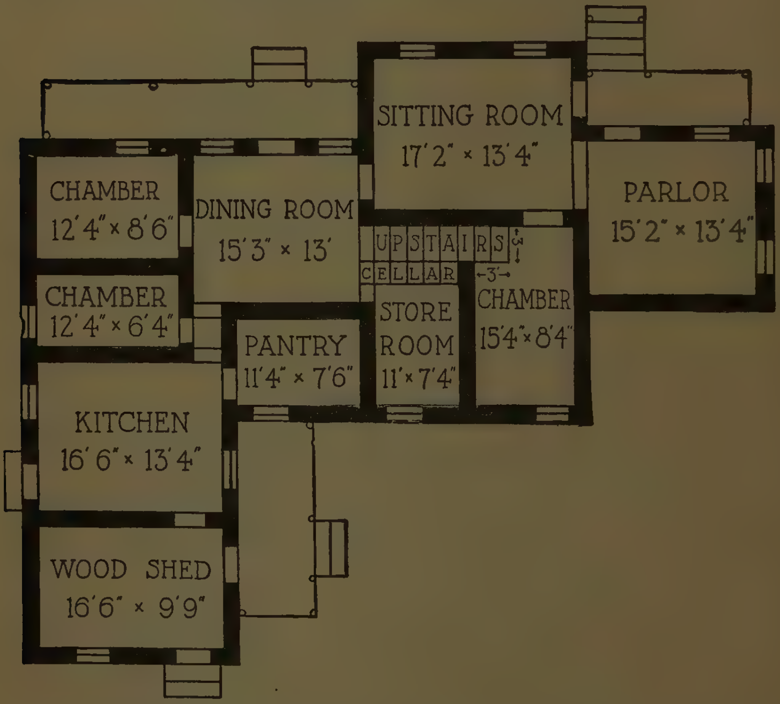
11. Find the cost of papering a room 18′ 6″ × 16′ × 8′ 6″ at 95¢ a double roll; allow for 1 window 4′ 6″ wide; 2 doors 4′ wide.

12. Find the cost of papering a room 24′ × 18′ 6″ × 10′ at $1.30 a double roll; allow for 3 windows 4′ wide; 2 doors 4′ 6″ wide.

13. Find the cost of papering a room 22′ 6″ × 19′ × 9′ 6″ at $1.65 a double roll; allow for 2 windows 3′ 6″ wide; 2 doors 4′ wide.

The diagram below is the first-floor plan of a modern farmhouse.

The rooms have a uniform height of 8′ 6″. The dining room and kitchen each have a wainscot 3′ high. All others have a base board 9″ wide. All windows are 5′ 9″ × 3′. Outside doors are 7′ × 3′ 4″; inside doors are 7′ × 3′ 2″. The double doors between the sitting room and the parlor are 7′ × 5′ 8″. (Measurements of windows and doors include casings.)



14. How many square yards of linoleum will cover the floors of the kitchen and the pantry?

15. Find the cost of ingrain carpet at $2.70 a yard for the dining room (carpet to be laid most economical way).

16. Find the cost of papering the walls and the ceiling of the parlor at $1.25 per double roll.

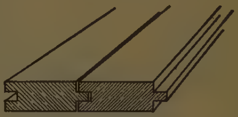
17. How many yards of Brussels carpet will cover the floor of the sitting room?

18. At \$4.20 per yard, find the cost of Wilton carpet for the parlor.

FLOORING

277. Flooring is estimated by the square or by the thousand board feet.

278. When lumber is "tongued" and "grooved," or matched, as it is called, there is some waste, as lumber dealers always measure the lumber at its full width before it is matched. The amount of waste depends upon the width of the boards. Carpenters generally allow one fifth for waste. That is, for 1000 sq. ft. of floor space, 1200 sq. ft. (board feet) of flooring is needed.



279. How many square feet of flooring are required for the parlor represented on page 140?

$15'\,2'' = 15\frac{1}{6}'$; $13'\,4'' = 13\frac{1}{3}'$;

$$\frac{91}{\cancel{6}} \times \frac{\overset{8}{\cancel{40}}}{3} \times \frac{\cancel{6}}{\cancel{5}} = \frac{728}{3} = 242\tfrac{2}{3},$$

Ans. 243 sq. ft.

Reduce feet and inches to feet and multiply the length of the room by the width. The allowance for waste is best reckoned by multiplying by $\frac{6}{5}$. Using the cancellation form of solution, the result is readily obtained.

PROBEMS

280. 1. A floor is 18′ 6″ × 14′ 4″. How many square feet of lumber are required to build it?

2. Find the total amount of lumber required for all the floors in the diagram on page 140, except the woodshed. How much will it cost at \$55 per 1000?

3. Find the cost of a mosaic floor 60′ × 32′ if the tile cost \$1.25 per square foot, and the cost of laying is \$8 per square.

ROOFING

281. Roofing is generally estimated by the square of 100 square feet.

282. The most common roofing materials are shingles, slate, tin, asphalt, asbestos, and copper.

283. Shingles are estimated as having an average width of 4 in. and are laid 4 in., 4½ in., 5 in., or 5½ in. to the weather, depending on the pitch of the roof. The steeper the pitch the greater the length exposed to the weather. The usual estimate per square is shown in the following table:

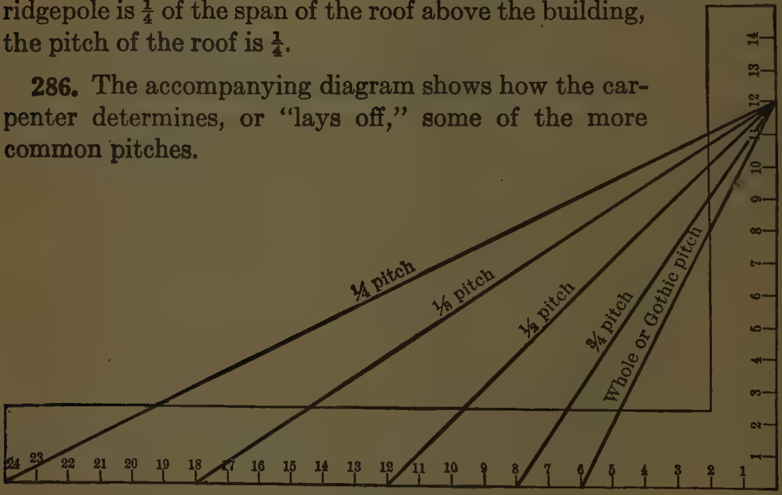
TABLE

LENGTH OF SHINGLE EXPOSED TO THE WEATHER	NUMBER OF SHINGLES PER SQUARE
4 inches	1000
4½ inches	900
5 inches	800
5½ inches	700

284. A bundle contains 250 shingles. A part of a bundle is not sold. Estimates, therefore, have to be made in whole bundles.

285. The pitch of a roof is numerically expressed by dividing the height by the span. When the height of the ridgepole is ¼ of the span of the roof above the building, the pitch of the roof is ¼.

286. The accompanying diagram shows how the carpenter determines, or "lays off," some of the more common pitches.



PROBLEMS

287. **1.** Using a span of 36 ft., make a diagram illustrating one third pitch; one fourth pitch; five twelfths pitch; five eighths pitch; equilateral or true pitch (60°).

2. If the foot of the rafters projects 14″ over the side of the building, find the length of the rafters for each of the pitches in 1.

The rafter is the hypotenuse of a right triangle.

3. A roof is 60 ft. long and 30 ft. wide on each side. How many shingles laid 5 in. to the weather must be bought to cover it?

$$\frac{2 \times 60 \times 30}{100} = 36 \text{ squares.}$$

$$36 \times 800 \text{ shingles} = 28{,}800 \text{ shingles} = 29{,}000 \text{ shingles.}$$

Each side of the roof is a rectangle 60 ft. by 30 ft. The area of both sides divided by 100 sq. ft. in a square, gives 36 squares. It takes 800 shingles for 1 square, and for 36 squares it takes 36 times 800 shingles or 28,800 shingles, making it necessary to buy 29,000 shingles.

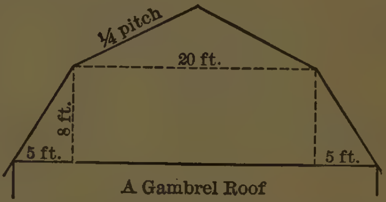
4. A building is 36 ft. wide. The pitch of the roof is one half and the rafters project 15 in. How many shingles laid 4½ in. to the weather are required if the ridgepole is 50 ft. long?

5. How many slates 6 in. wide, 5 in. to the weather, are required for a building 24 ft. wide, the pitch of the roof being five eighths, the ridgepole 36 ft. long, and the rafters projecting 15 in.?

6. A roof 44 ft. long and 18 ft. wide on each side is covered with tin. If each sheet is 18″ × 27″, how many sheets are there?

7. How many slates are needed to cover a roof 32′ × 16′ on a side if the slates are 6″ wide and are exposed 8″ to the weather?

8. The diagram shows part of the end of a barn having a gambrel roof. If the projection of the rafters is 18″ and the ridgepole is 59 ft. long, how many shingles are required for the roof if they are laid 5½ in. to the weather on the lower half of the roof and 4 in. to the weather on the upper half?



A Gambrel Roof

PAVING

288. Paving is estimated by the square foot or by the square yard.

PROBLEMS

289. **1.** Find the cost of paving a street 8 rd. long and 30 ft. wide, at $1.75 per square foot.

2. How many paving blocks 4″ × 8″ are required for a section of street 80 rd. long and 36 ft. wide? Find the cost of paving at $14 per square yard.

3. If paving stones are 5″ × 9″, how many will it take to pave a street 1 mi. long and 40 ft. wide?

4. A man who owns a corner lot with a frontage of 75 ft. and a depth of 100 ft., builds a cement walk 6 ft. wide on the side and front. If he puts the walk on this lot, find the cost at $3.75 per square yard.

PAPER AND BOOKS

290. Paper is of various kinds, and has many uses, some of which are:

1. Wrapping—made of straw, manila hemp, or wood pulp. It is used for wrapping bundles, making paper bags, etc. It is one of the cheapest kinds of paper.
2. News—made of wood pulp. It is of a higher grade than wrapping paper, and is used in making newspapers, and cheap books and magazines. It is sold chiefly in large rolls.
3. Book—made of wood pulp and cotton cloth (called "paper rags"). It is better than news and is used in making the better class of books and magazines. It is sold chiefly in large unfolded sheets; also in large rolls like news.
4. Flat—made of wood pulp and cotton or linen cloth, mixed with an animal size, or glue. The sizing produces a firm smooth surface and prevents the ink from spreading in the paper when written upon with a pen. It is used for correspondence, and for records and documents in which permanency is desired. It is sold wholesale in large unfolded sheets. For retail purposes it is cut into various-sized sheets dependent upon the use to which it is to be put.

291. Paper is sold in large quantities by weight; in small quantities, by the quire or ream. In some cases a ream consists of 500 sheets instead of 480.

292. Common Sizes of Paper

Book	Flat	
25″ × 38″	Cap	14″ × 17″
28″ × 42″	Demy	16″ × 21″
30½″ × 41″	Folio	17″ × 22″
32″ × 44″	Medium	18″ × 23″
35″ × 46″	Super Royal . . .	20″ × 28″

The 25″ × 38″ size is the basis from which the weights of other sizes of book paper are reckoned. A ream 25″ × 38″ weighs 40 lb., 50 lb., 60 lb., etc., according to the quality of the paper.

293. Formerly the terms folio, quarto, 8vo, 12mo, etc., indicated the number of times a sheet was folded in making a book. For instance, folio was the name given to a book made of sheets which had been folded but once, thus making two leaves or four pages from each sheet; a quarto indicated a book made of sheets folded twice, making four leaves or eight pages; etc.

At the present time the terms folio, quarto, octavo, or 8vo, have reference to the size of the page, regardless of the number of times a sheet is folded.

294. The following sizes are the commercial standard board (cover) measurements of various kinds of books:

Quarto	$12\frac{5}{8}'' \times 8\frac{1}{4}''$	16mo	$6\frac{7}{8}'' \times 4\frac{1}{4}''$
8vo	$9\frac{3}{4}'' \times 6''$	18mo	$6\frac{1}{8}'' \times 3\frac{7}{8}''$
12mo	$7\frac{5}{8}'' \times 4\frac{3}{4}''$	32mo	$4\frac{5}{8}'' \times 3\frac{1}{8}''$

In practice any of the above sizes may run large or small; hence it is impoossible to tell the exact size of a book by the terms folio, quarto, octavo, etc.

295. Most books, except very large ones, are printed in 16's or 32's; that is, each sheet has 16 or 32 pages on each side. The sheets are then folded and cut into two or four sections called "signatures." Each signature contains 16 pages. Hence a sheet is folded to make either 32 or 64 pages.

296. 1. If a sheet is folded into 4 signatures, each page $7\frac{1}{2}'' \times 4\frac{7}{8}''$, which of the above sizes of paper is it best to use?

4 signatures = 64 pages = 32 pages on one side of sheet.
32 pages = 8 pages by 4 pages.
$4 \times 7\frac{1}{2}'' = 30''$, width of sheet.
$8 \times 4\frac{7}{8}'' = 39''$, length of sheet.
Hence the size $30\frac{1}{2}'' \times 41''$ is best.

2. A book containing 448 pages is printed in 32's. If the pages are $7\frac{3}{4}'' \times 5\frac{1}{4}''$, what size paper would be used? How many reams (500 sheets) would be required to print 5000 copies of the book?

32's means 32 pages on one side of a sheet.
32 pages = 8 pages by 4 pages.
$4 \times 7\frac{3}{4}'' = 31''$, width of sheet.
$8 \times 5\frac{1}{4}'' = 42''$, length of sheet.
Hence, use sheets $32'' \times 44''$.
448 pages ÷ 64 pages (on 1 sheet) = 7, no. sheets in 1 book.
$\frac{5000 \times 7}{500} = 70$, no. reams.

3. If 70 sheets of strawboard 26″ × 38″ weigh 50 lb., what will be the cost of 1000 sheets of the same grade of strawboard $8\frac{1}{2}''$ × 17″ at $28 per ton?

First cause : Second cause :: First effect : Second effect.
70 sheets : 1000 sheets
26 in. : $8\frac{1}{2}$ in. :: 50 lb. :? lb.
38 in. : 17 in.

$$\frac{1000 \times 8\frac{1}{2} \times 17 \times 50}{70 \times 26 \times 38} \text{ lb.} = 104\tfrac{1}{2} \text{ lb.}$$

$$\$28 \div 2000 = \$.014 \text{ per pound.}$$

$$104\tfrac{1}{2} \times \$.014 = \$1.46, \text{ cost.}$$

By the "cause and effect" method of stating the proportion, it is readily seen that the first cause is the 70 sheets 26″ × 38″, and that the corresponding effect is 50 lb. The second cause is the 1000 sheets $8\frac{1}{2}''$ × 17″, leaving the second effect to be found. Dividing the product of the means by the given extreme gives the other extreme, or $104\frac{1}{2}$ lb., the weight of 1000 sheets $8\frac{1}{2}''$ × 17″. Multiplying by the price per pound gives $1.46, the required cost.

With a little practice the student should be able to write the above statement directly in the form for solution, thus:

$$\frac{50 \times 8\frac{1}{2} \times 17 \times 1000 \times \$28}{70 \times 26 \times 38 \times 2000} = \$1.46$$

reasoning in this way: If 70 sheets 26″ × 38″ weigh 50 lb., one sheet will weigh $\frac{1}{70}$ of 50 lb., and 1 sq. in. of 1 sheet will weigh as much as 26 × 38 is contained times in the weight of one sheet. Thus far the statement would be $\frac{50}{70 \times 26 \times 38}$ = weight of 1 sq. in. If 1 sq. in. of 1 sheet weighs that much, one sheet $8\frac{1}{2}''$ × 17″ will weigh $8\frac{1}{2}$ × 17 times that, and 1000 sheets will weigh 1000 times that result. To this point the statement is $\frac{50 \times 8\frac{1}{2} \times 17 \times 1000}{70 \times 26 \times 38}$. Having the weight, divide by 2000 lb. to a ton, and multiply by the price per ton. The complete statement is as given above.

PROBLEMS

297. 1. If two signatures are printed from one sheet, what size paper is best for a book whose pages are $11\frac{1}{4}''$ × $8\frac{1}{2}''$?

2. A 24-signature book is printed in 32's. If the pages are $6\frac{3}{4}''$ × 5″, what size of sheet would be best to use? How many volumes could be printed from 100 reams (480 sheets) of paper?

3. If 1 ream (500 sheets) of book paper 26″ × 38″ weighs 50 lb., find the cost of 70 reams of the same grade 28″ × 42″ at $85 per ton.

4. A ton of flat paper 17″ × 22″ costs \$.15 a pound and is made into 10¢ notebooks $5\frac{1}{2}$″ × $8\frac{1}{2}$″, of 64 pages each. If 12 sheets of paper weigh 1 lb., and the cost of manufacturing the books including the covers is \$1 per hundred, find the profit.

5. If a ream of Demy weighs 50 lb., and a firm is charged for 1075 lb. at 22¢ per pound for 80 reams 8″ × $10\frac{1}{2}$″, how much too great is the charge? (A discrepancy of 1 lb. in 20 lb., or 5%, is not considered an overcharge in the paper trade.)

The regulation size of letter paper is 8″ × 10″; of note paper is 5″ × 8″. (Commercial note is $5\frac{1}{2}$″ × $8\frac{1}{2}$″.)

6. How many reams of paper, letter size, can be cut from 2000 sheets 16″ × 21″?

7. If 24 reams 17″ × 22″ (500 sheets to a ream) are cut into commercial note size and sold at \$1.10 per ream (480 sheets to a ream), what is the amount received?

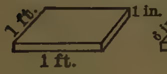
8. How many pads, 80 sheets each, $5\frac{3}{4}$″ × 9″, can be made from 4000 sheets 18″ × 23″?

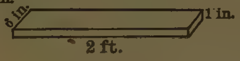
9. How many reams (500 sheets to a ream) Super Royal are required for 500 ledgers of 320 pages each, if the page is 10″ × 14″?

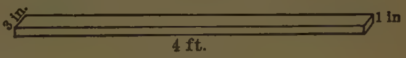
10. A publisher has an order for 10,000 copies of a book containing 512 pages, each page $8\frac{1}{2}$″ × $5\frac{1}{2}$″. He prints in 32's, and the paper costs \$100 per ton. If the same grade of paper 25″ × 38″ weighs 50 lb. to the ream, find the cost of the paper for the order.

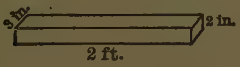
LUMBER

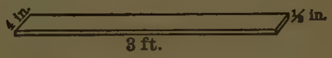
298. The unit of lumber measure is the **board foot**. A board foot is a square foot of board one inch (or less) thick. Each of these illustrations represents a board foot of lumber.











The volume of a board foot of lumber is thus seen to be 144 cu. in., except when the lumber is less than 1 in. thick, in which case a board foot is equal to a square foot of surface of the board.

299. How many board feet are there in 48 scantlings 2″ × 4″ × 16′ (2 in. × 4 in. × 16 ft. long)?

$$\frac{\overset{4}{\cancel{48}} \times 16 \times 2 \times 4}{\cancel{12}} = 512,$$ no. of board feet in 48 scantlings.

When there is more than one piece of lumber, the simplest method is to find the total length of all the pieces, and then multiply by the width and by the thickness, in inches, and divide the product by 12, using cancellation, as shown in the solution.

LUMBER YARD PRACTICE

300. To find the number of board feet in one piece of lumber.

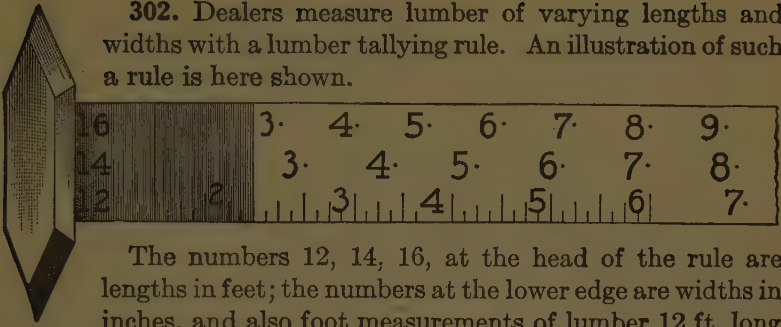
4 inches wide, take $\frac{1}{3}$ of the length.
6 inches wide, take $\frac{1}{2}$ of the length.
8 inches wide, take $\frac{1}{3}$ less than the length.
9 inches wide, take $\frac{1}{4}$ less than the length.
10 inches wide, take $\frac{1}{6}$ less than the length.
12 inches wide, take the length.
14 inches wide, add $\frac{1}{6}$ to the length.
15 inches wide, add $\frac{1}{4}$ to the length.
2 in. by 4 in. (2 × 4 = 8) take $\frac{1}{3}$ from the length.
2 in. by 8 in. (2 × 8 = 16) add $\frac{1}{3}$ to the length.
8 in. by 8 in. (8 × 8 = 64) take $5\frac{1}{3}$ times the length, etc.

NOTE. When no thickness is mentioned, lumber is understood to be one inch thick.

PROBLEMS

301. Find, mentally when possible, the number of board feet in the following:

1. 20 pcs. 3″ × 8″ × 14′
2. 44 pcs. 4″ × 4″ × 18′
3. 60 pcs. 12″ × 1″ × 16′
4. 90 pcs. 2″ × 4″ × 16′
5. 160 pcs. 6″ × 8″ × 12′
6. 16 pcs. 10″ × 12″ × 18′
7. 120 pcs. 9″ × 11″ × 16′
8. 75 pcs. 4″ × 6″ × 14′
9. 49 pcs. 8″ × 8″ × 22′
10. 200 pcs. 4″ × 8″ × 10′
11. 14 pcs. 3″ × 5″ × 14′
12. 18 pcs. 4″ × 9″ × 16′
13. 28 pcs. 6″ × 12″ × 18′
14. 38 pcs. 12″ × 12″ × 16′
15. 560 pcs. 10″ × 1″ × 16′
16. 960 pcs. 8″ × 1″ × 14′
17. 760 pcs. 6″ × 1″ × 18′
18. 1000 pcs. 5″ × 1″ × 16′



302. Dealers measure lumber of varying lengths and widths with a lumber tallying rule. An illustration of such a rule is here shown.

The numbers 12, 14, 16, at the head of the rule are lengths in feet; the numbers at the lower edge are widths in inches, and also foot measurements of lumber 12 ft. long and 1 in. thick. The other lines of numbers opposite 14 and 16 are the number of board feet in 14 ft. and 16 ft. lumber, respectively, for varying widths. For instance, a board 16 ft. long and 1 in. thick contains 8 board ft. if it is 6 in. wide, as shown by the 8 above the 6 in the lower line. As lumber is measured, the number of board feet is called off and recorded on a tally sheet. The numbers at the top of the tally sheet are foot measurements. The tallies in the columns are the number of pieces containing the number of board feet indicated at the top of the column. The total number of board feet in any column is the product of the number at the top by the number of tallies in the column.

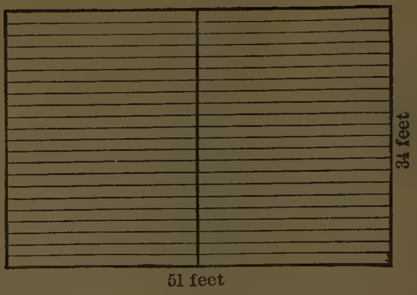
Tally Sheet

10	12	14	13	13	20	22	24
𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 111	𝍸 𝍸 𝍸 𝍸 𝍸 111	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 1	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 11	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 111	𝍸 𝍸 𝍸 𝍸 𝍸 11	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 11

The tallies are grouped in fives for convenience in counting. The oblique line through the group of four is the fifth tally in the group.

Problems of this sort are an excellent training in alertness and accuracy. This is essentially a class exercise, and it is suggested that teachers provide themselves with original problems for class use.

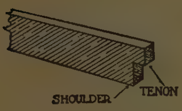
303. This diagram shows the framework of the ground floor plan of a house 34′ × 51′. The sills are 6″ × 8″. The side sills are spliced 5 ft. in the middle. The joists are 3″ × 8″, and are 18″ apart from center to center. Find the cost of the lumber at $38 per M.

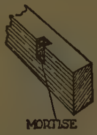


3 sills 6″ × 8″ × 34′ = 408 ft.
4 sills 6″ × 8″ × 28′ = 448 ft.
42 joists 3″ × 8″ × 26′ = 2184 ft.
Total, 3040 ft.

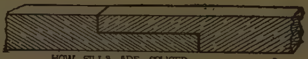
3.040 × $38 = $115.52, cost.

The three cross sills are each 34 ft. long, making 408 board feet. The side dimensions are so long as to make splicing necessary. The length of each piece of timber is found by adding the length of the splice to the length of the sill and dividing into two equal parts. (5 + 51) ÷ 2 = 28, no. ft. The four pieces contain 448 board feet.





Since the sills are 6″ thick, the width inside the sills is 33′. To find the number of joists divide 33′ by 18″, and subtract 1 from the quotient. 33′ ÷ 18″ = 22; 22 − 1 = 21. (The number of spaces is 22, but the number of joists is one less. Prove it by counting spaces and joists in the diagram.) As one joist reaches only to the middle sill, there are 2 times 21, or 42, joists in all. They are mortised into the sills 2″ at each end. The total width of the three cross sills is 18″, which, deducted from 51′, leaves 49′6″, net length of house exclusive of the sills. One half of 49′ 6″ equals 24′ 9″, length of joist exclusive of the tenon, — 2″ at each end,— which makes total length of a joist 25′1″. (24′9″ + 2″ + 2″.) But as lumber is sawed in even foot lengths, a joist 26′ long must be bought. 42 joists 3″ × 8″ × 26′ = 2184 board feet.



HOW SILLS ARE SPLICED

The total amount of lumber is 3040 ft., which at $38 per M is worth $115.52

NOTE. Lumber is practically never cut in an odd number of feet lengths, as 13′, 15′, 19′, 25′, but in even numbers, as 10′, 12′, 16′, 22′, etc. Hence, when the actual length required in building is an odd or fractional number, the nearest length in *even* number of feet greater is purchased.

PROBLEMS

304. 1. Estimate the cost of the lumber, at $44 per M, for a similar plan 25′ 4″ × 40′. The sills are 8″ × 8″, those on the sides being spliced 4 ft. The joists are 3″ × 8″, and are 16″ from center to center.

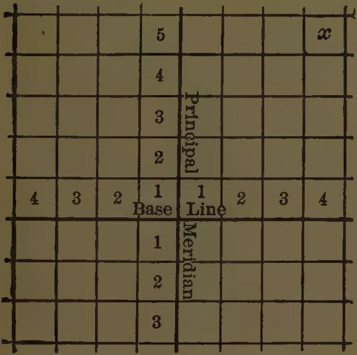
2. A house is 25 ft. by 30 ft. The sills are 6″ × 6″. A sill extends across the middle of the plan as in the preceding diagram. The joists are 3″ × 6″ and are 16″ from center to center. How many feet of lumber are required for the sills and joists?

3. Make a diagram of an original plan similar to those of the preceding problems, and estimate the amount of lumber necessary to construct.

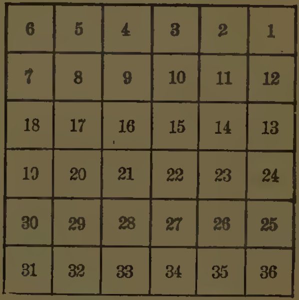
LAND

305. Land areas are estimated by the acre of 160 sq. rd.

306. In laying out public lands, surveyors select a north and south line as a **principal meridian**, and an east and west line as a **base line.**



Townships, 6 miles square



6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

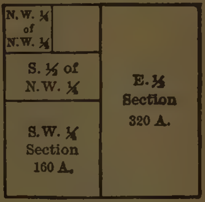
A Township divided into Sections

Other lines 6 mi. apart are run parallel to the principal meridian and to the base line, thus dividing the land into **townships**, as shown in the diagram.

Rows of townships north and south are called **ranges**, and are numbered both east and west from the principal meridian. The townships are also numbered north and south of the base line. Thus, Township *x* is 5 north, in the 4th range east of the principal meridian.

Each township is again divided into squares by parallel lines 1 mi. apart. Each square is called a section, and contains 640 A.

The sections in a township are always numbered as shown in the diagram. If the diagram of a township on p. 151 represents Township x of the preceding diagram, section 17 would be designated as section 17 of township 5 north, in the 4th range east of the principal meridian.



A Section

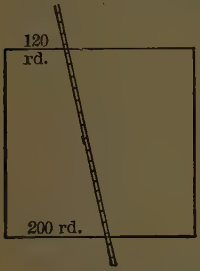
Sections are divided into halves, quarters, half-quarters, quarter-quarters, etc.

PROBLEMS

307. 1. A man owned the southeast quarter of a section of land He bought the south half of the northeast quarter, and the northeast quarter of the northeast quarter of the same section. Make a diagram of his original farm, adding thereto the areas bought, showing number of acres in each parcel.

2. A man owning the north half of a section bought the east half of the southeast quarter, and the west half of the southwest quarter of the same section. How many acres of land has he? Draw a diagram of his farm.

3. A railroad extends through a section of land as shown in the diagram. It occupies a strip 4 rd. wide. Find the land damage for the right of way at $40 an acre.



4. Find the cost at $.65 a rod of building a fence around a section of land; a quarter section of land.

5. Your deed shows that you own the southerly half of section 8 in Clark Co., S. Dak. Draw a township plan, and locate your land. Locate a house in the southeastern corner of your land. Locate a haystack in the northwestern corner of your land. Find distance in a direct line from the house to the haystack.

6. How many rods of fence are required to inclose each of the three parcels in problem 1?

7. How many rods of fence will inclose the combined tracts mentioned in problem 2?

308. An **estimate** is an approximate result obtained by making a rough calculation of a given problem. Estimates are often made by inspection, and in advance of careful, accurate solution.

1. Estimate the quantity of lumber required to build a porch 8 ft. wide along the front of a house 25 ft. wide. The flooring is of matched lumber and is laid on joists 2″ × 6″. The joists are 17 in number and are supported by sills 8″ × 8″. The porch is ceiled with matched lumber. The ceiling is nailed to 11 joists each 2″ × 4″. There are 13 rafters 2″ × 4″. The roof boards are 6″ wide.

2. Estimate the quantity of lumber required to build shelves on one side of a room 40 ft. long, if there are 7 shelves one above the other 14″ apart, and 12″ deep. The shelves are supported by board partitions 30″ apart.

3. Reckon the cost of painting (2 coats) a house 28′ × 36′, the eaves being 18 ft. from the top of the cellar wall. Use local prices of paint and labor. One gallon of paint will cover 400 sq. ft. first coat, and 500 sq. ft. second coat. One man will spread 3 gal. of paint first coat and 2 gal. second coat. Add 15% for contractor's profit.

4. Make a list of articles necessary for furnishing a 5-room apartment. Estimate the cost for each room, the total cost not to exceed $1000.

5. Assume that your house has been destroyed by fire. Make an insurance inventory of the articles burned with the values thereof, as a basis of settlement with the insurance company.

6. A quarter section of land in the form of a square is to be divided into 4 fields of 40 acres each, in such manner as to require the least amount of fencing. Make a diagram of the quarter section showing the position of the fences. Estimate the quantity of lumber required for a board fence inclosing the four fields with 6 in. boards, if the fence is 4 boards high. If the posts are 8 ft. apart, how many posts are needed?

7. A man owns a city lot 100 ft. deep and 75 ft. front. He desires to build a picket fence on the two sides and across the back end of the lot. Consult local builders and lumbermen for prices and materials, and estimate the cost of the fence.

8. Compare the cost of the picket fence in problem 7 with the cost of a moderately priced iron fence. Consult a hardware dealer for prices and lasting qualities, and determine which is the more economical, the picket or the iron fence.

9. Investigate and find the cost of a thorn hedge across the front of the lot mentioned in problem 7. Make allowance for gateways for a walk to the house and for a driveway leading to the garage.

10. Consult local contractors and determine the relative costs of covering a roof 36 ft. long having 20 ft. rafters, with #1 cedar shingles and with slate. Which is the more economical, considering durability?

11. A man desires to inclose his front porch with portable wire screens for summer use, and with portable glass sash for use as a sun parlor in winter. If the porch is 28 ft. long and 12 ft. wide, estimate the cost of the screens and sash.

12. It is estimated that there are 13,000,000 automobiles in the United States. Assuming that each automobile requires, on the average, four new tires a year, and that the average price of tires is $35, what is the approximate value of the tire industry in the United States per annum?

FARM PROBLEMS

309. 1. A field of potatoes is one quarter of a mile long. If the rows are 3 ft. 8 in. apart, how many rows make an acre?

2. If the field in problem 1 yields 10 bu. to the row, how many bushels will there be from 10 acres?

3. If potatoes shrink 1% a month for five months, what will be the loss in weight on the above crop of potatoes from October to March?

4. If the price of potatoes in October was 70 ¢ and in March it was 80 ¢, did it pay to hold the crop till March before marketing How much was gained or lost by holding the crop?

5. During the month of June one cow gives an average of 3 gal. of milk a day which tests 3.6% of butter fat; another cow gives an average of $2\frac{1}{2}$ gal. of milk testing 4% butter fat. Milk weighs $8\frac{5}{8}$ lb. per gallon. If butter is worth 48¢ a pound, find the value of the butter produced by each of the cows.

6 Complete the following ticket:

TICKET SHOWING WEIGHTS OF HAY

GROSS WEIGHT	WEIGHT OF WAGON	NET WEIGHT	VALUE AT $16.50 A TON
3674	1467	—	—
3860	1490	—	—
4135	1505	—	—
Totals			

7. Prepare a ticket like the above for 6 loads of coal weighing as follows: Gross weights, 4530, 4654, 4850, 5260, 4960, 5210; Weights of wagons: 1650, 1642, 1528, 1470, 1465, 1640. Coal is worth $10.50 per ton.

8. A farmer's corn field is 40 rd. long. If the rows are 3 ft. 8 in. apart each way, how many rows lengthwise of the field make an acre? How many hills of corn are there to the acre?

9. Compare two crops from the field in problem 8, under the following condition: (1st) 3 stalks to the hill, each stalk bearing one good ear of corn, 100 ears making a bushel, and the corn selling for 70 ¢ a bushel; (2d) 5 stalks to the hill, each stalk bearing a small ear of corn, 160 ears being required for a bushel, and the corn selling at 65 ¢ a bushel. Which is the better crop, and how much better per acre?

10. Estimate the number and size of pieces of lumber required to construct a corncrib 24 ft. long, 5 ft. wide at the bottom, 8 ft. wide at the top, and 6 ft. high to the foot of the rafters. The ridge of the roof rises 3 ft., and the rafters project 1 ft. Also estimate the number of board feet of lumber, and the number of shingles required.

GRAPHS

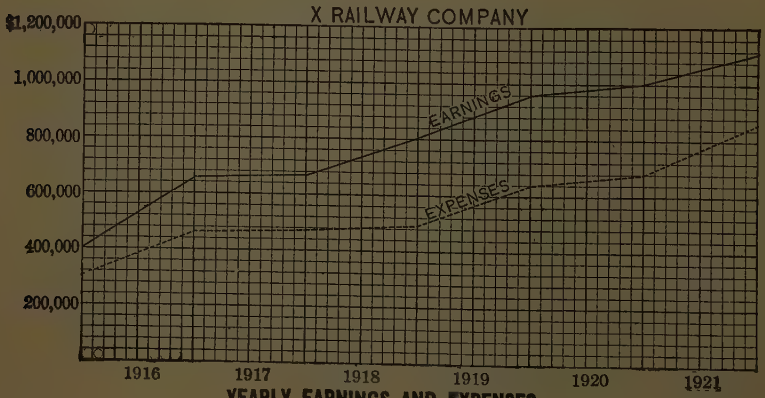
310. **A graph** is a diagram illustrating some relationship. The relationships illustrated by graphs are of several kinds. One kind is the relation existing among a series of values of the same kind occurring in chronological order, such as the monthly or annual income or disbursements of a business organization. Such a graph is spoken of as a "curve," and shows not only the relation of successive values to one another but also the "trend" of the series of values—*i.e.*, whether the tendency is for them to increase or decrease.

311. Another kind of graph shows the relation existing among each of several values and also of each value to the sum of all the values. A circle, bar or other form, used to represent the total, is used.

A third kind of graph is used to show the relation of several values to one another. Such a graph is called a "bar" graph.

312. General relationships are often illustrated by means of pictures showing comparative sizes, numbers, values, etc.

The following two graphs show the relation and the trend of the earnings and expenses of a railroad (1) for a series of years, and (2) each month for one year.



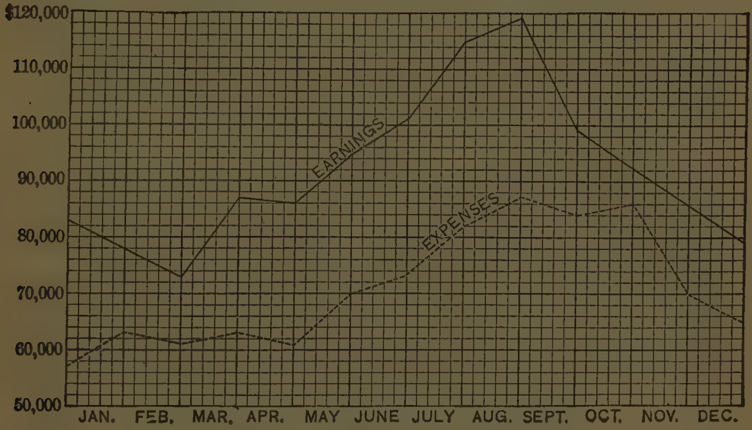
YEARLY EARNINGS AND EXPENSES

From 1916 to 1921 inclusive, the annual earnings and expenses of the X Railway were as follows:

	Expenses	Earnings
1916	$460,000	$ 660,000
1917	470,000	670,000
1918	485,000	800,000
1919	640,000	960,000
1920	680,000	1,000,000
1921	850,000	1,120,000

The monthly earnings and expenses of the same railway for 1921 were as follows:

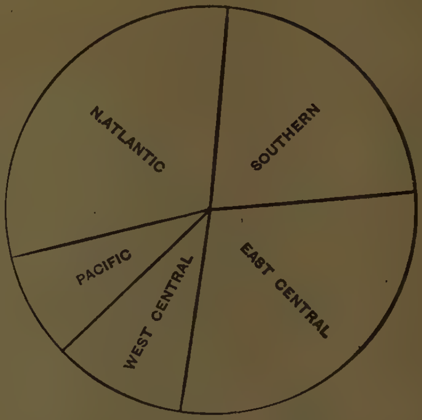
	Earnings	Expenses
January . .	$ 78,000	$63,000
February .	73,000	61,000
March . .	87,000	63,000
April . . .	86,000	61,000
May . . .	95,000	70,000
June . . .	101,000	73,000
July . . .	115,000	82,000
August . .	119,000	87,000
September .	98,500	84,000
October . .	92,000	86,000
November .	85,500	70,000
December .	79,000	65,000



MONTHLY EARNINGS AND EXPENSES 1921

For a recent year the Congress of the United States appropriated sums of money for the promotion of vocational education in the several groups of states as follows: 1. North Atlantic group $935,000. 2. Southern group $690,000. 3. East Central group $930,000. 4. West Central group $280,000. 5. Pacific group $215,000.

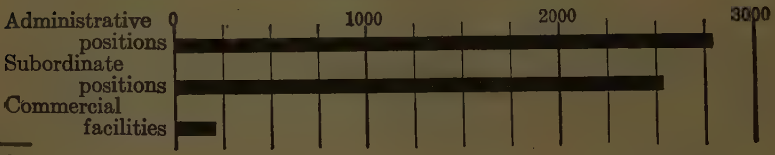
Graphically these results would appear as here illustrated.



At one time the number of disabled soldiers, sailors and marines pursuing certain vocational courses in Business and Commercial Training offered by the Government was as follows:

Administrative positions	2763
Subordinate positions	2581
Commercial facilities	210

The bar graph showing these figures is as follows:



313. 1. Illustrate graphically: During 1921 the monthly earnings and expenses of a given railway were as follows:

Earnings		Expenses	
Jan.	$19,000*	Jan.	$20,500†
Feb.	17,000	Feb.	18,500
Mar.	22,000	Mar.	19,000
April	28,000	April	20,500
May	25,000	May	24,000
June	24,000	June	23,500
July	25,500	July	24,500
Aug.	25,000	Aug.	23,000
Sept.	24,000	Sept.	22,500
Oct.	26,000	Oct.	25,500
Nov.	20,500	Nov.	20,000
Dec.	26,000	Dec.	25,000

*Increase of $1000 over Dec., 1920

†Decrease of $1000 from Dec., 1920

Make graph of chart.

2. The decennial public school population (age 5 to 18 yrs.) of the United States, and the amount expended for public education, since 1878 were as follows:

	Population	Expenditure
1878	14,356,000	79,000,000
1888	17,827,000	124,000,000
1898	21,572,000	194,000,000
1908	24,613,000	371,000,000
1918	27,686,000	763,000,000

Show these facts graphically.

3. Of the 27,686,000 persons, 5–18 yrs. of age, in the United States in 1918, 20,853,000 were enrolled in school, and the average daily attendance was 15,549,000. Show these facts by a bar graph.

4. The apple crop of the United States in a recent year was 147,000,000 bushels. Of that quantity Washington produced 23,000,000 bushels; New York, 17,000,000 bushels; Virginia, 10,000,000 bushels; California, 8,500,000 bushels; Pennsylvania, 8,000,000 bushels; Michigan, 6,500,000 bushels; Missouri, 5,750,000 bushels; Oregon, 5,500,000 bushels.

Show by means (*a*) of a circle, and (*b*) of a bar graph, the relation of the above values to each other and to the total crop.

5. The monthly customs receipts at the New York Custom House for the fiscal year ended June 30, 1920, were as follows: (in nearest quarter million dollars.)

1919		1920	
July	$15,250,000	January	$21,250,000
August	15,500,000	February	19,250,000
September	16,750,000	March	22,250,000
October	16,750,000	April	20,000,000
November	21,000,000	May	17,750,000
December	19,250,000	June	21,250,000

Prepare a graph of these facts.

6. Show by a diagram the relative earnings and expenses of a railway whose quarterly earnings and expenses for 1920 were as follows:

EARNINGS		EXPENSES	
Jan.-Mar.	$ 3,200*	Jan.-Mar.	$3,000†
Apr.-June	8,200	Apr.-June	8,900
July-Sept.	15,500	July-Sept.	8,200
Oct.-Dec.	5,400	Oct.-Dec.	5,800

7. Make a diagram showing the production of beet sugar in the United States for five consecutive years (in nearest 1000 tons).

	TONS
1915-16	874,000
1916–17	825,000
1917–18	765,000
1918–19	761,000
1919–20	764,000

8. Illustrate graphically the production of cotton in the United States since 1865, as shown by the following (numbers of bales given to the nearest 100,000):

	BALES (500 LB. EACH)
1865	2,100,000
1875	4,300,000
1885	6,400,000
1895	7,100,000
1905	10,800,000
1914	16,100,000
1919	11,000,000

9. The total corn crop of the United States for 1919 was 2,917,000,000 bushels. Of this amount, seven states produced as follows:

Iowa	416,000,000
Illinois	301,000,000
Nebraska	184,186,000
Indiana	175,750,000
Ohio	162,800,000
Missouri	155,412,000
Minnesota	118,000,000

Show these results graphically.

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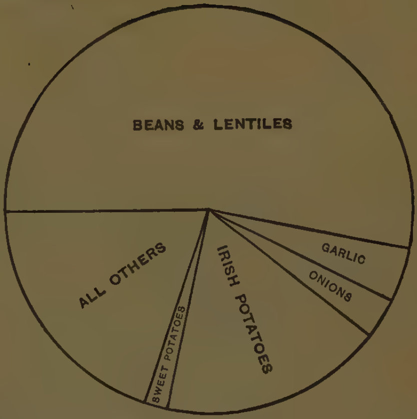
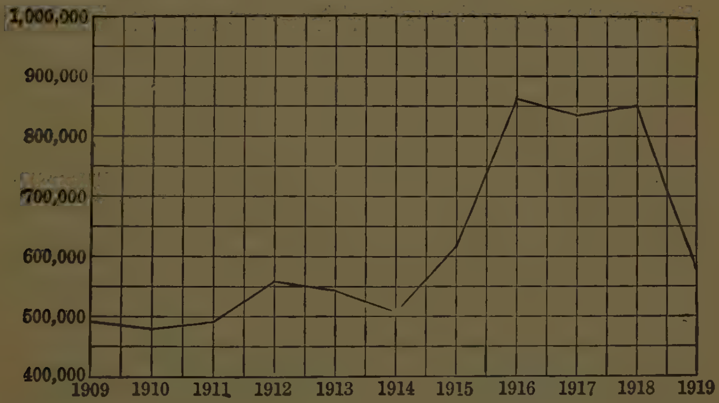
*Decrease from $5200, Oct.-Dec., 1919.

†Decrease from $4400, Oct.-Dec., 1919.

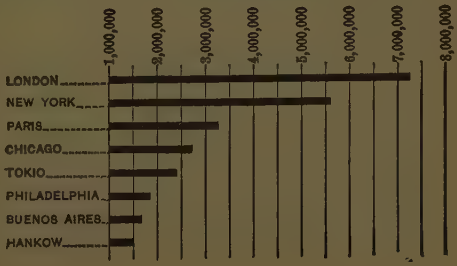
The total value of imports of vegetables into the United States for a recent year was $33,100,000. The proportion of the several kinds of vegetables is shown in the graph at the right. Tabulate, as nearly as possible, the value of each kind. Make estimates in round hundred thousands of dollars.



The following graph shows the amount, in long tons, of the copper mined in the United States from 1909 to 1919, inclusive. Make a tabulation of the number of tons for each year. Estimate values in nearest 10,000 tons.



From the graph at the right tabulate the list of cities given, with their population. Make estimate in nearest 100,000.



EXAMINATIONS

SPEED TEST

Minimum time, thirty minutes; maximum, one hour. Deduct one credit for every minute required beyond minimum time.

314. **1.** Perform the following operations:

$$\frac{\$1.25 \times 28 \times 77 \times 35}{4 \times 11 \times 7 \times 25} = ?$$

$12' \times 16'' \times 2\frac{1}{4}'' =$ board feet

20 planks $16' \times 9'' \times 2\frac{1}{2}''$ @ \$48 per M = \$

2. Find the total cost:

7,480 lb. bran	@	\$22 per ton.
15,870 lb. hay	@	16 per ton.
12,456 lb. meal	@	24 per ton.
9,498 lb. straw	@	12 per ton.

3. Find the number of yards of carpet $\frac{3}{4}$ yd. wide, for rooms,

16′ by 14′
13′ 6″ by 12′ 4″

4. How many double rolls of paper are required for rooms,

$16' \times 12' \times 8'\ 6''$, openings 11 ft. wide?
$18' \times 14' \times 9'$, openings 15 ft. wide?

5. How many paving blocks $12'' \times 5''$ are required for streets as follows:

$1\frac{1}{4}$ mi. long, 60 ft. wide?
$\frac{7}{8}$ mi. long, 56 ft. wide?

6. Find the compound time from Aug. 29, 1921, to July 1, 1923.

7. Find the exact time from May 17, 1922, to Feb. 28, 1923.

8. Count back 150 days from Dec. 1, 1924.

9. A rectangular field 320 rd. long and 240 rd. wide has a board fence around it. If the boards are 16 ft. long and 6 in. wide, and the fence is 4 boards high, how many feet of lumber are there in the fence?

10. Find the gain on each, and the total gain:

Cost	Selling Price	Gain
$ 1.75	$ 2.50	—
.88	1.45	—
135.40	163.50	—
27.75	36.40	—
	Total Gain	

WRITTEN TEST

315. **1.** Find the total cost of:

1350 ft. of pine @ $32.50 per M.
6240 ft. of hemlock @ 24.50 per M.
3650 cedar posts @ 9.50 per C.
4260 lb. of coal @ 7.25 per ton.

2. A railway train runs 240 rd. in $\frac{6}{7}$ of a minute. Find its velocity in miles per hour.

3. Find the cost of 16 pieces of timber 28 ft. long, 14 in. wide, and 9 in. thick, at $57.50 per 1000.

4. Find the cost of plastering the walls and ceiling of a room 18 ft. long, 14 ft. wide, and 8 ft. high, at $1.25 per sq. yd. Allow 100 sq. ft. for openings and baseboard.

5. Make a diagram, and estimate the quantity of lumber required for the framework of a ground floor plan of a house 44 ft. long and 28 ft. wide. The sills are 6 inches square, those on the sides being spliced 4 ft. The joists are 3 in. × 6 in. and are 18 in. apart.

6. A man had a yard 38 ft. long by 27 ft. wide; he reserved two grass plots each 8 ft. square, and had the rest paved with stone at $.45 a square yard. How much did the paving cost?

7. Find the value of a rectangular field $24\frac{1}{2}$ rd. long and $16\frac{1}{4}$ rd. wide, at $47.50 per acre.

8. Around the field in problem 7 is a wire fence. If the fence is 5 wires high and the posts average 8 ft. apart, how many feet of wire and how many posts are needed for the fence?

9. A park is 80 rd. long and 56 rd. wide. Around it is a brick walk 8 ft. wide. The exposed surface of each brick is 4″ × 8″. Find the cost of the brick at $15 per 1000.

PERCENTAGE

316. The term *percentage* includes those subjects in arithmetic in the computation of which $\frac{100}{100}$ is taken as the basis of comparison. All comparisons of values or quantities are expressed in hundreths. Instead of the term *hundredths*, the equivalent Latin expression *per cent* is used.

317. Per cent means by the hundred. The per cent sign (%) is generally used instead of the words per cent.

Observe these equations:

$$\frac{1}{4} = \frac{25}{100} = .25 = 25\%$$

$$\frac{1}{8} = \frac{12\frac{1}{2}}{100} = .12\frac{1}{2} = 12\frac{1}{2}\%$$

$$\frac{2}{3} = \frac{66\frac{2}{3}}{100} = .66\frac{2}{3} = 66\frac{2}{3}\%$$

$$\frac{5}{13} = \frac{38\frac{6}{13}}{100} = .38\frac{6}{13} = 38\frac{6}{13}\%$$

Note that in each case a common fraction has first been reduced to hundredths, and that next the fraction is in its decimal form.

The last step shows the fraction in its "per cent" form. Note also that the per cent form of the fraction is identical with its decimal form except that the *per cent sign is used instead of a decimal point.*

Percentage problems are, therefore, only a continuation of problems in fractions under a new name.

318. Nearly all problems in percentage can be reduced to one of the three type forms explained on page 118. In the language of percentage problems, the type forms would read thus:

1. Find 75 % of $48.
2. $62\frac{1}{2}$ % of what number equals $90?
3. $75 is what per cent of $90?

When reduced to their simplest arithmetical form, the types appear thus:

1. 75% of \$48 = ?
2. 62½% of ? = \$45.
3. ? % of \$90 = \$75?

In each of these examples there are three elements—the multiplicand, the multiplier, and the product. The corresponding percentage terms are **base, rate,** and **percentage.**

319. The **base** is the number or quantity represented by 100%, and is the basis of comparison in any given problem.

320. The **rate** is the number of hundredths or per cent, and shows the *ratio* of the *percentage* to the *base.*

321. The **percentage** is the *product* of the base multiplied by the rate. It is the number or quantity whose ratio or relation to the base is expressed in the rate.

322. What number is 24% of 300?

24% of 300 = 72.

300 is the base because it is the number with which comparison is made.

24% is the rate because it shows the ratio of the percentage to the base.

72 is the percentage because it is the product of the base and the rate. It is the number that is compared with the base.

323. If the percentage is added to the base, the sum is called the **amount.**

324. If the percentage is subtracted from the base, the result is called the **difference.**

325. Since any common fraction can be reduced to an equivalent decimal or per cent form, conversely, any per cent form of a fraction can be reduced to an equivalent decimal or common fraction form.

$\frac{1}{2}=\frac{50}{100}=.50=50\%$, and conversely, $50\%=.50=\frac{50}{100}=\frac{1}{2}$.

$\frac{5}{8}=\frac{62\frac{1}{2}}{100}=.62\frac{1}{2}=62\frac{1}{2}\%$, and conversely, $62\frac{1}{2}\%=.62\frac{1}{2}=\frac{62\frac{1}{2}}{100}=\frac{5}{8}$, etc.

The student must recognize the difference between $62\frac{1}{2}\%$ and $.62\frac{1}{2}\%$. The first expression equals $.62\frac{1}{2} = \frac{62\frac{1}{2}}{100} = \frac{5}{8}$, and the second expression equals $.0062\frac{1}{2} = \frac{62\frac{1}{2}}{10000} = \frac{5}{800}$.

326. When the per cent sign is used, it takes *the place* of the decimal point, or it is equivalent to moving the decimal point *two places to the right.*

To change a decimal to a rate per cent, move the point two places to the right and annex the per cent sign, and, conversely,

To change a rate per cent to a decimal, remove the per cent sign and move the point two places to the left.

$.045 = 4.5\% = 4\frac{1}{2}\%$ $\quad$ $15\% = .15$
$1.25 = 125\%$ $\quad$ $1\frac{1}{4}\% = .01\frac{1}{4} = .0125$, etc.

327. Reduce each of the following to the form indicated:

To the per cent form:

1. .18
2. .25
3. .56
4. .98
5. .05
6. .01
7. $.33\frac{1}{3}$
8. $.62\frac{1}{2}$
9. $.56\frac{1}{4}$
10. .125
11. .025
12. .0025

To both the decimal and the per cent forms:

13. $\frac{1}{4}$
14. $\frac{1}{5}$
15. $\frac{3}{8}$
16. $\frac{1}{16}$
17. $\frac{5}{6}$
18. $\frac{1}{7}$
19. $\frac{7}{9}$
20. $\frac{5}{12}$
21. $\frac{7}{10}$
22. $\frac{3}{2}$
23. $\frac{9}{8}$
24. $\frac{12}{3}$

To the decimal form:

25. 15%
26. 29%
27. 4%
28. 72%
29. 125%
30. $1\frac{1}{4}\%$
31. $12\frac{1}{2}\%$
32. $66\frac{2}{3}\%$
33. 37.5%
34. $\frac{1}{2}\%$
35. $\frac{5}{8}\%$
36. $\frac{2}{3}\%$

To both the decimal form and the common fraction form in its lowest terms:

37. 10%
38. 30%
39. 45%
40. 75%
41. $87\frac{1}{2}\%$
42. $93\frac{3}{4}\%$
43. $\frac{1}{8}\%$
44. $\frac{1}{8}\%$
45. $\frac{9}{16}\%$
46. 580%
47. $137\frac{1}{2}\%$
48. $101\frac{1}{2}\%$

328. In the chapter on Aliquot Parts many of the aliquot or fractional parts of one dollar were learned.

The aliquot or fractional parts of 100% are the same numerically as the like parts of a dollar.

329. In the following table of fractional equivalents, related rates are grouped, the more important rates being in heavy-faced type.

TABLE OF FRACTIONAL EQUIVALENTS

HALF QUARTERS EIGHTHS	SIXTEENTHS	THIRDS SIXTHS TWELFTHS	SEVENTHS NINTHS	MISCELLANEOUS RATES
$50\% = \frac{1}{2}$	**$6\frac{1}{4}\% = \frac{1}{16}$**	**$33\frac{1}{3}\% = \frac{1}{3}$**	**$14\frac{2}{7}\% = \frac{1}{7}$**	**$1\frac{1}{4}\% = \frac{1}{80}$**
	$18\frac{3}{4}\% = \frac{3}{16}$	**$66\frac{2}{3}\% = \frac{2}{3}$**	**$28\frac{4}{7}\% = \frac{2}{7}$**	**$1\frac{2}{3}\% = \frac{1}{60}$**
$25\% = \frac{1}{4}$	**$31\frac{1}{4}\% = \frac{5}{16}$**		$42\frac{6}{7}\% = \frac{3}{7}$	**$2\frac{1}{2}\% = \frac{1}{40}$**
$75\% = \frac{3}{4}$	$43\frac{3}{4}\% = \frac{7}{16}$	**$16\frac{2}{3}\% = \frac{1}{6}$**	$57\frac{1}{7}\% = \frac{4}{7}$	**$3\frac{1}{3}\% = \frac{1}{30}$**
	$56\frac{1}{4}\% = \frac{9}{16}$	**$83\frac{1}{3}\% = \frac{5}{6}$**	$71\frac{3}{7}\% = \frac{5}{7}$	$4\% = \frac{1}{25}$
$12\frac{1}{2}\% = \frac{1}{8}$	$68\frac{3}{4}\% = \frac{11}{16}$		$85\frac{5}{7}\% = \frac{6}{7}$	$5\% = \frac{1}{20}$
$37\frac{1}{2}\% = \frac{3}{8}$	$81\frac{1}{4}\% = \frac{13}{16}$	**$8\frac{1}{3}\% = \frac{1}{12}$**	**$11\frac{1}{9}\% = \frac{1}{9}$**	**$6\frac{2}{3}\% = \frac{1}{15}$**
$62\frac{1}{2}\% = \frac{5}{8}$	$93\frac{3}{4}\% = \frac{15}{16}$	**$41\frac{2}{3}\% = \frac{5}{12}$**	**$22\frac{2}{9}\% = \frac{2}{9}$**	$7\frac{1}{7}\% = \frac{1}{14}$
$87\frac{1}{2}\% = \frac{7}{8}$		**$58\frac{1}{3}\% = \frac{7}{12}$**	$44\frac{4}{9}\% = \frac{4}{9}$	$9\frac{1}{11}\% = \frac{1}{11}$
		$91\frac{2}{3}\% = \frac{11}{12}$	$55\frac{5}{9}\% = \frac{5}{9}$	$10\% = \frac{1}{10}$
			$77\frac{7}{9}\% = \frac{7}{9}$	
			$88\frac{8}{9}\% = \frac{8}{9}$	

330. The labor involved in solving percentage problems is materially diminished by using the common fractional equivalents of rate per cents wherever possible.

331. Problems in percentage are solved in accordance with the fundamental principles of multiplication and division of simple numbers, page 117. In the language of percentage they are stated thus:

PRINCIPLES: 1. Base multiplied by the rate = percentage.
2. Percentage divided by rate = base.
3. Percentage divided by base = rate.

332. To find the percentage, the base and rate being given.

1. Find 43% of $360.

$360 Base
.43 Rate
1080
1440
$154.80 Percentage

43% of $360 means .43 of $360. $360 multiplied by .43 equals $154.80.

10 10

2. Find $16\frac{2}{3}\%$ of \$540.

$$\begin{array}{r} \$540 \\ .16\frac{2}{3} \\ \hline 360 \\ 3240 \\ 540 \\ \hline \$90.00 \end{array}$$

Or, $16\frac{2}{3}\% = \frac{1}{6}$

$\frac{1}{6}$ of \$540 = \$90.

Multiplying \$540 by $.16\frac{2}{3}$ gives \$90. Or, since $16\frac{2}{3}\% = \frac{1}{6}$, take $\frac{1}{6}$ of \$540, which equals \$90.

3. Find $\frac{1}{8}\%$ of \$1250.

$$\begin{array}{l} 8)\underline{\$12.50} \\ \quad \$1.5625 = \$1.56 \end{array}$$

$\frac{1}{8}\%$ means $\frac{1}{8}$ of 1%. 1% of \$1250 is \$12.50; and $\frac{1}{8}$ of \$12.50 = \$1.56.

4. What number is $33\frac{1}{3}\%$ greater than \$480?

$$\begin{array}{rcr} 100\% & = & \$480 \\ 33\frac{1}{3}\% & = & 160 \\ \hline 133\frac{1}{3}\% & = & \$640 \end{array}$$

The base is always equal to 100%. Hence \$480 is equal to 100%. The required number is $33\frac{1}{3}\%$, or $\frac{1}{3}$, of \$480 greater than \$480. $33\frac{1}{3}\%$ of \$480 is \$160. By adding \$480 and \$160, the desired result, \$640, is obtained.

NOTE. It should be observed that the base and the percentage are like numbers.

333. Performing all operations mentally where possible, find:

1. 21% of 200.
2. 35% of 500.
3. $12\frac{1}{2}\%$ of 640.
4. 25% of 700. 175
5. 26% of 700. 182
6. $66\frac{2}{3}\%$ of 960. 640
7. $87\frac{1}{2}\%$ of 1200. 1050
8. 13% of 300.
9. 17% of 400. 6800
10. 6% of 475. 2850
11. 24% of 300. 72
12. $16\frac{2}{3}\%$ of 372. 62
13. $8\frac{1}{3}\%$ of 744. 62
14. $18\frac{3}{4}\%$ of 640. 120
15. 64% of 500. 320
16. 78% of 1000. 78
17. 93% of 800. 744
18. 125% of 960. 1200
19. 80% of 1200. 960
20. 75% of 1500. 1125
21. $6\frac{2}{3}\%$ of 1800. 12
22. $14\frac{2}{7}\%$ of 2149. 214
23. $22\frac{2}{9}\%$ of 1881. 250
24. $31\frac{1}{4}\%$ of 8000.
25. $56\frac{1}{4}\%$ of 3200.
26. 250% of 440. 1100
27. 375% of 600. 2250
28. $2\frac{1}{2}\%$ of 1600. 40
29. $3\frac{1}{3}\%$ of 1260. 42
30. $1\frac{1}{4}\%$ of 888. $11\frac{1}{10}$
31. $\frac{3}{4}\%$ of 84. 63
32. $\frac{5}{8}\%$ of 800. 500
33. $1\frac{1}{2}\%$ of 64.
34. 100% of 128. 128
35. 150% of 630. 945
36. 300% of 25. 75
37. 3% of 6140. 184.20
38. $12\frac{1}{2}\%$ of 8176. 102
39. $58\frac{1}{3}\%$ of 7200. 42
40. $6\frac{1}{4}\%$ of 6560. 410
41. $62\frac{1}{2}\%$ of 6560. 4100
42. $3\frac{1}{3}\%$ of 7230. 24
43. $33\frac{1}{3}\%$ of 7230. 24
44. $333\frac{1}{3}\%$ of 7230.
45. $\frac{1}{10}\%$ of 8400. 24

$1 \times 888 =$

84

Short Method

334. Find 28% of $7500.

$\frac{3}{4}$ of $2800 = $2100. By the commutative law of multiplication (page 117) the product of 28 times 75 equals the product of 75 times 28. Therefore, 28% of $7500 equals 75% of $2800. 75% = $\frac{3}{4}$, and $\frac{3}{4}$ of $2800 = $2100.

335. In like manner find, mentally, the value of each of the following:

1. 16% of $1250.
2. 18% of 333\frac{1}{3}$.
3. 26% of $2500.
4. 44% of $2500.
5. 56% of $5000.
6. 96% of $125.
7. 160% of $3750.
8. 180% of $750.
9. 32% of $375.
10. 36% of $75.
11. 240% of $2500.
12. 320% of $625.
13. 32% of $3125.
14. 124% of $6250.
15. 39% of 666\frac{2}{3}$.
16. 42% of 166.66\frac{2}{3}$.
17. 132% of $2500.
18. 12% of $25.
19. 36% of $87.50.
20. 52% of $75.
21. 48% of $375.

PROBLEMS

336. 1. In a certain school the attendance in 1922 was 425. What was the attendance in 1923 if there was an increase of 20%? 51

2. A house and lot cost $5000. If taxes, repairs, and other expenses amount to $180 per annum, what rent per month must the owner receive in order to clear 6% on his investment? 40

3. If a plant and machinery, valued at $75,000, depreciates at the rate of 5% of its value each year, find its value at the end of the fifth year.

4. The capital stock of a manufacturing company is $150,000. If the gross receipts amount to 32% of the capital stock, and the total expenditures amount to 75% of the receipts, find the profit for the year.

5. If a clerk's salary is $600 the first year of service, and he gets a 10% increase each year for 5 yr., what will be his salary the sixth year?

6. Chemical analysis shows that "Pennsylvania" Portland cement contains the following ingredients in the proportion here stated:

Silica	22%
Iron and Alumina	10.98%
Lime	61.50%
Magnesia	2.47%
Sulphur Anhydride	1.70%
Combustible materials	.85%

Find the amount by weight of each of the ingredients in 5 T. of cement.

7. The following table shows the per cent of the total of each kind of animal exported from the United States in a recent year. Find the value of each of the several kinds of animals.

KIND OF ANIMALS	PER CENT OF TOTAL	VALUE
1 Cattle	53.646	
2 Hogs	5.698	
3 Horses	23.796	
4 Mules	9.907	
5 Sheep	3.082	
6 All other (including fowls)	3.871	
Total	100.%	$12,003,684

8. Whole wheat contains the following:

Water	12%
Protein	13%
Fat	2%
Carbohydrates	71%
Mineral salts	2%

According to the U. S. Department of Agriculture, the average person consumes 5.3 bushels of wheat in a year. Find, to the nearest tenth of a pound, the weight of each of the substances named a person eats in a year in consuming the 5.3 bushels of wheat.

9. One of the large meat packing companies exported in one year 148,487,828 lb. of beef. Four years later the beef exports had increased 298.9%. Find the quantity exported in the latter year.

337. To find the rate, the base and percentage being given.

What is the ratio of 6 to 24? (See page 128.)
6 is what part of 24?
6 is what per cent of 24? or
What per cent of 24 is 6?

In each of these examples, 24 is the basis of comparison, or it is the base. 6 is the number compared with 24; hence it is the percentage.

The ratio of 6 to 24 is expressed as $\frac{6}{24}$ or $\frac{1}{4}$. In each of the examples, the desired relation between the numbers is determined by dividing 6 by 24. $6 \div 24 = \frac{1}{4} = 25\%$. That is, the percentage divided by the base gives the rate (Prin. 14, page 117).

338. 1. A man's income is $1800 a year. If he saves $630 a year, what per cent of his income does he save?

? % of $1800 = $630.
$630 ÷ $1800 = .35 = 35%

The problem is of the third type form as shown in the first equation. Dividing the percentage, $630, by the base, $1800, gives the rate, 35%.

2. $1500 is what per cent greater than $1200?

$1500 – $1200 = $300, amount which $1500 is greater than $1200.

$300 = ? % of $1200?
$300 ÷ $1200 = $\frac{300}{1200} = \frac{1}{4}$ = 25%.

First find how much $1500 is greater than $1200. The problem then becomes, $300 is what per cent of $1200? which, as shown in the solution, is 25%.

To THE STUDENT. Reduce each problem (either mentally or in writing) to its proper type form. Its solution will then be entirely clear to you.

Use the fractional form of division, when possible; reduce the fraction to its lowest terms mentally; and place the result equal to its equivalent rate per cent.

339. Solve, mentally when possible, each of the following:

What per cent of

1. 12 is 6?
2. 18 is 12?
3. 200 is 150?
4. 72 is 56?
5. 25 is 30?
6. 14 is 21?
7. 450 is 225?
8. 4 yd. is 6 ft.?
9. 2 bu. is 3 pk.?
10. $24 is $2.40?
11. $1.75 is $.75?
12. $15 is $.90?

What per cent greater than

13. 54 is 63?
14. 300 is 390?
15. 560 is 630?
16. 270 is 300?
17. $3000 is $4200?
18. 3 pk. is 1 bu.?
19. 1 lb. Troy is 1 lb. avoir.?
20. 1 oz. avoir. is 1 oz. Troy?

What per cent less than

21. 20 is 18?
22. 30 is 20?
23. 50 is 40?
24. 1 bu. is 3 pk.?
25. 1 oz. Troy is 1 silver dollar?
26. A long ton is a short ton?
27. $\frac{1}{2}$ is $\frac{1}{3}$?
28. $\frac{1}{3}$ is $\frac{1}{4}$?

PROBLEMS

340. 1. If in a school there are 160 boys and 192 girls, what per cent of the enrollment are boys? girls?

2. A man asked $180 for his horse. If he accepted $160, what per cent less than his asking price did he accept?

3. A building worth $12,500 is insured for $7500. For what per cent of its value is it insured?

4. In a spelling lesson of 60 words a pupil spelled 3 words incorrectly. What per cent of the words were spelled correctly?

5. A man bought a house for $8000. To pay for it he borrowed $4000 on bond and mortgage at 6%. The other $4000 he paid from his own money. If the house rents for $800 per annum, and he pays $200 a year for taxes, repairs, insurance, etc., besides the interest on the mortgage, what rate per cent does the investment pay him?

6. The following table shows the number of organized wage earners (males) in New York State employed one or more days during the first quarter of a given year. Find, to the nearest tenth per cent, the percentage for each limit of time. Prove the work.

PERIOD	NUMBER EMPLOYED
1–29 da.	15,497
30–59 da.	63,695
60–79 da.	189,756
80 da. or more	60,029

7. A merchant fails, having liabilities amounting to $28,500, and resources, to $19,000. What per cent of his debts can he pay? He owes Thos. Higbee $1846.50. How much will Mr. Higbee receive?

8. The total amounts of exports of agricultural implements from the United States for the six years ending 1919 were as follows:

1914	$21,649,523	1917	$33,264,649
1915	$13,582,849	1918	$32,849,164
1916	$21,229,774	1919	$41,195,494

Show in per cent the annual increase or decrease of exports.

9. The number of pupils registered in high schools in Sept., 1920, in New York City, was as follows:

		Per Cent of Total.
Borough of Manhattan	29058	1.0070
Borough of Brooklyn	27964	
Borough of Bronx	8829	
Borough of Queens	8315	
Borough of Richmond	1526	
Total		

Find the per cent of the total in each borough.

10. The average wholesale price per 100 lb. of dressed beef in different cities in a recent year was as follows: In New York $18.03; Washington $18.41; Chicago $16.33; Detroit $16.74; San Antonio $15.80; Atlanta $17.03; Des Moines $16.03; Cleveland $16.92; Indianapolis $15.48; Austin $15.19; Fort Worth $15.43. Find (*a*) the average for all the cities; (*b*) the per cent the highest price is above the average; (*c*) the per cent the lowest price is below the average.

341. To find the base, the percentage and the rate being given.

18 is $\frac{1}{4}$ of what number?
18 is 25% of what number?
18 is .25 × ?

The equation is in the form of the second type problem. The product (percentage) and one of the factors (rate) are given to find the other factor (base).

$$18 \div .25 = 72, \text{ the base.}$$

To find the base, divide the percentage by the rate.

342. 48 is $18\frac{3}{4}\%$ of what number?

$$18\tfrac{3}{4}\% \times ? = 48.$$
$$48 \div .18\tfrac{3}{4} = 256. \quad \text{Or,}$$
$$18\tfrac{3}{4}\% = \tfrac{3}{16}.$$
$$\tfrac{3}{16} \text{ of the number} = 48.$$
$$\tfrac{1}{16} \text{ of the number} = \tfrac{1}{3} \text{ of } 48 = 16.$$
$$\tfrac{16}{16} \text{ of the number} = 16 \times 16 = 256.$$

On reducing the problem to its simplest type form we have $18\frac{3}{4}\% \times ? = 48$. Since a product and one factor are given, the other factor (base) is found by dividing the product, 48, by the given factor, $18\frac{3}{4}\%$. The result is 256. Or,

By the use of the fractional equivalent of $18\frac{3}{4}\%$, the solution can be made mentally. $18\frac{3}{4}\% = \frac{3}{16}$.

If $\frac{3}{16}$ of the number is 48, $\frac{1}{16}$ of the number is $\frac{1}{3}$ of 48, or 16, and $\frac{16}{16}$ of the number, or the number itself, is 16 times 16, or 256.

To THE STUDENT. Wherever possible, use this fractional method of solution. It is far shorter and quicker than the solution first explained. The first method will have to be used in those problems containing rates that do not have convenient fractional equivalents.

343. Find the number of which

1. 42 is 6%.
2. 18 is 2%.
3. 25 is 20%.
4. 43 is 1%.
5. 86 is 43%.
6. 72 is $12\frac{1}{2}\%$.
7. 90 is $6\frac{2}{3}\%$.
8. 92 is 25%.
9. 106 is $33\frac{1}{3}\%$.
10. 125 is $16\frac{2}{3}\%$.
11. 130 is 50%.
12. 240 is 75%.
13. 280 is $62\frac{1}{2}\%$.
14. 224 is $87\frac{1}{2}\%$.
15. 378 is $56\frac{1}{4}\%$.
16. 663 is 13%.
17. 374 is 17%.
18. 836 is 19%.
19. 1236 is 6%.
20. 360 is $112\frac{1}{2}\%$.
21. 900 is $128\frac{4}{7}\%$.
22. 755 is 250%.
23. 848 is $266\frac{2}{3}\%$.
24. 833 is $116\frac{2}{3}\%$.
25. 48 is $\frac{1}{4}\%$.
26. 321 is $\frac{1}{2}\%$.
27. 424 is $\frac{8}{9}\%$.
28. 375 is $\frac{5}{8}\%$.
29. 15 is $\frac{1}{20}\%$.
30. $5\frac{1}{4}$ is $\frac{1}{8}\%$.
31. $3\frac{2}{3}$ is $\frac{1}{3}\%$.
32. $7\frac{1}{2}$ is $\frac{3}{4}\%$.
33. $4\frac{1}{2}$ is $\frac{3}{8}\%$.

PROBLEMS

344. 1. A young woman, by saving $31\frac{1}{4}\%$ of her earnings, paid for a house and lot costing $1875 in 6 yr. How much did she earn a year?

2. A parlor suite sold for $286, which was 30% more than it cost. How much did it cost?

3. A bookkeeper receives an increase of 10% in his salary each year for four years. His salary for the fifth year is $2196.15. What was his salary the first year?

4. A house rents for $900 a year. The expenses are $250 a year. If the net income is $6\frac{1}{2}\%$, what was the cost of the house?

5. A house was insured for 80% of its value at $\frac{3}{8}\%$. If the premium paid was $22.50, find the value of the house.

6. Of a consignment of eggs, 6% were broken. For the remainder, $45.12 was received at 32 ¢ a dozen. How many dozen eggs were in the consignment?

7. A merchant's sales for October were $28,460.50, for November were $35,827.43, and for December were $46,921.07. If the sales for these three months were 35% of the sales for the entire year, find the average monthly sales for the remainder of the year.

8. A bankrupt can pay 72% of his debts. If his resources amount to $13,475.28, what are his liabilities? How much will a creditor receive whom he owes $2485?

9. In an orchard 40% of the trees are winter apples; 10% are fall apples; 5% are harvest apples; 30% are pear trees; and the remaining trees are peach. If there are 30 peach trees, how many trees are in the orchard?

10. A house and lot cost $6400. The house cost $66\frac{2}{3}\%$ more than the lot. Find the cost of each.

11. A druggist's sales increased 15% the second year, 20% the third year, and 25% the fourth year. If the fourth year's sales were $29,023.13, what were the sales for the first year?

12. A merchant's sales increased $16\frac{2}{3}\%$ the second year, 20% the third year, 25% the fourth year, and $14\frac{2}{7}\%$ the fifth year. If the sales for the five years amount to $213,244.25, what were the sales the first year?

13. A salesman's sales increased 25% in November over October's sales, and 20% in December over November's sales. His January sales decreased 40% from those of December. If January's sales amounted to $3501, what were they for October?

14. In 1920 the population of Okmulgee, Okla., was 17,430, an increase of 317.4% in ten years. What was the population in 1910?

345. PERCENTAGE DRILL

	a	*b*	*c*	*d*	*e*	*f*	*g*
1.	$ 15	$ 18	$ 10	25 %	10 %	$ 6	$ 3
2.	16	22	14	20 %	$12\frac{1}{2}\%$	3	4
3.	21	24	15	40 %	30 %	4.20	6.30
4.	24	30	20	20 %	$33\frac{1}{3}\%$	2	8
5.	32	40	28	$16\frac{2}{3}\%$	25 %	6	12
6.	40	45	32	$12\frac{1}{2}\%$	$11\frac{1}{9}\%$	10	8
7.	48	56	36	$33\frac{1}{3}\%$	25 %	12	20
8.	50	60	45	25 %	30 %	15	18
9.	60	75	54	$16\frac{2}{3}\%$	25 %	20	18
10.	72	81	60	$33\frac{1}{3}\%$	$16\frac{2}{3}\%$	24	9
11.	80	96	70	20 %	$12\frac{1}{2}\%$	30	8
12.	96	120	84	$12\frac{1}{2}\%$	$8\frac{1}{3}\%$	24	16
13.	108	120	96	$16\frac{2}{3}\%$	5 %	9	36
14.	120	144	100	$37\frac{1}{2}\%$	50 %	20	12
15.	150	175	120	25 %	$14\frac{2}{7}\%$	50	40
16.	180	210	160	$66\frac{2}{3}\%$	$12\frac{1}{2}\%$	40	60
17.	240	270	210	$12\frac{1}{2}\%$	25 %	60	30
18.	300	360	250	20 %	25 %	25	100
19.	360	420	320	$33\frac{1}{3}\%$	$16\frac{2}{3}\%$	90	45
20.	450	540	360	20 %	$16\frac{2}{3}\%$	75	45

1. The numbers in column *b* are what per cent greater than those in column *a*?

2. The numbers in column *c* are what per cent less than those in column *a*?

3. Consider column *a* as cost, and column *d* as rate of gain. Find the selling price.

4. Consider column *a* as cost, and column *e* as rate of loss. Find the selling price.

5. If column *a* is cost and column *f* is gain, find the gain per cent.

6. If column *a* is cost and column *g* is loss, find the loss per cent.

7. If column *b* is cost and column *c* is selling price, find the rate of loss.

8. If column *c* is cost and column *b* is selling price, find the rate of gain.

9. If column *b* is selling price and column *d* is rate of gain, find the cost.

10. If column *b* is selling price and column *e* is rate of loss, find the cost.

PERCENTAGE REVIEW

Mental

346. 1. Two railroads carry freight 200 mi. The first carries it 75 mi., and the second 125 m.. What per cent of the distance does each carry it?

2. A man earns $1500 a year and spends $900. What per cent of his money does he spend?

3. A boy bought a watch for $12 and a chain for $4. Later he sold them both for the cost of the watch. What per cent of the cost was the selling price?

4. During the month of May a farmer sells 500 qt. of milk; during June he sells 600 qt. What is the per cent of increase?

5. Of a pile of potatoes containing 120 bu., one bushel out of five is bad. What per cent are bad? How many bushels are good?

6. From a half chest of Oolong tea containing 50 lb., 15 lb. were sold one day and 20 lb. the next day. What per cent of the tea remained unsold? What per cent was sold each day?

7. A bankrupt's resources are $12,000, and his liabilities are $20,000. What per cent of his debts can he pay? He owes Mr. Brown $600. How much will he receive?

8. Last year my house and lot was worth $4000. This year it is worth $4400. What per cent has it increased in value? If it increases in value at the same rate per cent next year, how much will the property then be worth?

9. In an invoice of 4 doz. crates of oranges, 10% are decayed. If there is an average of 8 doz. oranges to a crate, how many oranges are spoiled?

10. If coffee loses 16% of its weight in roasting, how many pounds of roasted coffee will there be in 500 lb. of green coffee?

11. A clerk's monthly salary was $80. Owing to hard times, his salary is reduced 5%. How much does he now receive per month?

12. A dairyman had 1000 qt. of milk a week in July. During August he increased the amount of grain fed to his cows $3 worth per week, and obtained 6% more milk. If he sold the milk at 8 ¢ a quart, did it pay to increase the amount of grain fed?

13. If coal has been selling at $12.50 a ton, a 10% increase will make it sell for how much?

14. A merchant's sales increased 33⅓% the second year over the first year. If the first year's sales were $72,000, what were the sales the second year?

15. I bought 56 yd. of cloth at $2.40 a yard and sold it to gain 25%. How much did I receive for all of it?

16. If merchandise cost $360, and increases in value 16⅔%, how much will it be worth? If later it decreases 16⅔% in value, how much will it then be worth?

17. In settling with a bankrupt, I received $2400, which was 75% of my claim. How much did I lose?

18. A man receives $50 a month rent for a house. His expenses for a year are $150. What is the value of the house if it nets him 9%?

19. During the second year a grocer's sales increased $5000, which was 20% of the first year's sales. What were the sales each year?

20. If, in problem 19, the profit was 20% of the cost of the goods, what were his profits each year? What would have been his profits if they were 20% of the selling price?

21. How much must a young man save so that at 6% he may have an annual income of $1200?

22. A merchant's average profit is 12½% of his sales. How much merchandise must he sell to make a profit of $6000?

23. If an enterprise pays 8%, what is the investment, the income being $1600?

24. A man sold $37\frac{1}{2}\%$ of his business for $3600. What was the entire business worth?

25. I sold a house and lot at a again of $200, which was 5% of the selling price. How much did the property cost me?

PROBLEMS

347. 1. If potatoes shrink 5% in weight during the winter, what is the shrinkage in weight of 1200 bu.?

2. A store is worth $18,000. It rents for $2000. Repairs cost 1% of the value of the store; taxes cost $1\frac{1}{2}\%$; insurance $\frac{1}{4}\%$ on a policy of $10,000; and other expenses amount to $85. The net income is what per cent of the gross rental? of the value of the store?

3. A bankrupt paid me 68% of my claim. If I received $1275.46, how much did he owe me?

4. If the duty on certain merchandise was 30%, and the present duty is 20% less than formerly, what is the present rate?

5. What per cent of the total area of the following cities is land? What per cent is water?

	LAND AREA	WATER AREA
New Haven, Conn.	11,460 acres	2,880 acres
Syracuse, N. Y.	10,639 acres	309 acres
Scranton, Pa.	12,362 acres	147 acres
Fall River, Mass.	21,862 acres	3,793 acres
Albany, N. Y.	6,914 acres	283 acres

6. In a school there are 400 girls, which is $14\frac{2}{7}\%$ more than the number of boys. How many pupils are there in the school?

7. After selling 560 sheep, a shepherd found he had $12\frac{1}{2}\%$ of his flock left. How many sheep were there in the flock?

8. What is the difference in pounds between $\frac{5}{8}$ of a ton and $\frac{5}{8}\%$ of a ton?

9. A man had $1460 and spent $365 for a year's board. What per cent of his money did his board cost him? The money he spent is what per cent of the amount he had left?

10. An agent sold three farms for $12,750. The price of the second was 50% more than the price of the first, and of the third was 30% less than the price of both the others. Find the price of each.

11. A bushel of wheat weighs 60 lb. $5\frac{1}{2}$ bu. will make a barrel of flour (196 lb.). What per cent of the wheat is flour?

12. The population New York, N. Y., in 1910 was 4,766,883, and in 1920 it was 5,620,048. Find the per cent of increase for the ten-year period.

13. On an invoice of merchandise valued at $672, a merchant paid freight $12.60, and cartage $9.80. The charges were what per cent of the value of the goods?

14. Find the sum of $\frac{1}{2}\%$ of $893.68; $\frac{3}{4}\%$ of $723.85; $\frac{1}{20}\%$ of $3487.60.

15. The number of automobiles in the United States for certain years were as follows:

1914 — 1,711,339
1916 — 3,512,996
1918 — 6,146,617
1920 — 7,904,271

Show the per cent of increase for each two-year period.

16. United States exports to and imports from Japan for 2 years were:

	Exports	Imports
1918	$323,012,765	$265,064,697
1919	$383,190,719	$414,048,810

Show in nearest tenth per cent, the per cent of excess of exports over imports, or of imports over exports for each year.

17. A manufacturer sells to a wholesaler at a profit of 20%; the wholesaler to the retailer at a 25% gain; and the retailer to the consumer at a gain of 60%. If the consumer pays $14.40, find the cost to the manufacturer.

18. A house and lot rents for $60 a month. The taxes, insurance, repairs, etc., are estimated at $180 per annum. How much can a purchaser afford to pay for the property so as to have a 6% investment?

19. A creditor receives from a bankrupt firm $735 on a debt of $1960. What per cent of their debt does the firm pay? If their total resources amount to $9474.60, how much are their liabilities?

20. A house and lot cost $6000; the insurance is $15, taxes are $60, and repairs are $45 annually. What rent per month must be received to realize 7% on the investment?

21. What per cent of 5 lb. avoirdupois is 7 oz.?

22. From a cask containing 45 gal., 2 gal. and 1 qt. are lost by leakage. What per cent is lost by leakage?

23. An agent charged $160 as his commission for selling real estate at $2\frac{1}{2}$%. For how much was the property sold?

24. A farmer's crop of grain this year is 15% less than it was last year. What was this year's crop if in the two years he raised 1554 bu. of grain?

25. A drover sold 330 sheep and had 1320 sheep left. What per cent of his flock did he sell?

26. A speculator increased his real estate holdings by 44%. He then sold 25% of all his property, and found he had 810 A. left. How many acres had he at first?

27. A man drew 65% of his money from the bank and with 40% of it bought a house and lot for $5200. How much money had he remaining in the bank?

28. From his income a young man saved 20%. Had he saved $200 more, the per cent saved would have been 30%. What is his income?

29. A merchant having 1848 yd. of cloth, sold $33\frac{1}{3}$% of it at $1.60 a yard, 25% of the remainder at $1.65 a yard, and then what was left at $1.55 a yard. How much did he receive for all?

30. Multiply each of the following percentages by its relative weight, and find the general average by dividing the sum of the products by the sum of the relative weights.

PERCENTAGES	RELATIVE WEIGHTS	PRODUCTS
76.5	6	——
93.25	5	——
87	4	——
91.65	12	——
71.81	9	——
	General Average	——

348. The following table gives the average composition of a few common food products.

FOOD MATERIALS (AS PURCHASED)	REFUSE	WATER	PROTEIN	FAT	CARBOHYDRATES	ASH	FOOD VALUE PER POUND	AVERAGE COST PER POUND
Animal Foods	%	%	%	%	%	%	Calories	Cents
Beef, ribs . .	20.8	43.8	13.9	21.2	—	.7	1135	20
Veal, hind quar.	20.7	56.2	16.2	6.6	—	.8	580	18
Mutton, loin chops . . .	16.0	42.0	13.5	28.3	—	.7	1415	18
Ham, smoked	13.6	34.8	14.2	33.4	—	4.2	1635	18
Chicken . . .	25.9	47.1	13.7	12.3	—	.7	765	20
Mackerel . .	44.7	40.4	10.2	4.2	—	.7	370	12
Oysters . . .	—	88.3	6.0	1.3	3.3	1.1	225	18
Eggs	11.2	65.5	13.1	9.3	—	.9	635	16
Butter . . .	—	11.0	1.0	85.0	—	3.0	3410	25
Whole milk .	—	87.0	3.3	4.0	5.0	.7	310	3
Vegetable Foods								
Oatmeal . . .	—	7.7	16.7	7.3	66.2	2.1	1800	4
White bread .	—	35.3	9.2	1.3	53.1	1.1	1200	5
Brown bread .	—	43.6	5.4	1.8	47.1	2.1	1040	5
Beans, dried .	—	12.6	22.5	1.8	50.6	3.5	1520	5
Potatoes . . .	20.0	62.6	1.8	.1	14.7	.8	295	1
Apples . . .	25.0	63.3	.3	.3	10.8	.3	190	1
Almonds . . .	45.0	2.7	11.5	30.2	9.5	1.1	1515	40

349. Energy is produced by fat and the carbohydrates.

350. Protein is the material of which the tissues are built.

351. Using the table given above, find the quantity of refuse and nutrients of the different classes in the following (to the nearest tenth of a pound):

1. 15 pounds of oatmeal.
2. 12 pounds of brown bread.
3. 1 bushel of potatoes (60 lb.).
4. 1 gallon of milk (8.6 lb.).
5. 5 pounds of beef ribs.
6. 6 pounds of chicken.
7. 24 pounds of smoked ham.
8. 18 pounds of mackerel.

The following dietaries have been determined by experiment:

QUANTITIES PER MAN PER DAY

	PROTEIN	FAT	CARBOHYDRATES	ENERGY VALUE
	Ounces	Ounces	Ounces	Calories
Farmers	3½	4½	16½	3415
Football players	8	12½	22½	6590
Lawyers, teachers, etc.	4	4½	15½	3220

Select two kinds of animal food and two vegetables that would be suitable for:

9. A street car motorman.

10. A clerk in an office.

11. A laboring man.

12. An athlete.

Find the quantity of nutrients (protein, fat, and carbohydrates), the energy in calories, and the cost of the following daily dietaries:

13. Beef ribs, 12 oz.; potatoes, 16 oz.; butter, 2 oz.; white bread, 24 oz.; oatmeal, 2 oz.; eggs, 4 oz.

14. Beans, 8 oz.; milk, 16 oz.; mackerel, 16 oz.; oatmeal, 4 oz.; potatoes, 16 oz.; brown bread, 16 oz.; mutton chops, 8 oz.

15. Almonds, 2 oz.; brown bread, 24 oz.; chicken, 16 oz.; potatoes, 16 oz.; apples, 16 oz.; butter, 4 oz.

16. Smoked ham, 6 oz.; white bread, 20 oz.; oatmeal, 4 oz.; beans, 4 oz.; eggs, 4 oz.; potatoes, 16 oz.; butter, 2 oz.

17. Prepare a daily dietary containing nutrients in the ratio (approximately) of protein 1 part, fat 1 part, and carbohydrates 4 parts, the total amount of nutrients not to exceed 24 ounces.

18. Find the amount of energy in calories, and the cost of the dietary prepared in problem 17.

19. Prepare a daily dietary containing nutrients in the ratio (approximately) of protein 1 part, fat 1 part, and carbohydrates 3½ parts, the total amount of nutrients not to be less than 24 ounces nor more than 32 ounces.

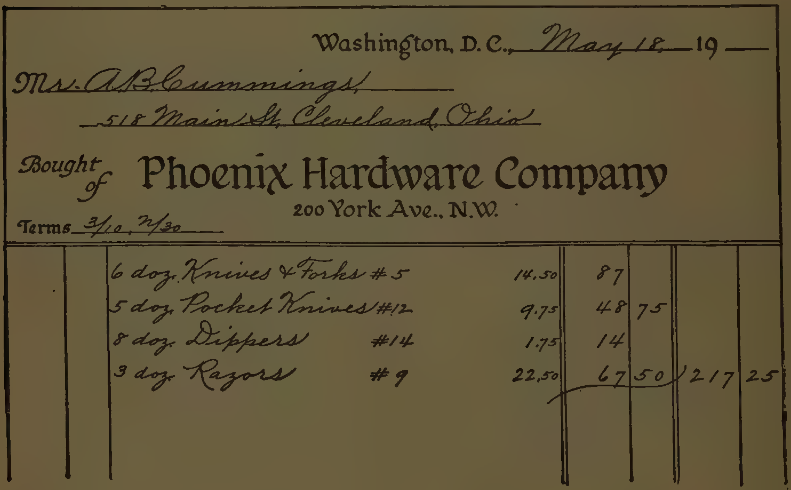
20. Find the amount of energy in calories, and the cost of the dietary prepared in problem 19.

BILLING AND COMMERCIAL DISCOUNT

352. **A bill or invoice** is a formal statement given by the seller of merchandise to the buyer. It contains the buyer's name and address; the seller's name and address; the date of the sale; a list of the items sold with the price of each and the total value of the entire bill; and the terms of the sale.

353. In some cases, the freight, or other transportation charge, is paid by the seller, and in other cases, by the buyer. If the buyer is to pay transportation charges, the seller frequently prepays it and then adds it to the bill.

354. Examine the following bill and make a tabulated list of its different parts.



Washington, D.C., May 18, 19__

Mrs. A.B. Cummings,
518 Main St., Cleveland, Ohio

Bought of **Phoenix Hardware Company**
200 York Ave., N.W.

Terms 3/10, n/30

6 doz. Knives & Forks	#5	14.50	87			
5 doz. Pocket Knives	#12	9.75	48	75		
8 doz. Dippers	#14	1.75	14			
3 doz. Razors	#9	22.50	67	50	217	25

The terms "3/10 n/30" mean that the buyer, if he pays within 10 days from May 18, may deduct 3% of the amount of the bill, but he may, if he chooses, delay making payment until 30 days from the date of the bill, at which time he must pay the full amount of the bill. N/30 (net 30 days) means no discount allowed if paid at end of 30 days.

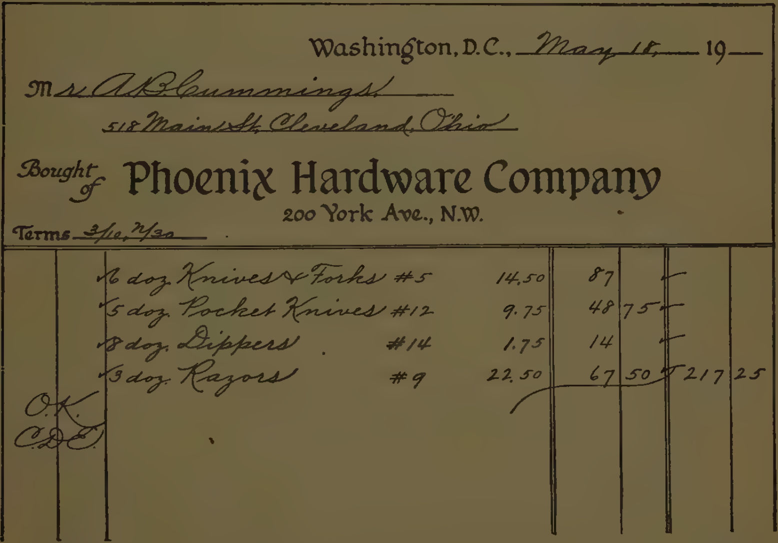
State, or write, the meaning of the following terms: 5/10, 3/30, n/60; 1% cash, net 30 days; 6/10, 3/30, n/90.

Discounts of this kind are called *Cash discounts.*

355. As soon as a bill of merchandise is delivered to the purchaser's place of business, a stock clerk inspects the goods to see whether the right kind and quantity of goods has been received. A check mark is placed on the invoice against each item if it agrees with the stock received.

A second checking is necessary to verify the accuracy of the price charged for each item, and total cost of the entire invoice. If an error is found, the seller is notified at once and an adjustment is made with the buyer.

The invoice on page 184 properly checked would appear as follows:



Washington, D.C., May 18, 19

Mr. A. B. Cummings

518 Main St., Cleveland, Ohio

Bought of **Phoenix Hardware Company**

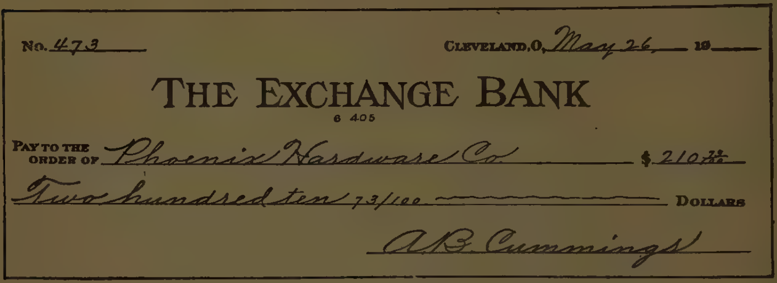
200 York Ave., N.W.

Terms 3/10, n/30

	✓6 doz. Knives & Forks	#5	14.50	87		✓		
	✓5 doz. Pocket Knives	#12	9.75	48	75	✓		
	✓8 doz. Dippers	#14	1.75	14		✓		
	✓3 doz. Razors	#9	22.50	67	50	✓	217	25
O.K. C.D.E.								

If Mr. Cummings decides to pay the above bill on or before May 28, he will deduct 3% from the face of the bill and send his check for the difference. With the check he will send the invoice to be receipted.

The following is a form of the check Mr. Cummings would send.



No. 473

Cleveland, O. May 26, 19

THE EXCHANGE BANK

6 405

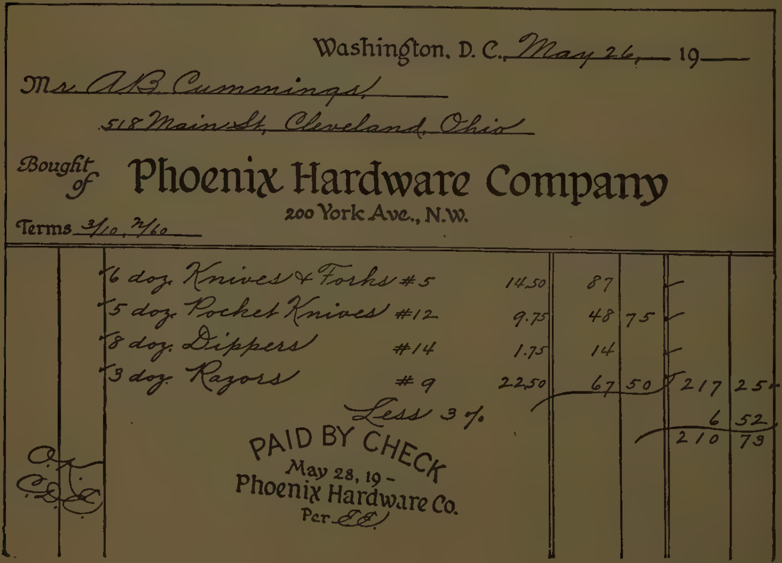
Pay to the order of Phoenix Hardware Co. $210 73/100

Two hundred ten 73/100 Dollars

A. B. Cummings

Upon receiving the check and invoice, some authorized person of the Phoenix Hardware Company will mark or stamp the invoice paid, with date of payment, and sign the Company name "by" or "per" his own name or initials. The receipted invoice will then be sent back to Mr. Cummings, who will place it on file.

The following illustration shows the invoice after it has been paid and receipted.



Washington, D. C. May 26, 19

Mr. A. B. Cummings

518 Main St., Cleveland, Ohio

Bought of **Phoenix Hardware Company**

200 York Ave., N.W.

Terms 3/10, n/60

6 doz. Knives & Forks #5	14.50	87	—		
5 doz. Pocket Knives #12	9.75	48	75		
8 doz. Dippers #14	1.75	14	—		
3 doz. Razors #9	22.50	67	50	217	25
Less 3%				6	52
				210	73

O.K.

PAID BY CHECK
May 28, 19 -
Phoenix Hardware Co.
Per E.E.

356. The acceptance of a cash discount is usually of advantage to the buyer. To illustrate: On an invoice of $1800, with the terms 2/10 n/30, and dated June 10, the purchaser would save 2¢ on each dollar of the bill, or 2% of its face value, by paying on June 20. If he lets the bill run to July 10, only 20 days longer, he must pay the full $1800. That is, he makes 2% on the bill in 20 days by paying on June 20. There are 18 times 20 days in a year. 18 times 2% equals 36%. Hence, by paying early, the buyer is getting in the discount the equivalent of 36% interest on his money. The buyer might better borrow money, if need be, to enable him to pay promptly and secure the benefit of the discount.

Determine the rate of interest received by the buyer if he takes the discount on a bill in which the terms are 1%, n/30; 3/10, n/60; 4% cash, n/60; 5% cash, n/90.

It is of advantage also to the seller if the buyer takes advantage of the discount offered. He receives promptly funds due him on sales and is not required to have so much capital invested or tied up in outstanding accounts. He is spared the expense of collecting overdue accounts. The "bad debt" customer is generally the one who lets his bills run as long as possible, and finally is unable to pay, thus causing serious loss to the seller. Hence, the seller can well afford to offer a generous discount as an inducement to the buyer to pay promptly.

357. It is suggested in the following exercises that one half of the class prepare the first invoice and the other half of the the class write the second invoice and so on alternately. As a part of a class session, have pupils exchange papers so that those who prepared invoice No. 1 will receive invoice No. 2, and vice versa. Let each pupil now check the invoice he has received, and if it is correct, O. K. it. He will then deduct the cash discount, and write a check in settlement thereof. The invoice, properly checked and O.Kd., with discount deducted, should then, with the check, be returned to the "seller." If the check is correct as to form and amount, the invoice will be receipted and returned to the "buyer."

PROBLEMS

1. The Eureka Grocery Company, Poughkeepsie, N. Y., sold to The Kingston Grocery Company, Kingston, N. Y., terms 3/10 n/30,

8 doz. Van Camp's Evaporated Milk at $2.75
6 doz. Borden's Evaporated Milk at $2.80
4 doz. pkg. Cream of Wheat at $2.85
6 doz. pkg. Quaker Oats at $1.35
42 cans Crisco, 1 lb. cans, at $.22
16 cans Crisco, 3 lb. cans, at $.65
36 lb. Yuban Coffee at $.38

2. A. B. Johnson & Company, Reading, Pa., bought from Gristow Bros., Pittsburgh, Pa., terms 4/10 n/60,

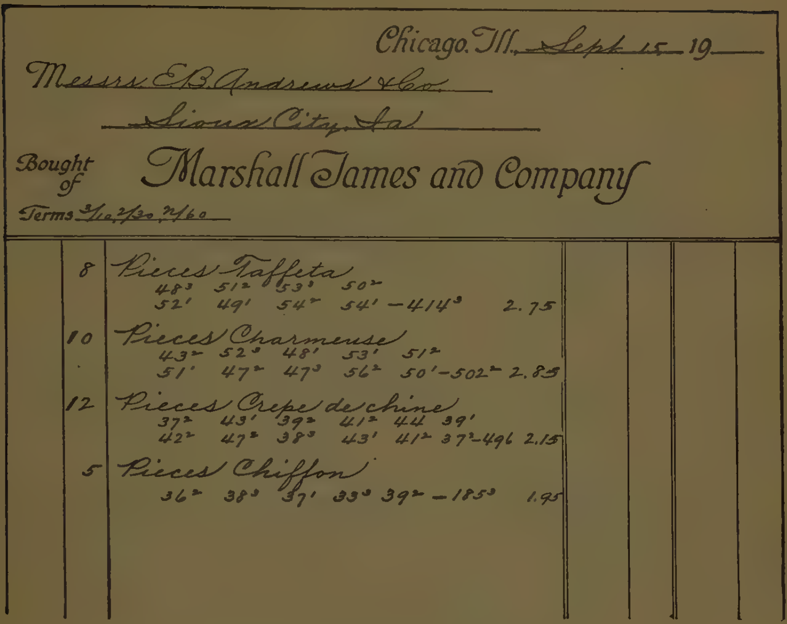
8 cases, 16 doz., Maryland Tomatoes No. 2 size, at $1.22 per doz.
6 cases, 12 doz., Maryland Tomatoes, No. 3 size, at $1.66 per doz.
48 cans, 8 oz. size, Columbia River Salmon, at $.18
5 cases, 10 doz., Wisconsin Peas, No. 2 size, at $1.64 per doz.
3 tub, 156 lb., Pure Leaf Lard, at $.17

3. McCarthy & Redmond, Joliet, Ill., sold to Mills & Son, Louisville, Ky., on account 60 days, with 5% off if paid in 10 days,

4 doz. No. 16, One piece Dish Pans, at $11.25
1½ doz. No. 9, Wash Boilers, at $46
5 doz. No. 11, 1 qt. cups, at $1.15
3 doz. No. 74, Colonial Percolators, at $48.50
2 doz. No. 5, Electric Toasters, at $22.50
¾ doz. No. 8, Aluminum Tea Kettles, at $56

4. A. R. Jameson, Vicksburg, Tenn., bought from S. S. Patterson, St. Louis, Mo., terms 5/10, n/90,

16 M No. 5 Envelopes at $2.75
14 M No. 6 Envelopes at $2.85
2 C No. 3 Pads, ruled at $6.00
6 gross Penholders at $3.25
½ gross Superior Writing Fluid at $3.60



Chicago, Ill., Sept 15 19

Messrs E. B. Andrews & Co.

Sioux City, Ia.

Bought of Marshall James and Company

Terms 3/10, 2/30, n/60

8	Pieces Taffeta 48^3 51^2 53^3 50^2 52^1 49^1 54^2 54^1 – 414^3	2.75
10	Pieces Charmeuse 43^2 52^3 48^1 53^1 51^2 51^1 47^2 47^3 56^2 50^1 – 502^2	2.85
12	Pieces Crepe de chine 37^2 43^1 39^2 41^2 44 39^1 42^2 47^2 38^3 43^1 41^2 37^2 – 496	2.15
5	Pieces Chiffon 36^2 38^3 37^1 33^3 39^2 – 185^3	1.95

The number of yards varies in the different pieces. Hence the number of yards in each piece must be indicated. The small figures indicate quarter yards. Thus, 48^3 means $48\frac{3}{4}$; 51^2 means $51\frac{2}{4}$, that is $51\frac{1}{2}$, etc. The total number of yards in one kind of cloth is also written in the invoice, thus, the 8 pieces of taffeta contain $414\frac{3}{4}$ yards, at $2.75 per yard.

6. Stone & Son, Chicago, Ill., bought of The American Woolen Company, Boston, Mass., terms 3/30, n/90,

8 pcs. Cheviot 43^1 38^2 41^2 42^3 44^3 47^1 39 38^1	at $4.75
6 pcs. Broadcloth 38^2 37^1 39^2 36^2 38^3 37^2	at $5.80
7 pcs. Serge 47^2 43^1 44^3 45^1 42^1 46^2 47^1	at $5.25
4 pcs. Jersey Cloth 42^2 47^1 39^2 46	at $2.25

358. Manufacturers, wholesalers and jobbers issue catalogues containing names, descriptions and prices of articles they have to sell. The catalogues are sent to retailers and others to whom they wish to sell their goods.

359. The price quoted in the catalogue is called the catalogue, or list, price. It is generally a higher price than it is expected the article will sell for. Oftentimes it is the price at which the retailer is expected to sell the article.

360. In order that the retailer may buy from the manufacturer or wholesaler at a price that will enable him to resell the goods at a fair price and make a reasonable profit, the wholesaler allows him a so-called *trade discount* from the catalogue price. To illustrate:

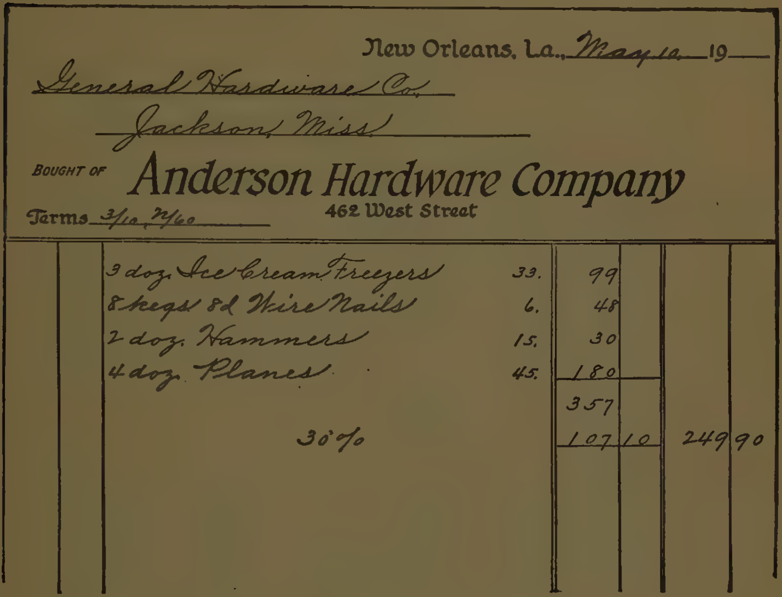
The manufacturer's catalogue price of a furnace is $150. He sells the furnace to a retail dealer at a discount of 25% from the catalogue price. Find the net cost to the retailer.

$150 is catalogue price
25% of $150 = 37.50, the trade discount
$112.50 is the net price or net cost.

Find the net amount of the following bills:

GROSS AMOUNT	TRADE DISCOUNT	GROSS AMOUNT	TRADE DISCOUNT	GROSS AMOUNT	TRADE DISCOUNT
1. $ 8	$12\frac{1}{2}\%$	11. $ 1.25	20 %	21. $436	25 %
2. 10	20 %	12. 1.50	$16\frac{2}{3}\%$	22. 127.50	$16\frac{2}{3}\%$
3. 12	25 %	13. 3.60	25 %	23. 342.30	20 %
4. 6	$33\frac{1}{3}\%$	14. 4.50	10 %	24. 98.75	30 %
5. 9	50 %	15. 48	$18\frac{3}{4}\%$	25. 233.60	40 %
6. 14	$14\frac{2}{7}\%$	16. 45	$11\frac{1}{9}\%$	26. 207.80	35 %
7. 16	$6\frac{1}{4}\%$	17. 72	$8\frac{1}{3}\%$	27. 216.75	$33\frac{1}{3}\%$
8. 20	10 %	18. 50	4 %	28. 328.40	$27\frac{1}{2}\%$
9. 24	$16\frac{2}{3}\%$	19. 60	3 %	29. 892.85	30 %
10. 30	$6\frac{2}{3}\%$	20. 75	8 %	30. 943.36	$33\frac{1}{3}\%$

361. The following illustration shows an invoice with the trade discount deducted.



New Orleans, La., May 10 19

General Hardware Co.
Jackson, Miss.

BOUGHT OF **Anderson Hardware Company**
462 West Street

Terms 3/10, n/60

3 doz. Ice Cream Freezers	33.	99			
8 kegs 8d Wire Nails	6.	48			
2 doz. Hammers	15.	30			
4 doz. Planes	45.	180			
		357			
30%		107	10	249	90

$357 is called the "gross amount" of the bill.

$249.90 is called the "net amount" of the bill.

A trade discount is deducted by the seller from the amount of the bill before it is sent to the buyer.

If the General Hardware Co. pays the bill within 10 days of its date, they will deduct an additional discount of 3% of $249.90 for early payment.

The discount allowed by a wholesaler or manufacturer varies from time to time depending on market conditions. If the price of an article becomes less because of lower cost of manufacturing, the seller offers an additional discount instead of changing his list or catalogue price.

A manufacturer has been selling clothing to a retail clothier at $40 a suit less 25%. Because of a decrease in the cost of materials he is able to sell the suit for less money, and offers an additional discount of 12½%. What is the cost of the suit to the retailer?

$40 = list price
25% of $40 = 10 = 1st discount
30 = price less 1st discount.
12½% of $30 = 3.75 = 2nd discount
$26.25 = net cost.

It is to be noted that when two or more discounts are offered on the same bill, the second discount is deducted from the value *after* the first discount has been deducted. A third discount would be taken from the amount remaining after the second discount is taken off, and so on.

Bills from which a trade discount has been deducted are subject to a cash, or time, discount, the same as illustrated on page 186.

Two or more discounts offered on the same bill are called a "discount series."

Find the net cost in each of the following:

GROSS AMOUNT	TRADE DISCOUNT	GROSS AMOUNT	TRADE DISCOUNT
1. $60	20% and 16⅔%	11. $1145.50	22½% and 10%
2. 75	20% and 33⅓%	12. 1265.80	15% and 10%
3. 80	25% and 10%	13. 1376.80	12½% and 5%
4. 120	33⅓% and 25%	14. 1891.84	16⅔% and 12½%
5. 150	20% and 10%	15. 1938.64	20% and 16⅔%
6. 240	20% and 12½%	16. 2463.70	37½% and 25%
7. 540	50% and 20%	17. 3284.65	50% and 12½%
8. 840	37½% and 20%	18. 4136.75	20% and 10%
9. 960	40% and 16⅔%	19. 2348.57	25% and 5%
10. 1200	50% and 25%	20. 3167.86	33⅓% and 20%

362. The discount series varies on different articles, hence one seller may offer a great variety of discounts. To enable the billing clerk to compute the net cost of an invoice quickly, a table of Net Cost Rate Factors is used. Such a table shows the net cost per cent for each of a larger number of discount series.

The net cost rate factor is found as follows:

Assume the discount series is 25% and 20%.

100%   100%
 25%   20%
75% × 80% = 60%, N. C. R. Factor.

Subtract each rate separately from 100%. Multiply the remainders together. The product is the net cost rate factor. Or

or
4)100%
   25%
5) 75%
   15%
  60% N. C. R. Factor.

Deduct the first discount, 25%, from 100%, leaving 75%. The second discount, 20%, is deducted by taking 20% (⅕) of 75% from 75%. ⅕ of 75% is 15%, which, taken from 75%, leaves 60%, the net cost rate factor.

The latter method is excellent when the rates of discount are familiar aliquot parts of 100%.

363. The following table shows the net cost rate factors for a limited number of discount series.

Table of Net Cost Rate Factors

	10%	12½%	15%	16⅔%	20%	25%	33⅓%
2½%	.87750	.85313	.82875	.81250	.78000	.73125	.65000
5%	.85500	.83125	.80750	.79167	.76000	.71250	.63333
10%	.81000	.78750	.76500	.75000	.72000	.67500	.60000
10%, 5%	.76950	.74813	.72675	.71250	.68400	.64125	.57000
10%, 10%	.72900	.70875	.68850	.67500	.64800	.60750	.54000
16⅔%	.75000	.72917	.70833	.69444	.66667	.62500	.55556
20%, 10%	.64800	.63000	.61200	.60000	.57600	.54000	.48000
25%, 10%	.60750	.59063	.57375	.56250	.54000	.50625	.45000

In the preceding table a discount series is made up of a combination of the rate or rates at the left and the rate at the top of a column. Thus, at the left on the 4th line is 10% and 5%, and at the top of the first column is 10%. Hence the discount series is 10%, 10%, and 5%, and its equivalent net cost rate factor is .7695, or 76.95%.

It is often desirable to know what single per cent discount is equivalent to a discount series. In every case the single discount equal to a series is the difference between the net cost rate factor and 100%. That is, the single discount equal to a series of 10%, 10%, and 5% is 23.05% (100% − 76.95%).

Some firms prefer a table of single discounts equal to a series in place of a net cost rate factor table.

364. Using the form and rates given below complete a table showing the single discount equal to the indicated series.

Table of Single Discounts Equal to Discount Series

	20%	25%	30%	33⅓%	37½%	40%	
5%							
10%							
12½%							
15%							
16⅔, 10%							
20%, 12½%							

Find the net cost of an invoice of $1625 less a trade discount of 20%, 15% and 10%.

The net cost rate factor for 20%, 15% and 10% is .612 = 61.2%.
.612 × $1625 = $994.50, net cost.

Multiplying the gross cost by the net cost rate factor gives the net cost.

Find the net cost of an invoice of $1860 less 20%, 5% and 2½%.

100% − 20% = 80%
100% − 5% = 95%
100% − 2½% = 97½%

80% of 95% of 97½% = .741, or 74.1%, net cost rate factor.
74.1% of $1860 = $1378.26

or by aliquots,

5	$1860
	372
20	1488
	74.40
40	1413.60
	35.34
	$1378.26, net cost.

If the discount series is not found in the net cost rate factor table, deduct each rate in the series separately from 100%. Then multiply the gross amount of the bill $1860 by the product of the several remainders as illustrated.

Or, by using aliquots, deduct from the gross amount $\frac{1}{5}$ (20%); then from the remainder deduct $\frac{1}{20}$ (5%), and finally from that remainder deduct $\frac{1}{40}$ (2½%). The net cost is the same as by the other processes.

A motor boat listed at $1125 is sold at a discount of 25%, 20% and 12½%. How much is the total discount and the net cost?

The single discount equal to 25%, 20%, and 12½% is 47½%.

47½% of $1125 = $534.37, trade discount.
$1125 − $534.37 = $590.63, net cost.

or by aliquots,

5	$1125
	225
4	900
	225
8	675
	84.37
	$590.63

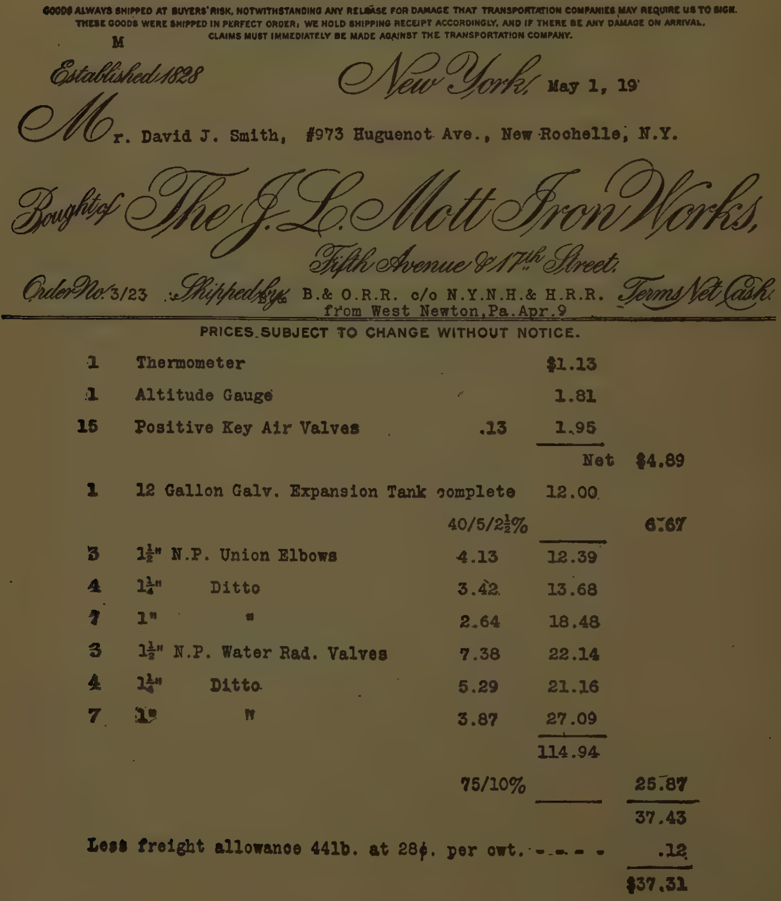
$1125 − $590.63 = $534.37 trade discount.

When the amount of the discount is desired, find (from a table or by calculation) the single discount equal to the series, and multiply the gross amount of the bill by it. The result is the trade discount. Deducting the trade discount from the gross amount of the bill gives the net cost.

Or, by using aliquots, find the net cost, as illustrated, and deduct net cost from the gross amount to find the trade discount.

Note.—In deducting the several discounts of a given series it is immaterial which discount is deducted first. It sometimes shortens the calculation to change the order of the rates.

365. The following illustration shows the form of invoice used by The J. L. Mott Iron Works. Verify the results given.



GOODS ALWAYS SHIPPED AT BUYERS' RISK, NOTWITHSTANDING ANY RELEASE FOR DAMAGE THAT TRANSPORTATION COMPANIES MAY REQUIRE US TO SIGN.
THESE GOODS WERE SHIPPED IN PERFECT ORDER; WE HOLD SHIPPING RECEIPT ACCORDINGLY, AND IF THERE BE ANY DAMAGE ON ARRIVAL,
CLAIMS MUST IMMEDIATELY BE MADE AGAINST THE TRANSPORTATION COMPANY.

M

Established 1828 New York, May 1, 19

Mr. David J. Smith, #973 Huguenot Ave., New Rochelle, N.Y.

Bought of The J. L. Mott Iron Works,
Fifth Avenue & 17th Street.

Order No. 3/23 Shipped by B.& O.R.R. c/o N.Y.N.H.& H.R.R. from West Newton, Pa. Apr. 9 Terms Net Cash.

PRICES SUBJECT TO CHANGE WITHOUT NOTICE.

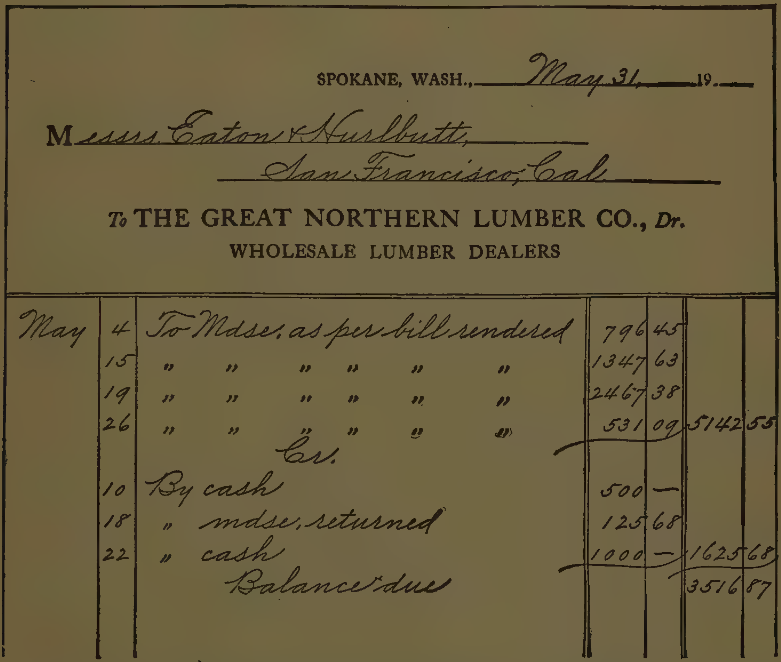
1	Thermometer		$1.13	
1	Altitude Gauge		1.81	
15	Positive Key Air Valves	.13	1.95	
			Net	$4.89
1	12 Gallon Galv. Expansion Tank complete		12.00	
		40/5/2½%		6.67
3	1½" N.P. Union Elbows	4.13	12.39	
4	1¼" Ditto	3.42	13.68	
7	1" "	2.64	18.48	
3	1½" N.P. Water Rad. Valves	7.38	22.14	
4	1¼" Ditto	5.29	21.16	
7	1" "	3.87	27.09	
			114.94	
		75/10%		25.87
				37.43
	Less freight allowance 44lb. at 28¢. per cwt.			.12
				$37.31

366. Make out a bill for each of the following purchases:

1. A. C. Bolton, Cincinnati, Ohio, bought of the Rochester Hardware Co., Rochester, N. H., Sept. 1, 1924 on account 60 da., 3% 10 da.:

24 doz. C. S. Axes	@	$10.
12 doz. Handsaws	@	$14.
Less 20%.		
8 doz. Wrenches	@	$12.
Less 25% and 10%.		

MODEL STATEMENT



SPOKANE, WASH., May 31, 19

Messrs. Eaton & Hurlbutt,
San Francisco, Cal.

To THE GREAT NORTHERN LUMBER CO., *Dr.*
WHOLESALE LUMBER DEALERS

May	4	To Mdse. as per bill rendered	796	45		
	15	" " " " " "	1347	63		
	19	" " " " " "	2467	38		
	26	" " " " " "	531	09	5142	55
		Cr.				
	10	By cash	500	—		
	18	" mdse. returned	125	68		
	22	" cash	1000	—	1625	68
		Balance due			3516	87

2. B. S. Shenkman & Co., Wheeling, W. Va., bought of Norfolk Grocery Co., Norfolk, Va., Dec. 1, 1924, on account 90 da., 2% 30 da., 5% 10 da.:

20 mats Java Coffee, 1500,	@	25¢.
30 bags Maracaibo Coffee, 3750,	@	20¢.
30 bags Rio Coffee, 3750,	@	15¢.

A discount of 15% and 10% is allowed on the above.

25 half-chests Japan Tea, 1875 lb.,	@ 25¢.
30 half-chests Oolong Tea, 1500 lb.,	@ 45¢.
25 half-chests Eng. Breakfast Tea, 1500 lb.,	@ 30¢

A discount of 20% and 10% is allowed on the tea. Find the net amount of the bill if paid on Dec. 11, 1924.

3. A. J. Cammeyer, New York, bought of New England Shoe Co., Boston, Mass., Sept. 18, 1924, on account 30 da., 5% cash:

100 cases Patent Calf,	3600 pr.,	@ $2.50.
80 cases Tan Calf,	2880 pr.,	@ 2.25.
60 cases Kangaroo,	2160 pr.,	@ 2.25.
120 cases French Kid,	4320 pr.,	@ 3.50.

If a discount of 25% and 12½% is allowed on the entire bill, find amount required to pay the bill at once. Receipt the bill.

4. Find the net amount of each of the following:

160 yd. Silk,	@ $3.50.
120 yd. Broadcloth,	@ 2.25.
240 yd. Duchess Satin	@ 2.50.
80 yd. French Crépon	@ 1.75.

Discount of 33⅓%, 12½%, and 5%.

5. 18 grindstones at $2.25; 24 ice cream freezers at $4.50; 4 doz. jack planes at $7; and 12 doz. mortise locks at $3.50. Discount, 37½%, 25%, and 10%.

6. 24 boxes lemons at $4.50; 30 boxes oranges at $5.40; 18 doz. pineapples at $1.25; 20 bunches bananas at $1.25. Discount 20%, 12½%.

7. 1800 yd. denim at 12¢; 1400 yd. cashmere at 50¢; 2000 yd. kersey at $1.10; 3000 yd. storm serge at 62½¢. Discount 20% and 12½%.

8. 360 yd. Brussels carpet at $1.20; 800 yd. ingrain carpet at 75¢; 1200 sq. yd. linoleum at 62½¢; 900 yd. moquette carpet at $1.25. Discount 22½% and 5%.

9. One dealer offers a piano at $650, less 33⅓% and 20%. Another dealer offers the same grade of piano for $700 less 37½%, 20%, and 5%. Which is the better offer, and how much?

367. There are many persons who like the following solution because of its simplicity.

1. Find the net cost of goods listed $480, less 25%, 20%, and $16\frac{2}{3}$%.

$$\frac{\overset{80}{\cancel{\$480}} \times 3 \times \cancel{4} \times \cancel{5}}{\cancel{4} \times \cancel{5} \times \cancel{6}} = \$240.$$

By deducting each rate in the series from 100% and reducing the remainders to their fractional equivalents, and placing the resulting fractions as shown in the solution, the problem resolves itself into one of simple cancellation. In many cases this is a very practical solution.

2. Which is the better for the purchaser, and how much per cent, a series of 30%, 20%, and 10%, or a straight discount of 50%? How much on a bill of $325?

3. A bedroom suite is invoiced at $420 less 20% and $14\frac{2}{7}$%. What is the net price?

4. A gentleman bought an automobile for $2400 less $12\frac{1}{2}$%, 10%, and 5%. What was the net cost?

5. Goods listed at $128 are sold subject to a discount of 75%, $12\frac{1}{2}$%, and 5%. Find the net price.

6. You are offered 400 suits of ready-made clothing at $18 a suit, less 20% and 5%, by A, and at $16 a suit, less 10%, by B. If you buy of A, you pay the freight, $4.50. B offers to pay the freight. Which offer is better, and how much?

7. The list price of plows is $12. If you buy them at that price less $33\frac{1}{3}$% and 10%, and sell them at the same list price less 20% and 5%, how much do you make on 48 plows?

8. I can buy a bathroom outfit of one dealer for $150, less $33\frac{1}{3}$% and 20%. I can get the same outfit of another dealer for $125, less 25% and $12\frac{1}{2}$%. The terms in each case are net 30 da., 2% cash. What is the least amount of ready money for which I can get the outfit?

9. A merchant bought goods at 30% and 10% less than the list price, and sold them at the same list price less 10%. Find his profit on goods listed at $1000.

10. An overcoat in a clothier's windows is marked "Was $27.50. Now $21." What is the rate of discount?

11. The merchants in a certain town gave a ticket of admission to the county fair costing fifty cents to each customer who purchased $3.50 worth of merchandise. This was equivalent to what rate of discount?

12. The fare one way between two cities is $48.75. If a round-trip ticket costs $85, what is the rate of discount?

13. Which is the better offer, and how much, on a bill amounting to $1480, a discount series of 33⅓% 25%, and 20%, or a series of 40%, 20%, and 12½%?

14. A bill of $1260.50 is subject to discounts as follows: $450 of the bill to 25% and 12½%; $375.60 to 37½% and 8⅓%; and the remainder of the bill to a discount of 16⅔%. Find the net amount of the bill.

15. Find the net amount of a bill of the following amounts: $80 less 25% and 20%; $126.50 less 16⅔% and 12½%; and $172.75 less 62½% and 25%. The whole bill is subject to a cash discount of 5%.

16. At what price must goods be marked to net $4.80 after allowing a discount of 20%?

In each of the following, find the list price:

	Net Price	Discount		Net Price	Discount
17.	$3.60	20%, 10%	**22.**	$3.32	25%, 20%, 16⅔%
18.	$4.20	25%, 20%	**23.**	$29.70	25%, 20%, 10%
19.	$4.50	33⅓%, 10%	**24.**	$28.26	20%, 15%, 12½%
20.	$3.90	20%, 10%, 5%	**25.**	$52.69	15%, 10%, 5%
21.	$12.14	25%, 12½%	**26.**	$31.50	30%, 20%, 10%

Using the form of statement on page 197 as a model, prepare a monthly statement for each of the following accounts, showing the amount due at the end of the month in each case.

27.

J. D. Christopher & Co., Pittsburgh, Pa.

19—						19—					
Oct.	4		S 17	426	50	Oct.	7	Check	C 24	225	
	14		S 24	327	60		18	Mdse. returned	J 11	74	50
	22		S 27	831	45		28	Check	C 30	500	

28.

Mrs. L. B. Ingalls, 22 Sand St.

19—						19—					
Nov.	5		S 13	15	22	Nov.	9	Cash	C 28	10	
	10		S 19	4	28		20	Cash	C 32	5	
	15		S 23	7	50		25	1 lb. Coffee returned	J 43		40
	24		S 30	6	75						

29.

D. E. Lewis, Buffalo, N. Y.

19—						19—					
Dec.	9		S 43	1125		Dec.	15	Check	C 92	900	
	17		S 51	1575	60		22	Allowance for defects	J 111	75	
	23		S 60	931	75		28	Check	C 107	1000	
	29		S 67	832	50						

30.

E. B. Newell, Richmond, Va.

19—						19—					
Jan.	4		S 47	527	50	Jan.	10	Check	C 120	300	
	11		S 53	673	85		17	Check	C 135	400	
	19		S 60	1147	80		24	Mdse. returned	J 170	125	
	27		S 67	931	75		30	Check	C 150	1000	

PROFIT AND LOSS

368. If eggs are bought at 25¢ a dozen and sold at 30¢ a dozen, what is the profit? The profit is what part of the cost? of the selling price?

If oranges cost 30¢ a dozen and are sold to gain $33\frac{1}{3}\%$ of the cost, how much is gained per dozen? What is the selling price per dozen?

An overcoat was sold for $30. How much did the retail clothier pay for it if he made a profit of 20% of the selling price? 20% of the cost?

369. The subject of profit and loss has for its object the reckoning of gains, losses, costs and selling prices which arise in business transactions. The treatment of the subject will be in two parts:

First, from the standpoint of merchandising business.

Second, from the standpoint of the manufacturing business.

370. By merchandising business is meant a business in which the chief purpose is buying and selling merchandise. In such a business **prime cost** is the first or direct cost of the goods.

371. **Gross cost** is the prime cost plus all buying expenses, such as inward freight, cartage, storage, commission, etc.

372. Gross cost, then, includes any and every expenditure necessary to render the goods ready for sale.

373. Any expenditure incurred in selling the goods is a **selling expense** and should be deducted from income. Some such expenses are rent, heat, light, salaries and wages of the selling force, advertising, delivery expense, depreciation, etc.

374. **Net sales** is the difference between the total amount of sales and the value of goods returned.

375. **Gross profit** is the difference betwen the net sales and the gross cost of the goods sold.

376. **Net profit** is the difference between the gross profit and the total selling expense.

Profits are reckoned as per cent of

a. Gross cost, or
b. Net sales.

377. To find the gain or loss and the selling price.

An article cost a retailer $15. Find his selling price if he makes a gross profit of (a) 25% of the cost: (b) 25% of the selling price.

(a) 25% of $15 = $3.75 gain
$15 + $3.75 = $18.75 selling price.

(b) Selling price = 100%
Gain = 25%
Cost = 75% of S.P.
$15 ÷ .75 = $20 S. P.

(b) Since the profit is 25% of the selling price, the selling price must represent unity, or 100%. Selling price, in this case, is made up of cost and gain. Hence, cost is the difference between selling price and gain. Therefore, the cost, $15, divided by 75%, gives the selling price, or $20.

Find the gain or loss and the selling price in each of the following:

	Cost	Gross Profit or Loss on			Prime Cost	Buying Exp.	Gross Profit or Loss on	
		Cost	Sales				Cost	Sales
1.	$0.20	20% gain	20% gain	**13.**	$2.50	$0.14	33⅓% gain	12% gain
2.	.24	25% gain	25% gain	**14.**	3.45	.24	12½% loss	18% gain
3.	.30	40% gain	40% gain	**15.**	4.50	.06	8⅓% gain	14% gain
4.	.32	12½% gain	20% gain	**16.**	4.60	.20	18¾% gain	20% gain
5.	.36	25% gain	20% gain	**17.**	6.00	.10	50% gain	22% gain
6.	.40	12½% loss	20% gain	**18.**	17.00	.20	15% loss	14% gain
7.	.48	16⅔% gain	4% gain	**19.**	33.25	.35	22½% gain	16% gain
8.	.60	33⅓% gain	25% gain	**20.**	42.20	.30	18% gain	15% gain
9.	.75	40% gain	25% gain	**21.**	91.75	1.25	37½% gain	22½% gain
10.	.90	10% gain	10% gain	**22.**	103.60	1.40	40% gain	30% gain
11.	1.12	37½% gain	30% gain	**23.**	192.50	2.50	45% gain	35% gain
12.	1.20	15% gain	25% gain	**24.**	276.20	2.80	60% gain	40% gain

In many cases the buying expense is expressed as a per cent of the value of the goods purchased. This percentage is found by dividing the total buying expenses for a given period (usually a year or more) by the total prime cost of goods for the same period. The gross cost of a given article is then found by adding to its prime cost the predetermined percentage of expenses.

Last year a merchant's total purchases (prime cost) of merchandise amounted to $328,475.60. His total buying expenses (commissions, inward freight, cartage, storage, etc.) were $16,422.50. Dividing $16,422.50 by $328,475.60 gives very nearly 5%. As a result of last year's experience he estimates that his buying expenses this year will be 5% of the prime cost of merchandise.

Allowing 5% of prime cost as buying expense, find the gross cost of articles bought for,

1. $12
2. $16
3. $20
4. $24
5. $ .20
6. $ .60
7. $1.40
8. $2.60
9. $15.80
10. $18.40
11. $25.60
12. $30
13. $125
14. $140
15. $128
16. $160
17. $225
18. $250
19. $275
20. $360

PROBLEMS

378. 1. Last year Mr. Amos bought merchandise invoiced at $428,750, and paid buying expenses amount to $12,862.50. Using that experience as a guide for the year's business, find the gross cost of an article invoiced at $24.

2. Allowing 4% of purchase price for buying expense, find the gross cost and the correct selling price of an article bought for $20, and sold to make a gross profit of 20% of the selling price.

For a period of three years, Mr. Kane's purchases and buying expenses were as follows:

	PURCHASES	BUYING EXPENSES
1st year	$28,493.50	$1162.64
2nd year	$36,592.55	$1804.97
3rd year	$38,694.73	$1721.43

Using the average percentage of buying expenses for the three years, complete the following table:

	ARTICLE	PRIME COST	% BUYING EXPENSE	GROSS COST	% PROFIT ON SALES	SELLING PRICE
3.	Overcoat	$18			$33\frac{1}{3}\%$	
4.	Derby hat	4			25%	
5.	Neckties	9 (doz.)			20%	Each
6.	Umbrella	3			10%	
7.	Pr. shoes	5.60			16%	
8.	Handkfs.	4.50 (dz.)			37%	Each

9. A stock of goods valued at $13,500 was damaged by fire and water so as to cause a loss of 30%. What was the loss, and for how much did the goods sell?

10. A piece of real estate costing $2400 increased in value $41\frac{2}{3}\%$, when it was sold. Find the gain and the selling price.

11. A vegetable dealer buys potatoes at 60¢ a bushel and sells them at a gross profit of $66\frac{2}{3}\%$ of the cost. What is the gain per bushel, and the selling price per peck?

12. A fruiterer bought a carload of berries for $1380. He was obliged to sell them at a loss of $12\frac{1}{2}\%$ of the cost. One retailer who purchased $375 worth of the berries on credit failed in business and paid his creditors 40¢ on the dollar. Find the wholesaler's total loss.

13. A grocer buys sugar at 5¢ a pound. He puts it up in bags of $3\frac{1}{2}$ lb. each. At what price per bag must he sell it to gain $2\frac{6}{7}\%$ of the cost?

14. A grocer buys 6 bags Rio coffee, 125 lb. in a bag, at 12¢ a pound, and 9 bags Java coffee, 75 lb. in a bag, at 20¢ a pound. He makes a mixture of the two kinds, and sells it at 30¢ a pound. If his cost of doing business is 27.37% of gross sales, find his rate of profit, based on sales.

15. An invoice of merchandise cost $74.25 less $33\frac{1}{3}\%$ and 10%. Freight and cartage were $7.45. Find the gain and the selling price if the goods were sold at a gross profit of $37\frac{1}{2}\%$ of cost.

16. A shoe dealer buys shoes at $3 less $16\frac{2}{3}\%$ and 10% per pair. The expenses of the purchase amount to 4% of prime cost. At what price must he sell them to clear 15% of sales if selling expense is 25% of sales?

In a given year a merchant's selling expenses amounted to $21,475.65, and his gross sales amounted to $95,447.33. The selling expenses were what per cent of the gross sales?

$21,475.65 ÷ $95,447.33 = .225 = 22½%.

The total selling expense is frequently spoken of as the "cost of doing business." (C. D. B.) It is expressed as a rate per cent and is found by dividing the total selling expense by the gross sales.

The gross cost of an article is $5.25. If the C. D. B. is 22½%, find the selling price if a net profit of 7½% of sales is realized.

The selling price = 100% of itself
Deductions:
  C. D. B. = 22½% of selling price
  Net profit = 7½% of selling price
Total deductions = 30% of selling price
  Leaving cost ($5.25) = 70% of selling price

$5.25 ÷ .70 = $7.50, selling price.

379. 1. A dry goods merchant's gross sales for a given year were $225,763.85. His annual statement showed the following items charged as selling expenses: Salaries and wages, $31,425; advertising, $2400; delivery expense, $4500; rent, $7200; insurance, $500; depreciation, $3500; general expense, $11,000; bad debts, $1560. Find his cost of doing business.

Using the per cent of C. D. B. in the above problem, find the selling price of each article in the following list:

	ARTICLE	PRIME COST	% OF BUYING EXPENSE	GROSS COST	C. D. B.	% NET PROFIT ON SALES
2.	Silk, yard	$3.60	5%			9½%
3.	Satin, yard	2.80	5%			12%
4.	Shirt Waist	6.40	4%			10%
5.	Gloves	1.75	8%			12½%
6.	Sweater	4.75	6%			9%

Find the selling price of an article whose gross cost is \$3.40, to make a net profit of 12% of the cost, if the C. D. B. is 20% of the sales.

The profit is 12% of \$3.40 = \$.408.

The cost plus the profit is \$3.40 + .408 = \$3.808.

The selling price is made up of the gross cost + the profit + the selling expenses. (C. D. B.)

That is, the selling price = 100% of itself
The C. D. B. = 20% of the Selling Price
The difference (cost + profit) = 80% of the Selling Price
Hence \$3.808 = 80% of Selling Price.
\$3.808 ÷ .80 = \$4.76, the Selling Price.

380. Find the selling price of each article:

	Article	Prime Cost	Buying Exp.	Rate of Profit on Cost	Cost of Doing Business
1.	Fur neckpiece	\$45.00	8%	12½%	16⅔%
2.	Sealskin coat	500.00	8%	10%	12%
3.	Auto Robe	28.00	7%	8⅓%	22%
4.	Auto tire	27.50	6%	20%	20%
5.	Handbag	14.80	5%	9%	15.3%

To find the gain or loss per cent, the cost and the gain or loss being given.

Find the gain or loss per cent of cost in each of the following:

	Cost			Cost			Cost	
1.	\$0.15	\$0.05 gain	**9.**	\$ 3.50	\$ .70 loss	**17.**	\$603.00	\$201.00 gain
2.	.20	.05 gain	**10.**	5.00	1.50 gain	**18.**	74.50	14.90 loss
3.	.36	.06 loss	**11.**	3.60	.90 gain	**19.**	38.50	3.85 gain
4.	.45	.05 gain	**12.**	4.50	.90 loss	**20.**	94.50	31.50 gain
5.	.50	.05 loss	**13.**	16.00	8.00 gain	**21.**	18.00	22.50 S. price
6.	.80	.20 gain	**14.**	28.00	4.00 gain	**22.**	27.00	33.75 S. price
7.	1.00	.20 loss	**15.**	450.00	75.00 gain	**23.**	40.00	36.00 S. price
8.	2.50	.40 gain	**16.**	425.00	170.00 gain	**24.**	64.00	80.00 S. price

381. The following cost and sales prices are from a "monthly statement of sales" in a hardware store. Find the rate of gain in each case. In 1–8 use cost as a basis; in 9–16 use selling price as a basis.

	Cost Price	Sales Price		Cost Price	Sales Price
1.	$16.38	$20.25	**9.**	$ 8.65	$10.00
2.	22.50	29.45	**10.**	41.50	48.75
3.	12.50	15.00	**11.**	25.00	31.25
4.	35.25	41.50	**12.**	54.00	65.50
5.	5.65	7.50	**13.**	2.50	3.20
6.	4.50	6.00	**14.**	23.50	26.65
7.	15.25	20.00	**15.**	21.50	26.00
8.	12.50	15.00	**16.**	15.00	18.50

PROBLEMS

382. 1. An article cost $4.80, less $16\frac{2}{3}$% and $12\frac{1}{2}$%. Buying expenses were 6% of prime cost. It was sold for $5.60 less $12\frac{1}{2}$%. If the cost of doing business was $14\frac{2}{7}$%, what per cent of the sales was the net profit?

The net selling price is $\frac{7}{8}$ of $5.60 . . . . = $4.90
The prime cost is $\frac{5}{6}$ of $\frac{7}{8}$ of $4.80 . . . = $3.50
The buying expenses are 6% of $3.50 . = .21
The gross cost . . . . = 3.71
The gross profit . . . . = 1.19
The C. D. B. is $14\frac{2}{7}$% of $4.90 . . . . = .70
The net profit . . . . = .49
The per cent of net profit is $.49 ÷ $4.90 = .10 = 10%.

2. A merchant paid $6.40 less $12\frac{1}{2}$% and 10% for a given article. He sells it at $12 less 25%. If his buying expenses are 5%, and his cost of doing business is 31.4%, what per cent of the gross cost was the net profit?

The net selling price is 75% of $12 . . . . . . = $9.00
The prime cost is $\frac{7}{8}$ of $\frac{9}{10}$ of $6.40 . = $5.04
The buying expense is 5% of $5.04 . = .252
The gross cost . . . . = 5.292
The gross profit . . . . = 3.708
The C. D. B. is 31.4% of $9 . . . . = 2.826
The net profit . . . . = $ .882
The per cent of net profit is $.882 ÷ $5.292 = .16$\frac{2}{3}$ = 16$\frac{2}{3}$%.

3. Find the rate of gross profit on cost when merchandise is bought for $6.40 less 25% and is sold at $6.40 net.

4. An article cost $120 less 16$\frac{2}{3}$% and 10%. Buying expenses were 6$\frac{2}{3}$%. If cost of doing business was 11$\frac{1}{2}$%, what per cent of sales was made by selling the article at $120?

5. A bookseller bought a book for $8, less 33$\frac{1}{3}$% and 10%. If his cost of doing business is 13$\frac{1}{3}$%, what per cent of sales does he make by selling the book for $8, less 10%?

6. Hats cost $54 a dozen less 20% and 16$\frac{2}{3}$%. Buying expenses are 5%. Cost of doing business is 27.55%. If the hats are sold at $5 each, find rate of profit on gross cost.

7. A statement of the merchandise account at the close of the year showed the following: Inventory at the beginning of the year, $3500; purchases $22,500; sales $28,000; inventory at the close of the year, $6000. Make a statement showing the gain or loss on sales. What per cent of the sales was the profit or loss?

	RETURNS		
	Merchandise Sales		28000
	COST		
	Merchandise, inv. at beginning	3500	
	Merchandise purchases	22500	
	Merchandise, total cost	26000	
	Less inventory at closing	6000	
	Net cost of sales		20000
	Merchandise gain		8000

$8000 ÷ $28,000 = .28$\frac{4}{7}$ = 28$\frac{4}{7}$%, rate of profit.

8. At the beginning of the year a merchant had on hand merchandise amounting to $5400. During the year he bought $38,000 worth, and sold $45,000 worth of merchandise. His buying expenses amounted to $750. His selling expenses were $2250. At the end of the year he had on hand an inventory of merchandise of $7400. Make a statement showing the gross selling profit, the net profit. What per cent of the sales was the gross selling profit? the net profit?

9. Make a statement, showing net gain or loss and find the gain or loss per cent, based on gross cost, from the following facts: Jan. 1, 1923, inventory $7925; Dec. 31, 1923, purchases for the year $53,000; inward freight and cartage $796.50; sales for the year $54,837.50; selling expenses $2662.50; December 31, 1923, inventory $1721.50.

383. To find the cost, the gain or loss, or selling price and the per cent of gain or loss being given.

If a profit of 10¢ a dozen is made on oranges, and the profit is ¼ of the cost, what is the cost?

If a loss of 3¢ a dozen on eggs is a loss of 10% of the selling price, what is the cost?

384. Find the cost in each of the following:

	Gain	Gain % on Cost		Gain	Gain % on Selling Price		Selling Price	Gain %
1.	$0.06	25%	11.	$12.00	25%	21.	$ 3.75	16⅔% loss on C.
2.	.07	20%	12.	15.00	3⅓%	22.	4.50	20% gain on S. P.
3.	.15	33⅓%	13.	18.00	2½%	23.	6.40	25% gain on S. P.
4.	.25	12½%	14.	30.00	75%	24.	18.00	33⅓% loss on C.
5.	.45	16⅔%	15.	160.00	31¼%	25.	22.50	25% less on C.
6.	.75	6¼%	16.	220.00	5%	26.	37.50	25% gain on S. P.
7.	1.25	14 %	17.	28.00	33⅓%	27.	31.00	3⅓% gain on S. P.
8.	2.40	37½%	18.	29.00	14 %	28.	39.00	2½% loss on C.
9.	3.20	50%	19.	38.00	66⅔%	29.	59.00	1⅔% loss on C.
10.	3.60	56¼%	20.	54.00	9%	30.	162.00	1¼% gain on S. P.

A merchant's cost of doing business is 19.4% of his sales, and his net profit is 8.6% of his sales. What should be the gross cost of an article which he sells for $75?

The sales . . . . = 100% of themselves
The C. D. B. . . = 19.4% of sales
The net profit . . = 8.6% of sales
A total of . . . . 28% of sales
Hence the cost . . . . = 72% of sales
72% of $75 = $54, cost.

PROBLEMS

385. A grocer's sales amounted to $125,750.60 for a given year. His net profit was $11\frac{1}{4}\%$ of his sales, and his cost of doing business was $22\frac{1}{2}\%$ of sales. Find his gross cost of purchases.

2. If the buying expenses amount to 8% of prime cost, find the prime cost of an article whose gross cost was $28.08.

3. A merchant's gross profit was $13,650, which amount was 26% of the sales. Find the amount of sales, and the gross cost of goods sold.

4. In a special sale a merchant marked a quantity of goods down so that his loss was 10% of the cost. If his loss was 75¢ per article, what was the selling price of each article?

5. Rather than carry over a stock of straw hats, a dealer sold his stock of hats at one half his former price. His loss on each hat was then 25¢ which was 10% of his original selling price. Find the cost of each hat.

6. My sales for 1 year were $72,375.90. The cost of doing business was $17\frac{1}{2}\%$, and the net profit was $13\frac{1}{2}\%$ of sales. Find the gross cost of goods sold.

7. A merchant sold a sideboard at a gain of 25% of the cost, which was a gain of $18. When the merchant bought the sideboard, he was allowed discounts of 20% and 10% from the list price. Find the net cost and the list price.

8. A stock of goods was damaged 15% by fire and water. If the loss was $12,000, what was the value of the goods?

9. A speculator bought a lot and sold it so as to gain 20% of the cost. He immediately bought a second lot with the selling price of the first, which he afterward sold at a profit of $16\frac{2}{3}\%$ of its cost. If his entire gain was $1400, how much did he pay for each lot?

10. In selling a quantity of oranges and pineapples, a huckster gained $12.50, gaining 25% on the cost of the oranges, but losing 10% on the cost of the pineapples. If the loss on the pineapples was 20% of the net gain, find the cost of each.

11. I bought a house for 10% below its estimated value and sold it for 10% more than its estimated value. My profit was $800. How much did I pay for the house? What rate of profit did I make?

12. If you buy at $.485 per pound less 25% and $12\frac{1}{2}\%$, and sell at the same list price less $16\frac{2}{3}\%$ and 10%, what rate of discount can you allow so that you can have the same rate of profit if you have to buy at $.485 less 20% and 10%?

386. PROFIT AND LOSS DRILL CHART

	Cost	% Gain	% Loss	Gain	Loss	Selling Price	Selling Price
	1	2	3	4	5	6	7
1.	$ 16.00	$.12\frac{1}{2}$	.10	$ 4.00	$ 1.00	$18	$12
2.	18.00	$.16\frac{2}{3}$	$.11\frac{1}{9}$	6.00	2.00	24	15
3.	22.50	$.33\frac{1}{3}$	$.16\frac{2}{3}$	3.75	11.25	30	15
4.	28.00	25	$.12\frac{1}{2}$	4.00	7.00	30	20
5.	36.00	$.11\frac{1}{9}$	.25	12.00	3.00	42	30
6.	48.00	$.08\frac{1}{3}$	$.16\frac{2}{3}$	4.00	2.00	64	36
7.	62.50	.20	.10	25.00	12.50	75	50
8.	96.00	$.12\frac{1}{2}$	$.37\frac{1}{2}$	6.00	24.00	128	80
9.	120.00	$.37\frac{1}{2}$	$.06\frac{2}{3}$	15.00	12.00	160	108
10.	150.00	$.33\frac{1}{3}$	.04	15.00	7.50	210	141
11.	240.00	.15	$.06\frac{1}{4}$	15.00	24.00	280	228
12.	360.00	.05	$.03\frac{1}{3}$	54.00	18.00	450	324
13.	500.00	.24	.03	25.00	10.00	575	490
14.	750.00	.18	.12	7.50	45.00	900	720
15.	900.00	.30	$.06\frac{2}{3}$	135.00	72.00	1080	837
16.	1200.00	$.08\frac{1}{3}$	$.02\frac{1}{2}$	180.00	84.00	1400	1056
17.	1500.00	$.06\frac{2}{3}$	.06	300.00	45.00	1550	1350
18.	1800.00	$.22\frac{2}{9}$	.02	450.00	90.00	2100	1500

1. Combine columns 1 and 2, and find selling price.
2. Combine columns 1 and 3, and find selling price.
3. Combine columns 1 and 4, and find gain per cent.
4. Combine columns 1 and 5, and find loss per cent.
5. Combine columns 1 and 6, and find gain per cent.
6. Combine columns 1 and 7, and find loss per cent.
7. Combine columns 2 and 4, and find cost and selling price.
8. Combine columns 3 and 5, and find cost and selling price.
9. Combine columns 4 and 6, and find gain per cent.
10. Combine columns 5 and 7, and find loss per cent.

REVIEW OF PROFIT AND LOSS

MENTAL

387. 1. If milk costs 24¢ a gallon, what per cent of the cost is gained by selling it at 10¢ a quart?

2. If eggs cost 50¢ a dozen, how many can a grocer sell for 25¢, if he gains 20% of the cost?

3. When potatoes cost wholesale 80¢ a bushel, what per cent of the selling price does a retailer make who sells them at 15¢ a basket (8 baskets to the bushel)?

4. An article that cost $4 sells for $5. Will it pay to reduce the selling price 10%, if by so doing there are three times as many sales daily?

5. A dining table cost $40. Find the selling price to gain 20% of the cost; 25%; 15%; 10%; 40%.

6. An article cost $12. Find selling price to gain 20% of the selling price; 25% of the selling price; to lose 20% of the selling price; 25% of the selling price.

7. The cost is $48 per dozen. Find selling price of 1 to gain 25% of the cost; of 2 to gain 15%; of 3 to gain 20%; of 2 to lose 5%; of 6 to lose 10%.

8. If chairs cost $72 per dozen and 4 of them sell at $9.00 each, and the remainder sell at $7.50 each, what per cent of the cost is the profit?

9. You buy an article for $4.50 and sell it for $6. If the cost drops to $4.20, at what price will you sell to make the same rate of profit?

10. A firm has uncollected accounts of $2400. In closing their books for the year, they allow 5% for bad debts. How much do they allow for bad debts?

11. An article which cost $2.50 is sold to gain 20% of the cost. Find the gain on 50 such articles.

12. A dealer bought handsaws at $24 a dozen, less 16⅔%. What per cent of selling price does he make if he sells them at $2.50 each?

13. If coal costs $8 a ton wholesale, and sells at 30% above that price, what is the gain on 1000 T., if it costs $1 a ton to deliver it?

14. The raw material in a rocking chair costs $3.50, the labor of making costs $8.50. If the profit is 25% of cost, for how much does it sell?

15. An American bought a horse in Canada for $130. He brought him to the United States, paying the duty, $30, and sold him for $200. What per cent of the selling price did he gain?

16. The man (in problem 15) returned to Canada and bought another horse with the $200 and paid the duty, 25%. At what price must he sell this horse to gain 20% on total cost?

17. If apples cost 90¢ a bushel, and 10% of them spoil, at what price per bushel must the remainder be sold to gain 20% on the entire cost?

18. A grocer buys 30 doz. eggs at 40¢ a dozen. If, by an accident, 10% of them are broken, what per cent does he gain or lose by selling the remainder at 50¢ a dozen?

19. I buy a carpet at $3.60 a yard, less 25% What per cent of cost do I make if I sell it at $3.60 net?

20. A room is 20′ × 15′, and requires 360 board feet of matched lumber to lay the floor. What is the percentage of waste?

21. 10 lb. of 15¢ coffee are mixed with 10 lb. at 20¢. At what price per pound should it sell to gain 20% of the cost?

22. Equal quantities of 30¢ and 40¢ tea are mixed. Find selling price to gain 30% of the selling price.

23. A 40¢ tea is mixed with a 70¢ grade in the ratio of 2 to 1. Find the per cent of profit on cost if the mixture is sold at 65¢.

PROBLEMS

388. 1. A merchant bought 440 yd. of cloth for $1100, less 20% and 10%. At what price per yard must he sell the cloth to gain 33⅓% of the cost?

2. Hats cost $45 a dozen, less 20% and 16⅔%. At what price should they retail to gain 37½% of the selling price?

3. If a grocer mixes 28 lb. of one grade of coffee costing 22½¢ a pound with 32 lb. of another grade costing 17½¢ a pound and sells the mixture at 35¢ a pound, what per cent of the selling price is gained?

4. I bought goods for $435, less 20%, 12½%, 10%. I sold them for $560, less 25%, and 16⅔%. What per cent of cost did I make?

5. If a desk costs $18 less 16⅔% and 10%, how many desks can be bought for $270?

6. A fruit dealer bought 60 doz. oranges for $12. He sold 40 doz. at 30¢ a dozen. Of those remaining 5 doz. spoiled. At what price must the rest be sold to gain 45% on the cost of all?

7. The prime cost of a quantity of merchandise was $22,836.54. The buying expenses were 4% of prime cost. If the cost of doing business was 13⅓% and the net profit was 7½% of sales, find the selling price of the entire quantity of goods.

8. If 75% of a stock of goods is sold for what the entire stock cost, what is the rate of gain on cost? If the remainder of the goods is sold at cost, what per cent of cost is gained on the whole stock?

9. At what per cent of the list price should goods be marked that are bought at discounts of 25% and 20%, if a profit of 33⅓% of cost is to be realized?

10. An article cost $24, less 20%, 16⅔%, and 6¼%. What is the retailer's price mark if he marks it to gain 40% of the selling price?

11. At what price shall each of the items in the following bill be sold to gain 25% of cost?

Boston, Mass., Aug. 3, 1924.

Mr. James Doyle,
Pittsfield, Mass.
Bought of WELLS, FLOWER & CO.
Terms: Net 60 days.

6	Roll Top Desks	$45.00	270			
8	Flat Top Desks	25.00	200			
12	China Closets	37.50	450			
9	Sideboards	62.50	562	50		
12	Dining Tables	25.00	300			
12	Sets Dining Chairs	20.00	240			
			2022	50		
	Less 25% and 20%		809		1213	50

Suggestion. Find at what per cent of the list price the goods must be sold to make the required profit.

12. Find the net amount of the following items, and determine a selling price mark for each, if sold to gain 33$\frac{1}{3}$% of cost.

2 doz. Smoothing Planes @ $10.50, less 14$\frac{2}{7}$%.
4 doz. Screwdrivers @ $5.40, less 25%
200 Carriage Bolts, each $\frac{3}{8}$" × 2", @ $5.60
$\frac{3}{8}$" × 2$\frac{1}{2}$", @ $5.80
$\frac{1}{2}$" × 4$\frac{1}{2}$", @ $6.25,
less 37$\frac{1}{2}$% and 20%
400 Machine Bolts, $\frac{3}{4}$" × 6", @ $8.40,
less 50% and 10%.

13. I bought 60 bbl. of apples at $2.25 per barrel. Upon sorting them, I found there were 35 bbl. first grade and 21 bbl. second grade, the rest being decayed. The first grade sold at $3.50, and the second grade at $2.15. If it cost 10¢ a barrel to sort, what per cent of the selling price was the gain or loss?

14. By marking goods at $4.50 each, a merchant sold on the average 4 of the articles per day. Could he afford to reduce the price to $4 if by so doing the sales increased to 8 articles a day, the cost being $3.25? What per cent would his profits be increased or decreased each day?

15. Dining tables selling for $16.50 at the rate of 6 a day are reduced to $14, whereupon the sales increase to 10 a day. If the tables cost $10.50, does the merchant gain or lose, and what per cent, by the cut in the price?

16. At a sale the price of a chiffonier was reduced from $28 to $21. The regular sales were 3 a day. During the sale there were 12 sales a day. If the cost was $16.80, were the daily profits increased or decreased, and what per cent?

17. At what price should goods costing $216 be marked to make a gross profit of 25% of the cost after allowing a discount of 25%, 20%, and 10%?

18. If coal can be bought at $4.75 per long ton and the cost of handling is 80 ¢ a short ton, what rate of profit on cost is realized if the coal is sold at $6.50 per short ton?

19. At the beginning of a certain year a firm had on hand $16,800 worth of merchandise. They bought during the year $47,500 worth, and paid freight on it amounting to $700. They sold $62,500 worth of merchandise and paid freight on it to the amount of $850. If at the close of the year they had an inventory of $19,500, what rate of profit did they make? Prepare a statement showing all essential facts of the merchandise account.

Find the gain or loss per cent on sales in each of the following:

20.	Mdse. purchases	$13,250.00
	Mdse. sales	12,687.50
	Mdse, inventory at closing	4,893.80

21.	Mdse. inventory at beginning	$ 3,890.00
	Mdse. purchases	10,860.00
	Mdse. sales	9,640.00
	Mdse. inventory at closing	6,460.00

22.	Jan. 1, 1924.	Inventory	$ 3,200.00
		Purchases during the year	9,600.00
		Sales during the year	10,500.00
		Returned to creditors	450.00
		Returned by debtors	600.00
	Dec. 31, 1924.	Inventory	4,500.00

MANUFACTURING COSTS, PROFITS, ETC.

389. A manufacturing business is one which takes raw material and converts it into manufactured goods. The goods manufactured in a given factory are called its finished product. The finished product of one factory may be the raw material of another. For instance, a cotton mill may spin the cotton into yarn or thread, and weave it into cloth, which is the finished product of the mill. A clothing manufactory will take the cloth as its raw material and make it into clothing.

390. There are several elements or parts entering into the cost of manufactured goods. In general, they are:

1. Prime cost, which consists of (a) cost of the raw material entering into the finished product, and (b) the cost of labor employed directly in the process of manufacture.

2. Factory expense, which consists of rent, or interest on investment, depreciation, fuel, heat, light, insurance, foremen's salaries, office salaries, etc.

3. Selling expenses, which consist of salesmen's salaries, commissions, advertising, or any other expense incurred in making sales.

391. **1.** A shoe manufacturing company has a special order for 15,000 pairs of shoes. If their records show the following facts as to cost, find the average cost per pair of shoes.

Raw material cost		$18,000
Direct labor cost		19,000
Factory expense		
Indirect labor cost	$3,000	
Rent	3,000	
Power, heat, etc.	12,000	
Depreciation, repairs	2,000	
Foremen's salaries	12,000	
Office and adm. exp.	6,000	
Gen. exp., ins., etc.	1,500	39,500

If the company charged $6 per pair for the shoes, their profits were what per cent of their sales?

What per cent of the total sales was the prime cost? was the factory expense?

2. A clothing manufacturer, in filling an order for 25,000 boys' suits, found his costs as follows:

Raw material $35,452.60; direct labor $42,138.95; indirect labor $7043.65; rent $5052.50; power, heat and light $17,960; depreciation on plant and machinery $2632.65; repairs $2456.33; office salaries $9500; administrative expenses $2892.75; general expenses $2860.57. (a) Find the cost of one suit. (b) The cost of raw material was what per cent of the whole cost? (c) Direct labor was what part per cent of the total cost?

If the manufacturer charged a price to net him a profit of $12\frac{1}{2}$% of his total cost, find his selling price of one suit, and his gain on the whole lot. His net profit was what per cent of his sales?

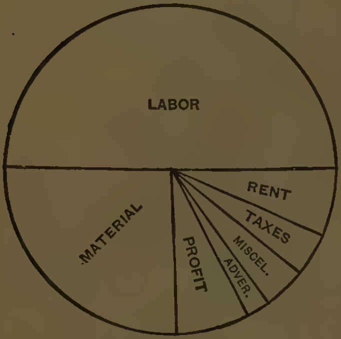
3. A cabinet maker pays for the raw material in a certain chair, $6.50. The cost of making is $16.90. His overhead expense is 15% of his sales. Find his selling price if he clears 10% of his sales.

4. The raw material in 100 pieces of machinery cost $960. There were six separate processes in their manufacture. The first required 86 hr. at 75¢ per hour; the second 132 hr. at 64¢; the third 164 hr. at 70¢; the fourth 140 hr. at 68¢; the fifth 216 hr. at 68¢; and the sixth 96 hr. at 80¢ an hour. Factory expense was 26%, and selling expense was 15%, of sales. The machines were sold at $30.88 each. The net profit was what per cent of the sales?

5. In a given factory are four departments with floor areas as follows: Department A 1920 sq. ft.; B 3240 sq. ft.; C 4320 sq. ft.; and D 2880 sq. ft. The overhead expense for the whole factory for a year is $24,750. If this expense is divided among the several departments according to floor space, find the weekly overhead for each department.

PROBLEMS

392. A prominent clothing firm recently made public the following facts and figures, based on six months' actual experience, to show what becomes of a customer's dollar when spent for clothing.



Allow 20% of material for linings, buttons, etc.

Material		$0.2661
Labor—Wages for making and selling		.4862
Rent		.0720
Taxes		.0362
Miscellaneous:		
"Moneyback"	$0.0073	
Delivery and Freight	.0072	
Postage, carfare, etc.	.0060	
Containers, Twine, etc.	.0053	
Printing, Stationery	.0050	
Fixture Depreciation	.0029	
Building Repairs	.0018	
Insurance	.0016	
Bad Debts	.0012	
Telephone	.0008	
		$0.0391
Advertising		.0183
Profit—for dividends and surplus		.0821
		$1.00

The same firm gives the following prices of woolen cloth for overcoats and suits.

	Price before the War	Price 1920
Overcoating per yard	$1.20	$4.55
Serge for spring suits, per yd.	1.27\frac{1}{2}$	$4.50
A typical wool cloth, per yd.	1.07\frac{1}{2}$	$3.75

The average number of yards required for an overcoat is $2\frac{5}{8}$, for a suit is $3\frac{3}{8}$.

393. **1.** Using the above facts, find (to nearest quarter dollar) (a) the selling price of an overcoat in 1920. How much of the selling price was paid to labor? How much of it was profit? (b) the cost to the consumer of a spring suit of serge (1) in 1920; (2) before the War. (Assume the same ratio of labor and other costs to material before the War as given above.)

2. Assume the average weekly cost of telephone service was $28. Find the average sales per week; the average net profit per week.

3. A spring suit of serge was made from material costing $4.50 a yard. Linings, buttons, etc., cost $3.50 for the suit. It was priced to sell at $75. During a special sale suits of this grade were sold at a discount of 20%. Using the distribution of costs, etc., given on the preceding page, determine the amount of gain or loss per suit sold during the sale.

4. A manufacturing concern sold in one year $109,176.83 worth of goods. Of that amount $22,400 was spent for raw material; $36,750, for direct labor; $5,400, for indirect labor; $8,400, for rent; $10,500, for advertising; $4,500, for miscellaneous expenses; and $1,575, for bad debts; the balance being profit. Using the illustration on the preceding page as a guide, prepare a schedule showing what becomes of the customer's dollar. Carry results out to the nearest tenth of a mill. Prepare a graph illustrating the several percentages or ratios.

MARKING GOODS

394. It is necessary in many lines of business to have the selling price of goods marked on them. This process requires careful, accurate calculation, otherwise merchandise may be marked incorrectly and sold at a loss.

395. Find the selling price of each of the following:

COST	RATE OF GAIN	COST	RATE OF GAIN	COST	RATE OF GAIN
1. $12.00	25%	7. $22.50	$33\frac{1}{3}\%$	13. $ 1.20	$16\frac{2}{3}\%$
2. 6.40	$18\frac{3}{4}\%$	8. 15.00	20%	14. 1.50	20%
3. 4.50	20%	9. 4.40	25%	15. 2.20	25%
4. 3.30	$33\frac{1}{3}\%$	10. 6.60	$16\frac{2}{3}\%$	16. 17.00	25%
5. 2.50	20%	11. 8.80	$12\frac{1}{2}\%$	17. 14.00	20%
6. 1.60	50%	12. 1.10	10%	18. 19.00	$12\frac{1}{2}\%$

396. Many kinds of goods are bought and sold wholesale by the dozen. The retailer sells them by the piece.

In marking goods, all calculations, as far as possible, should be mental. In dividing the price of a dozen by 12 to find the cost of one article, twelfths of a dollar from $\frac{1}{12}$ to $\frac{11}{12}$ must frequently be reduced to cents. Hence the equivalents of twelfths of a dollar should be memorized. See page 17.

397. Find the selling price per article of each of the following:

Cost per Dozen	Rate of Gain	Cost per Dozen	Rate of Gain	Cost per Dozen	Rate of Gain
1. \$12.00	25%	**6.** \$16.00	25%	**11.** \$32.00	$12\frac{1}{2}\%$
2. 9.60	$33\frac{1}{3}\%$	**7.** 18.00	$33\frac{1}{3}\%$	**12.** 33.60	$33\frac{1}{3}\%$
3. 8.40	25%	**8.** 20.00	25%	**13.** 34.00	25%
4. 14.00	20%	**9.** 25.00	40%	**14.** 45.00	20%
5. 15.00	40%	**10.** 27.00	$33\frac{1}{3}\%$	**15.** 56.00	$14\frac{2}{7}\%$

398. To find the marked price, having given the cost with discount, the rate of gain or loss, and the selling discount.

A jobber buys bicycles at \$24, less $12\frac{1}{2}\%$. He sells to gain $33\frac{1}{3}\%$. What is his marking price if he allows 20% discount?

$\frac{1}{8}$ of \$24 = \$ 3, discount.
\$24 — \$3 = \$21, net cost to the jobber.
$\frac{1}{3}$ of \$21 = \$ 7, gain.
\$21 + \$7 = \$28, jobber's net selling price.
\$28 ÷ 80% ($\frac{4}{5}$) = \$35, jobber's marked price.

The discount is $\frac{1}{8}$ of \$24, or \$3, which deducted leaves \$21 as the net cost of the bicycle. $33\frac{1}{3}\%$ gain is equal to $\frac{1}{3}$ of \$21, or \$7, making the net selling price \$28. Since the jobber allows a 20% discount, he sells for 80% of his marked price. Hence \$28 ÷ .80 gives \$35, the list price (Prin. 14).

PROBLEMS

399. **1.** A sewing machine catalogued at \$40 is sold to a jobber subject to a discount of 25% and 20%. Find the jobber's marked price if he sells to gain 25% after deducting 25% and 20%.

2. A manufacturer sells bathtubs at \$35, less 40%. At what price should the retailer mark them to sell at 25% discount and still make 50%?

3. A merchant pays \$10.80, less 25% and 20%, a yard for lace. Find his marked price if he sells to gain 25% after allowing a discount of 10%.

4. Complete the following invoice, and determine the marking price of each chiffonier to clear 20%. (Fosdick pays cash.)

CLEVELAND, OHIO, Aug. 10, 1924.

PETER R. FOSDICK,
New Haven, Conn.

BOUGHT OF OHIO FURNITURE COMPANY

Terms: n/60 3/10.

18	#5287 Oak Chiffoniers @ 22.50 . .				
	Less 10% and 5%				
	Freight prepaid			13	61

5. Complete the following, and find the retail selling price to gain 30%.

KALAMAZOO, MICH., Sept. 14, 1924.

G. A. HUBBARD,
Chicago, Ill.

BOUGHT OF EDWARDS & CHAMBERLAIN

Terms: Net Cash

2	doz. Shovels	$12.50				
3	" Wheelbarrows	15.00				
60	ft. 2″ Lead Pipe	.25				
	Less 20% and 10%					

6. If hats cost $48, less 20% and $16\frac{2}{3}$%, per dozen, what should be the list price to gain 35%, and allow $16\frac{2}{3}$% and 10%?

7. When hosiery costs $9, less 20% and $12\frac{1}{2}$% per dozen pairs how shall they be marked per pair to gain 20%, if $12\frac{1}{2}$% discount is offered?

8. A rug 12′ by 15′ is imported, the price being £5. If the duty and other expenses amount to $27.60, and the cost of a pound sterling is $4.8665, at what price should the rug sell to make a profit of 20% of the selling price?

9. The list price of saws was \$54 per dozen, subject to a discount of 12½% and 5%; the freight was \$2.11. At what price must each saw be sold to make a profit of 40% of the cost?

10. A retail grocer has found by experience that his losses based on selling price are 2% from bad debts, 3% from perishable goods and 3% from other sources; at what per cent above cost must he mark all goods so that, after allowing for the above losses, and for 12% for cost of doing business, he may still make a profit of 15% of the selling price?

11. An automobile manufacturer lists an automobile at \$1275, subject to a trade discount of 20% and 16⅔%. At what price must the dealer who buys the automobile sell it in order to gain 25% of his cost?

12. A retailer finds that the selling and overhead expenses of his business approximate 15% of his total sales. If he buys an article for \$21.50 less 20% and 10%, at what price must he sell it to make a net profit of 25% of the sales?

13. The cost of manufacturing an article is as follows: materials \$4.72, labor \$2.35, and overhead \$1.03. At what price must the article be sold to realize a profit of 20% of the total cost, and also to provide 15% of the selling price for selling expenses?

14. An implement dealer bought a grain drill for \$96 less a discount of 10% and 5%. At what price, in nearest whole dollars, must he mark the drill, so that he may allow a discount of 20% from his asking price and still make a profit of 20% of cost?

15. To manufacture a given article costs \$17.50, and to sell it costs \$6.50. Find the selling price to gain 25%, (a) of cost plus selling expense; (b) of selling price.

16. The wholesale price of lamps is \$56 per dozen, less 12½% and 10%. The freight is \$1.40 per dozen. At what price per lamp should they be sold to make 16⅔% of the selling price?

17. A machine is in 3 parts, A, B, and C. Its cost is \$20. A is 40% of the cost, B is 35%, and C, 25%. The machine sells at a profit of 25% of its cost. If the cost of A is increased 10%, and of B 20%, find new selling price to maintain the same rate of profit.

COMMISSION AND BROKERAGE

400. A **commission merchant** is a person who buys or sells merchandise, or transacts other business, for another person.

401. The person for whom business is transacted by the commission merchant is called the **principal.**

402. Comission merchants usually charge a rate per cent on the volume of business transacted. Such a charge is called **commission** or **brokerage.** In some cases the commission is reckoned at a certain price per unit of quantity of merchandise, as the bushel, barrel, etc.

403. An additional charge called **guaranty** is sometimes made by commission merchants for undertaking to be responsible for sales on credit or for the quality and quantity of goods bought, etc.

404. Merchandise shipped by a principal (also called **consignor**) to be sold by a commission merchant is called (by the consignor) a **shipment.** The commission merchant (also called the **consignee**) calls the merchandise a **consignment.**

405. The **gross proceeds** of a sale or collection is the entire amount received by the commission merchant. The **net proceeds** is the amount remaining after all expenses have been paid.

406. The **prime cost** of a purchase is the first cost, or the amount actually paid for the merchandise. The **gross cost** is the prime cost plus all the expenses of the purchase.

407. An **account sales** is an itemized statement of the sales of merchandise, the charges thereon, and the net proceeds of the sale. The statement is made by the commission merchant and sent to his principal. (For illustration, see problem 22, page 227.)

408. An **account purchase** is an itemized statement of the quantity, quality, and price of merchandise purchased, the expenses incurred, and the gross cost of the purchase. It is made by the purchasing agent and sent to his principal. (See problem 26, page 228.)

409. The principles of percentage apply in commission.

The **gross sales** or prime cost is the *base.*

The **rate of commission** is the *rate.*

The **commission** is the *percentage.*

The **net proceeds** is the *difference.*

The **gross cost** is the *amount.*

410. To find the commission, the cost or selling price, and the rate of commission being given.

1. Find the commission and the net proceeds of a sale of merchandise amounting to \$4500; commission 5%.

$$5\% \text{ of } \$4500 = \$225, \text{ commission.}$$
$$\$4500 - \$225 = \$4275, \text{ net proceeds.}$$

The commission is 5% of the gross sales, \$4500, which equals \$225. The net proceeds is the difference between the gross sales and the expenses.

2. Find the commission and the gross cost of a purchase or merchandise amounting to \$3400; commission 4%, guaranty 2%, cartage \$20.

$$4\% \text{ of } \$3400 = \$136, \text{ commission.}$$
$$2\% \text{ of } \$3400 = \$68, \text{ guaranty.}$$
$$\$3400 + \$136 + \$68 + \$20 = \$3624, \text{ gross cost.}$$

The commission and the guaranty are found by taking 4% and 2%, respectively, of \$3400. The gross cost is the sum of the prime cost, \$3400, and all the charges.

411. Find the commission and the net proceeds, or the gross cost, in each of the following:

	Gross Sales	Com.	Guar.	Other Expenses		Prime Cost	Com.	Guar.	Other Expenses
1.	\$2400	5%		\$15	**11.**	\$1030	4%	3%	\$8.50
2.	3000	3%	2%	25	**12.**	1150	3%	2%	9.60
3.	1500	6⅔%	1%	2%	**13.**	1250	4%		12.50
4.	1800	6¼%		\$20	**14.**	1440	6¼%		2%
5.	2250	3⅓%	3%		**15.**	1880	12½%		50.00
6.	3600	8⅓%		30	**16.**	3360	16⅔%	5%	
7.	4500	10%		28	**17.**	4150	5%	2½%	
8.	1600	2½%	2%	12	**18.**	9600	2½%	2½%	100.00
9.	2700	5%		50	**19.**	12400	10%	2½%	350.00
10.	3200	4%	2%	60	**20.**	16800	12½%	5%	500.00

21. An agent sold 950 bbl. pork at \$17.50 a barrel, on a commission of 2%. The other charges were for storage \$25, freight \$60, and insurance \$5. Find the commission and the net proceeds

22. Find the net proceeds of the following:

ACCOUNT SALES HAY

Albany, N.Y., Oct. 7, 1924.

Sold for Account of

August Stadlich, Wellsville, N. Y.

By F. A. Mead & Co.

1924							
Sept.	10	25 tons Hay	$15.00				
	14	35 tons Hay	15.50				
	20	40 tons Hay	14.75				
		Charges					
Sept.	6	Freight, $100; Cartage, $75;					
		Insurance, $5		180			
	20	Storage		10			
		Commission, 50¢ a ton					
		Net proceeds					

23. Prepare an account sales from the following data, using your own name as agent, and any name you wish as principal:

On August 15, you receive a consignment of 3000 bu. of corn. The same day you pay freight, 2¢ a bushel; cartage, 3¢ a bushel; insurance, $7.50. August 20, you sell 1200 bu. at 60¢; August 24, 900 bu. at 62¢; and August 28, 900 bu. at 61¢. Your commission is 2¢ a bushel, and the storage charge is $15.

24. A consignment of 460 bbl. of flour was received Nov. 1, 1924. Freight charges were $56; cartage, $14; cooperage, $3.70; advertising, $11.50. November 5, 65 bbl. at $6.40 were sold; November 7, 185 bbl. at $6.45; November 9, 50 bbl. at $6.35; and November 16, the remainder at $6.50. The storage charge was $15.80; commission 2%; guaranty, 1%. Prepare an account sales from the foregoing data.

25. If it requires $5\frac{1}{2}$ bu. of wheat to make 1 bbl. of flour, and the cost of grinding is 15¢ a barrel, and the barrels cost 35¢ each, find the gain or loss per cent to the consignor in problem 24, wheat costing 80¢ a bushel.

26. Find the gross cost of the following:

ACCOUNT PURCHASE

PHILADELPHIA, PA., Aug. 4, 1924.

Purchase for the account of

J. L. COOK, Watkins, N.Y.

BY H. M. DONOVAN & Co.

		25 M White Pine Lumber	$72				
		30 M Georgia Pine Ceiling	90				
		18 M Hemlock Boards	32				
		22 M White Oak	96				
		CHARGES					
		Cartage		250			
		Commission $2\frac{1}{2}\%$					
		Amount charged to your account					

27. Prepare an account purchase from the following facts:

A. B. Carver & Co., Baltimore, Md., bought for the account of Howard Cole of Scranton, Pa., 100 gal. oysters at 75¢; 200 doz. clams at 8¢; 600 lb. bluefish at 5¢; and 800 lb. white fish at 8¢. Charges are 3% commission and drayage $4.50.

COMMISSION REVIEW

MENTAL

412. **1.** An agent sold $5000 worth of clothing on a commission of 5%. What was his commission?

2. I bought for my principal sugar worth $1250 on a commission of 4%. What was the total cost of the sugar?

3. A lawyer collected 75% of a claim of $1600 and charged 10% for collecting. What was his commission?

4. A book agent's commission was 40%. He sold 120 books at $2.50 each. How much did he earn?

5. An auctioneer received 5% of his sales, which amounted to $2400. Find his commission.

6. My agent sold for me 100 tubs of butter averaging 50 lb. each at 50¢ a pound, and charged 2%. How much did I receive for the butter?

Find the proceeds in each of the following:

7. 20 T. hay at $15; commission 8%; other expenses $25.

8. 12 T. cabbage at $12.50; commission 6%; other expenses $11.

9. 300 bbl. flour at $6; commission 3%; storage 3¢ a barrel; freight 20¢ a barrel.

10. 20,000 ft. lumber at $40 per M; commission 2½%; freight $30.

11. I bought through an agent 300 yd. of carpet at $1.50, paying him 2%. How much did the carpet cost me?

12. An agent buys 1000 bu. of wheat at $1.10, and charges ½¢ a bushel. What is his commission, and the total cost?

13. Find the gross cost of 4000 brick at $15 per M; commission 50¢ a thousand; other charges $2.50.

Find the gross cost of the following purchases:

14. 1200 bu. corn at 75¢; commission 3%; freight 1½¢ a bushel.

15. 700 T. coal at $12.50; commission 2%; freight $1 a ton.

16. 800 yd. silk $2.50; commission ¾%; freight $2.50.

17. 10 half chests Young Hyson, 700 lb., at 25¢; commission 2%; freight $1.50.

18. 10 bbl. sugar, 3500 lb., at 5¢; commission 4%; freight and drayage $1.25 per barrel.

19. A broker bought for me 800 lb. of leather and charged $8, commission being at 4%. What was the price of the leather, and what was the gross cost including freight at 25¢ a hundred pounds?

20. My agent charges 2% commission, and 2% guaranty, for buying coffee at 20¢ a pound. If the commission and guaranty amount to $40, how many pounds were bought, and what was the gross cost?

21. Which is better for the agent, a commission of 1½¢ a bushel or 2%, when a bushel sells for 80¢?

22. I bought a job lot of muslin consisting of 1500 yd. bleached and 800 yd. unbleached for $120; commission 5%; and other expenses $9. If I sell the bleached muslin at 7¢ a yard and the unbleached at 5¢ a yard, what is my profit?

23. I have 75 bbl. of flour which cost me \$360. I send them to my agent, who sells them at \$6.20, and charges 2% commission, and \$5.70 for other expenses. What per cent profit do I make?

24. Find the per cent profit realized by buying potatoes at 80¢ a bushel, and shipping them to New York, if I pay 5¢ a bushel for freight, 3¢ a bushel commission, 2¢ a bushel for other expenses, and the potatoes sell at \$1.10.

REVIEW PROBLEMS

413. 1. An agent remitted to me \$247.38 after retaining a commission of 5% for collection. What sum did he collect? What was the amount of his commission?

2. The gross cost of a purchase of sweet potatoes was \$864.55. The expenses were for drayage \$12.50, cooperage \$3.35, and commission $2\frac{1}{2}$%. If the potatoes cost \$.72 a bushel, how many bushels were bought?

3. An agent charged his principal \$106.25 (commission being $2\frac{1}{2}$%), for buying 5000 bu. of wheat; the freight charges, etc., amounted to \$43.75. How much a bushel did the wheat cost the principal?

4. An agent sold 3000 bu. of wheat, and after deducting his commission of $2\frac{1}{2}$%, sent his principal the proceeds, \$2808. For how much a bushel was the wheat sold?

5. A commission merchant sold a consignment of cottonseed oil for \$12,500 on a commission of 4%. Other expenses of the sale amounted to \$375. What amount was due to the principal?

6. My agent purchased for me 180 bbl. of sugar, averaging 275 lb. each, at $5\frac{1}{4}$¢ a lb. He charged 2% commission and \$71.77 for other expenses. Find the gross cost of the sugar. If it is sold at 6¢ a pound, what rate of gain is realized?

7. An agent sold for his principal 6500 bu. of potatoes at \$1.20 a bushel, charging 3¢ a bushel for selling. The freight charges were 2¢ a bushel, and other expenses amounted to \$85. He purchased for his principal an invoice of silks for \$2475, charging 5% for buying, and remitted the proceeds. What was the amount of the remittance?

8. At what price shall I mark an article that cost me $18, so that I can instruct my agent to allow 25% discount on the marked price, pay him 15% commission for selling, allow 5% of sales for bad debts, and still make a profit of $33\frac{1}{3}$%?

9. To manufacture a certain style of automobile costs $800. What must be the catalogue price of this machine, so that the manufacturer can give the retailer 25% discount on the list price, pay an agent $16\frac{2}{3}$% commission for selling, and still make a profit of 25%?

10. A broker sold for me 400 bales of cotton (500 lb. in a bale) at 12.52¢ a pound on a commission of $5 per hundred bales. He invested the proceeds of the sale in flour at $5 a barrel, charging 2% for buying. How many barrels were purchased, and what sum was left unexpended in the broker's hands?

SUGGESTION:—Find the gross cost of 1 barrel of flour before dividing.

11. An agent received $7500 to invest in apples at $2.40 a barrel after paying all expenses of the purchase. Charges were as follows: commission 3%; guaranty 2%; drayage 5¢ a barrel; and freight 12¢ a barrel. Find the agent's commission, the number of barrels purchased, and the unexpended balance.

12. An agent sells $8000 worth of hardware on a commission of $2\frac{1}{2}$%. He pays freight charges, $162.50; drayage $68.50; and charges $2\frac{1}{2}$% for guaranteeing payment. What are the net proceeds of the sale?

13. You are a commission merchant and have received, Sept. 1, 1924, 840 bbl. of flour to be sold for the Twin City Milling Co. You pay the freight, $131.50; drayage $210; advertising $27.50. September 8, you sell 225 bbl. at $6.75; September 14, 300 bbl. at $6.85; and September 21, the balance at $6.80. Storage charge is $16.74, insurance $\frac{1}{4}$% of gross sales, and your charge for commission is $2\frac{1}{4}$%. Prepare an account sales, showing all facts here given, and the net proceeds.

14. You buy through a purchasing agent 42 Ideal refrigerators at $17.50; 24 dining tables at $26.40; and 18 dressers at $32.75. The agent's commission charge is 5%, and he pays $15 for drayage. Prepare an account purchase using the current date and any name of agent and place you wish.

EXAMINATIONS

SPEED TEST

(Minimum time, 30 minutes; maximum, 1 hour): Deduct one credit for each minute over minimum time.

414. 1. Find the selling price of each of the following:

	Cost		Selling Price
(*a*)	$ 0.16	37½% gained	——
(*b*)	6.00	20 % lost	——
(*c*)	56.00	25 % gained	——
(*d*)	4.50	16⅔% lost	——
(*e*)	828.00	¼% gained	——

2. Find in each of the following the number of hours and minutes from the time commenced to time finished:

	Time Commenced	Time Finished	Hours	Minutes
(*a*)	9 : 50 A.M.	4 : 13 P.M.	——	——
(*b*)	9 : 16 A.M.	3 : 55 P.M.	——	——
(*c*)	9 : 45 A.M.	12 : 16 P.M.	——	——
(*d*)	10 : 15 A.M.	1 : 00 P.M.	——	——
(*e*)	9 : 14 A.M.	10 : 02 P.M.	——	——

3. Divide the numbers in column 1 by the numbers in column 2, and express your answer as per cent correct to two figures only:

	Column 1	Column 2	Per Cent
(*a*)	245,326	981,304	——
(*b*)	5,253	30,900	——
(*c*)	12,596	59,923	——
(*d*)	257,515	476,878	——
(*e*)	18,487	55,460	——

4. Multiply each of the following percentages by its relative weight, and find the general average by dividing the sum of the products by the sum of the relative weights:

	Percentages	Relative Weights	Products
(*a*)	97.51	8	——
(*b*)	81.33⅓	12	——
(*c*)	65.	5	——
(*d*)	69.16	7	——
		——	——
	General average		——

5. Perform the following indicated operations:

$\frac{1}{8}$ of 6876.88 =
\$79,487.98 × 6% =
9569.64 ÷ $\frac{1}{8}$ =
\$6734.17 ×.5% =

7. Find the amount to be remitted to the employer in each case:

	Amount Collected	Rate of Commission	Amount Remitted
(*a*)	\$3,400	5 %	——
(*b*)	91,500	$3\frac{1}{3}$%	——
(*c*)	81,600	$2\frac{1}{2}$%	——
(*d*)	30,501	$\frac{1}{3}$%	——

6. Find the gain per cent in each of the following:

	Cost	Selling Price	Gain %
(*a*)	\$4.50	\$4.95	——
(*b*)	7,500.00	8,750.00	——
(*c*)	5,280.00	7,040.00	——
(*d*)	9,600.00	10,200.00	——

8. Find the net cost of each item and total net cost of all:

	List Price	Discount	Net Cost
(*a*)	\$270.00	25%	——
(*b*)	480.00	25 % and 20%	——
(*c*)	6.40	$12\frac{1}{2}$% and 10%	——
(*d*)	960.00	$33\frac{1}{3}$% and 25%	——
		Total	

9. Exports from the United States for two years; find the total for each year:

Article	1918	1919	Excess
Agricultural implements . . .	\$35,076,911	\$41,195,494	....
Books, maps, etc.	11,433,064	18,239,016	....
Oils, Minerals	289,040,072	328,867,886	....
Wheat	80,802,542	356,898,296	....
Leather & manufactures of . .	100,880,843	303,176,539	....
Meats and meat products . .	593,924,928	1,014,165,889	....

10. Find the excess of each kind, and the total excess of exports for 1919 over those for 1918 in problem 9.

WRITTEN TEST

415. 1. Harry Fenton, a commission merchant, has sold the following consignment of goods for Andrew Crowl: 55 bbl. apples at \$3.50; 60 bbl. potatoes at \$2.75; and 80 bbl. cabbage at \$1.50. Fenton paid expenses as follows: Freight \$17.50; cartage \$7.50, and charged 5% commission. Find the net proceeds of the consignment.

2. In a given five year period factory accidents in a certain state were as follows: Number of men killed 864; number of men permanently disabled 6580; and number temporarily disabled 31,800. Find to the nearest tenth per cent the per cent (a) killed, (b) permanently disabled, and (c) temporarily disabled.

3. A manufacturing concern can make a piece of machinery at a cost of $22.50. To sell it costs an additional amount of $8.75 The selling price is $50. Find the per cent profit based (a) on cost, (b) on selling price.

4. A lumber dealer's books show the following records for one year: Value of lumber on hand at the beginning of the year $15,280; stock purchased during the year $105,450; freight paid on purchases $1124.50; sales for the year $134,652.50; value of stock on hand at close of the year $21,475. Amount spent for labor and delivery expense was $3975.50. Find (a) the cost of the lumber sold; (b) the profit on sales; (c) the net profit; and (d) the per cent of net profit on total sales.

5. A man bought a house for $9600. He paid $3600 in cash and gave a mortgage at 6% for the remainder of the purchase price. The house rented for $90 a month. Taxes averaged $220 a year, and other incidental expenses were 12½% of the amount received for rent. What rate of income did he receive on his actual investment?

6. A. M. Orton bought of the Excelsior Hardware Co., Dec. 15, 1924, terms 2/10 net 30, 600 pairs barn-door hangers at $2.75, less 25% and 10%; 175 doz. strap hinges at $3.25, less 20% and 12½%; and 15 doz. wrought iron wrenches at $14.50. Find the net cost if paid for Dec. 24, 1924. At what price should these items be sold retail to gain 25%?

7. A coal dealer bought a carload of coal containing 40 long tons at $3.60 a ton, and paid freight at $2.60 a ton. He sells the coal at $10.50 a short ton. His selling expenses average 30% of his total sales. His net profit is what per cent of his gross cost? (Express the result in the nearest tenth per cent.)

8. What must be the list price for an article that cost $20 in order to realize a profit of 30% of the cost and still allow a discount of 10% on the asking price? What per cent of the cost will be gained or lost if the purchaser becomes bankrupt and pays to creditors only 60¢ on the dollar?

INTEREST

416. If a man lends $1000 for 1 yr. and receives at the end of the year $1060, why is he paid $60 more than he lent?

417. The compensation paid for the use of money is called **interest.**

418. That sum of money for the use of which interest is paid is called the **principal.** In the above illustration, $1000 is the *principal* and $60 is the *interest.*

419. The amount of interest to be paid is reckoned as a certain rate per cent per annum. The most common legal rate is 6% That means that for the use of $1 for a year a man pays 6% of $1, or $.06. For the use of $1 for 2 yr. a man pays 2 times 6% of $1, or $.12.

420. The **rate of interest** is the fractional part of $1 (expressed as hundredths, or as a per cent) that is paid for the use of $1 for 1 yr.

421. The **unit of time** used as a basis in interest calculations is 1 yr. Hence, the expressions 4%, 5%, and 6% interest mean, respectively, 4%, 5%, and 6% interest for 1 *year's* use of a given principal.

422. The sum of the principal and the interest is called the **amount.**

423. The principles of percentage apply to all solutions of interest problems.

424. Since the rate of interest is a rate for 1 yr., a new element —time—has to be taken into consideration in interest computations. For the purpose of solving problems the rate is equal to the rate per cent of interest multiplied by the time expressed as years. That is, from a percentage standpoint, if the rate of interest is 6%, and the time is 2 yr. and 6 mo., the rate is 2½ times 6%, or 15%. Hence,

The **principal** is equivalent to the *base.*

The **rate of interest** multiplied by the time in years is equivalent to the *rate.*

The **interest** is equivalent to the *percentage.*

The **amount** is equivalent to the *amount.*

425. The legal rate of interest in the several states is determined by their legislatures. Valid contracts to pay more than the legal rate of interest cannot be made, except in certain cases provided for by statute.

426. Charging more than the legal rate is called **usury.** Most, if not all, the states impose a penalty for charging usury.

427. There are several kinds of interest, viz., **ordinary, accurate,*** and **compound.***

428. Ordinary interest is simple interest reckoned on a 360-day-year basis.

METHODS

429. There are a number of good methods of reckoning ordinary interest. One method is explained in this work, viz., the **banker's 60-day method**—that being the best adapted to all kinds of problems.

For a preliminary discussion of this method, see pages 32-37.

430. To find the interest for any number of days.

Find the interest at 6% on $1660 for 45 da.

$16|60 = int. for 60 da. at 6%.
4|15 = int. for 15 da. at 6%.
$12|45 = int. for 45 da. at 6%.

Interest for 60 da. is found by pointing off 2 places in the principal, which gives $16.60. 45 da. is 15 da. less than 60 da.; hence, the interest for 45 da. is ¼ less than the interest for 60 da. ¼ of $16.60 is $4.15, which, taken from $16.60, leaves $12.45.

DRILL EXERCISE

431. The student should practice until he can find the total interest in each of the following groups in four minutes or less.

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*For definition, see page 241.

432. Find the total amount of interest at 6% on:

1. $1400 for 90 da.
$1600 for 75 da.
$1560 for 80 da.
$1300 for 20 da.
$1200 for 24 da.
$ 875 for 50 da.
$ 930 for 65 da.
$ 870 for 40 da.
$1100 for 150 da.
$1370 for 180 da.
$ 920 for 36 da.
$1670 for 42 da.

2. $2900 for 75 da.
$3200 for 45 da.
$2800 for 15 da.
$2400 for 21 da.
$3300 for 25 da.
$4400 for 33 da.
$1800 for 85 da.
$1200 for 55 da.
$1000 for 72 da.
$2000 for 84 da.
$3000 for 80 da.
$4000 for 48 da.

3. $7340 for 6 da.
$8970 for 12 da.
$5980 for 3 da.
$3890 for 9 da.
$9660 for 10 da.
$4520 for 7 da.
$3360 for 5 da.
$2180 for 9 da.
$1890 for 10 da.
$3280 for 18 da.
$4170 for 2 da.
$7320 for 1 da.

4. $1172 for 40 da.
$1260 for 30 da.
$1440 for 8 da.
$1240 for 3 da.
$ 890 for 45 da.
$ 680 for 48 da.
$ 840 for 65 da.
$ 960 for 55 da.
$1020 for 70 da.
$1210 for 90 da.
$1620 for 80 da.
$ 740 for 3 da.

5. $ 720 for 14 da.
$ 840 for 16 da.
$ 960 for 18 da.
$1200 for 21 da.
$1500 for 27 da.
$1800 for 28 da.
$3600 for 36 da.
$7200 for 54 da.
$5400 for 45 da.
$6300 for 75 da.
$4500 for 90 da.
$7500 for 12 da.

6. $ 80 for 15 da.
$ 90 for 60 da.
$ 60 for 90 da.
$ 72 for 45 da.
$ 45 for 72 da.
$ 40 for 45 da.
$ 300 for 20 da.
$ 200 for 30 da.
$ 180 for 21 da.
$ 210 for 18 da.
$ 360 for 40 da.
$ 500 for 12 da.

433. To find the interest for years, months, and days.

1. Find the interest at 6% on $840.50 for 5 mo. 10 da.

$ 8	40.50	= int. for 2 mo. at 6%.
16	81.00	= int. for 4 mo. at 6%.
4	20 25	= int. for 1 mo. at 6%.
1	40 08	= int. for 10 da. at 6%.
$22	41 33	= int. for 5 mo. 10 da. at 6%.

Interest for 2 mo. is found by pointing off *two places* in the principal, 4 mo. is twice 2 mo., and the interest for 4 mo. is twice $8.405, or $16.81. Similarly, the interest for 1 mo. is one half of $8.405, or $4.2025 and the interest for 10 da. is $\frac{1}{3}$ of $4.2025, or $1.4008 On adding, $22.41 is found to be the interest for the entire time.

2. Find the interest at 6% on $1360.75 for 1 yr. 7 mo. 25 da.

$13|60.75 = int. for 2 mo. at 6%.
136|075 = int. for 20 mo. at 6%.
1|1339 = int. for 5 da. at 6%.
$134|9411 = int. for 1 yr. 7 mo. 25 da. at 6%.

1 yr. 7 mo. 25 da. = 19 mo. 25 da. = 20 mo. less 5 da. From the interest for 2 mo., it is easily found for 20 mo. to be $136.075. 20 mo. is 5 da. too much; hence, find the interest for 5 da., $1.1339, and deduct it from $136.075, leaving $134.94, the desired result.

3. Find the interest at 6% on $1500 for 2 yr. 3 mo., 23 da.

$15|00 = int. for 2 mo. at 6%.
195|00 = int. for 26 mo. at 6%.
7|50 = int. for 1 mo. at 6%.
5|00 = int. for 20 da. at 6%.
|75 = int. for 3 da. at 6%.
$208|25 = int. for 2 yr. 3 mo. 23 da. at 6%.

The time is 27 mo. 23 da. 13 times the interest for 2 mo. gives the interest for 26 mo. The interest for the remaining 1 mo. is $\frac{1}{2}$ of $15, or $7.50. 20 da. is $\frac{1}{3}$ of 60 da.; hence, the interest is $\frac{1}{3}$ of $15, or $5. For 3 da. the interest is $\frac{1}{10}$ of the interest for 1 mo., or $.75. The total is $208.25.

NOTE. Keep partial results to the fourth decimal place to insure accurate results.

434. Find the interest at 6% on the following:

PRINCIPAL	TIME	PRINCIPAL	TIME
1. $ 38.40	2 mo. 15 da.	**14.** $560	45 da.
2. $126.75	4 mo. 20 da.	**15.** $640	29 da.
3. $248.50	6 mo. 12 da.	**16.** $832	24 da.
4. $360	8 mo. 15 da.	**17.** $975	18 da.
5. $375.60	9 mo. 20 da.	**18.** $760	19 da.
6. $890.50	11 mo. 10 da.	**19.** $480	1 mo. 18 da.
7. $429.60	1 yr. 4 mo. 15 da.	**20.** $760	3 mo. 20 da.
8. $387.50	2 yr. 6 mo. 20 da.	**21.** $1840	9 da.
9. $840	90 da.	**22.** $1920	28 da.
10. $660	100 da.	**23.** $1500	42 da.
11. $930	4 mo.	**24.** $7290	48 da.
12. $740	6 mo.	**25.** $4000	54 da.
13. $425	30 da.	**26.** $3000	50 da.

	Principal	Time		Principal	Time
27.	$125	5 mo. 15 da.	**40.**	$582	1 yr. 3 mo. 5 da.
28.	$115.75	9 mo. 13 da.	**41.**	$2135	2 yr. 6 mo.
29.	$825.40	7 mo. 14 da.	**42.**	$4240	10 mo. 11 da.
30.	$1260	3 mo. 23 da.	**43.**	$1325	9 mo. 8 da.
31.	$1340	5 mo. 29 da.	**44.**	$2150	5 mo. 13 da.
32.	$1696	1 mo. 20 da.	**45.**	$6322	8 mo. 15 da.
33.	$1440	70 da.	**46.**	$4178	132 da.
34.	$5000	65 da.	**47.**	$7500	96 da.
35.	$2840	36 da.	**48.**	$8800	84 da.
36.	$1000	75 da.	**49.**	$8100	78 da.
37.	$2375	90 da.	**50.**	$5400	154 da.
38.	$1024	95 da.	**51.**	$4300	150 da.
39.	$2717	7 mo. 15 da.	**52.**	$3300	165 da.

435. To find the interest at any rate per cent.

What is the interest on $4800 for 60 da. at 6%? at 5%? at 4%? at 7%?

(*a*)	$48	00 int. at 6%	(*b*)	$48	00 int. at 6%	(*c*)	$48	00 int. at 6%
	8	00 int. at 1%		16	00 int. at 2 %		8	00 int. at 1%
	$40	00 int. at 5%		$32	00 int. at 4%		$56	00 int. at 7%

In solution (*a*), the interest at 6% is found by pointing off two places. Since 5% is 1% less than 6%, the interest at 5% is found by subtracting the interest at 1% from the interest at 6%. To find the interest at 1%, divide the interest at 6% by 6. On subtracting, the interest at 5%, $40, is found.

In solution (*b*), the interest at 2% is subtracted from the interest at 6% to find the interest at 4%. The interest at 2% is found by dividing $48 by 3. Why?

In solution (*c*), the interest at 1% is added to the interest at 6% to find the interest at 7%; and so on.

NOTE. The student should observe that in each case the interest is first found for the given time at 6%. Then the interest at 6% is increased or diminished in proportion as the required rate is greater or less than 6%.

436. The interest at rates other than 6% may be found as follows:

To find the interest at:

$6\frac{1}{2}\%$ . . . add $\frac{1}{12}$ to the interest at 6%.
$6\frac{3}{4}\%$ . . . add $\frac{1}{8}$ to the interest at 6%.
7 % . . . add $\frac{1}{6}$ to the interest at 6%.
$7\frac{1}{5}\%$ . . . add $\frac{1}{5}$ to the interest at 6%.
$7\frac{1}{2}\%$ . . . add $\frac{1}{4}$ to the interest at 6%.
8 % . . . add $\frac{1}{3}$ to the interest at 6%.
9 % . . . add $\frac{1}{2}$ to the interest at 6%.
12 % . . . multiply the interest at 6% by 2.
$5\frac{1}{2}\%$ . . . deduct $\frac{1}{12}$ from the interest at 6%.
$5\frac{1}{4}\%$ . . . deduct $\frac{1}{8}$ from the interest at 6%.
5 % . . . deduct $\frac{1}{6}$ from the interest at 6%.
$4\frac{4}{5}\%$ . . . deduct $\frac{1}{5}$ from the interest at 6%.
$4\frac{1}{2}\%$ . . . deduct $\frac{1}{4}$ from the interest at 6%.
4 % . . . deduct $\frac{1}{3}$ from the interest at 6%.

To find the interest at any other rate, divide the interest at 6% *by* 6, *to find the interest at* 1%, *and then multiply the interest at* 1% *by the given rate.*

437. ILLUSTRATIONS

1. If $16.3875 is the interest at 6%, find the interest at $5\frac{1}{2}\%$.

12) $16.3875 = int. at 6%
1.3656 = int. at $\frac{1}{2}\%$
15.0219 = int. at $5\frac{1}{2}\%$

2. If $58.63 is interest at 6%, find the interest at 5%.

6) $58.63 = int. at 6%
9.7716 = int. at 1%
48.8584 = int. at 5%

3. If the interest at 6% is $137.24, find the interest at $7\frac{1}{2}\%$.

4) $137.24 = int. at 6%
34.31 = int. at $1\frac{1}{2}\%$
171.55 = int. at $7\frac{1}{2}\%$

438. Find the interest at $6\frac{1}{2}\%$ and at 7% in Art. 434, problems 1-12 inclusive; at $7\frac{1}{2}\%$ and at $4\frac{1}{2}\%$ in problems 13-24 inclusive; at $5\frac{1}{2}\%$ and at 5% in problems 25-38 inclusive; and at 10% and 4% in problems 39-52 inclusive.

ACCURATE INTEREST

439. Accurate interest is computed for exact time in days, and on a basis of 365 da. to the year. It is used chiefly by the United States government, and by some bankers. Business men and bankers generally find the computation of accurate interest too inconvenient, though some merchants and bankers use it with the help of interest tables.*

440. Since by the "ordinary" interest methods, 360 da. are counted as a year, 360 days' interest by the ordinary method is greater than 360 days' interest by the exact interest method, by as much as $\frac{360}{360}$ is greater than $\frac{360}{365}$, or $\frac{1}{72}$. That is, for any given number of days, the ordinary interest is $\frac{1}{72}$ greater than the accurate interest at the same rate per cent. Hence, to change accurate interest to ordinary interest, add $\frac{1}{72}$ of the accurate interest to itself; and to change ordinary interest to accurate interest, deduct $\frac{1}{73}$ of the ordinary interest from itself.

441. In practice, however, accurate interest is computed as shown in the following illustrative example:

1. Find the accurate interest on $790 at 5% for 140 da.

$$\frac{\$790 \times .05 \times \overset{28}{\cancel{140}}}{\underset{73}{\cancel{365}}} = \frac{1106}{73} = \$15.15, \text{ the accurate interest.}$$

Multiplying $790 by 5% gives the interest for one year. Dividing that product by 365 gives the interest for 1 da., which multiplied by 140 gives the interest for 140 da.

442. Find the accurate interest on:

1. $750 for 148 da. at 6%.
2. $675 for 213 da. at 4%.
3. $876 for 219 da. at 5%.
4. $1314 for 58 da. at 7%.
5. $1825 for 64 da. at 6½%.
6. $1600 for 13 da. at 4½%.
7. $1500 for 18 da. at 3%.
8. $1700 for 38 da. at 5%.
9. $4380 for 51 da. at 6%.
10. $3825 for 47 da. at 5%.
11. $4564 from Aug. 11, 1924, to May 15, 1925, at 5%.
12. $3870 from June 1, 1925, to Dec. 30, 1925, at 6%.

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* Bankers and others use interest tables for both ordinary and accurate interest. The tables are too long to permit of illustration in this book.

BANK DISCOUNT

443. Bank discount is a deduction made from the amount due at maturity on notes and drafts, in consideration of their being cashed or bought before maturity.

444. If a merchant has received from one of his debtors a draft payable in 60 da., and does not wish to wait for payment till it matures, he can take it to a bank and sell it, or discount it, as it is called. The discount is determined by reckoning the simple interest on the amount of the draft from the day it is discounted to the day of maturity. The merchant will receive the **proceeds**, which is the difference between the amount due on the draft at maturity and the discount.

445. As a rule, banks charge interest, or discount, for the *exact number of days* from the day of discount to the day of maturity. This time is called the **term of discount.**

NOTES. (1) In a few states, viz., Ky., La., Md., Pa., and Va., the day of discount, as well as the day of maturity, is included in the term of discount, thus adding one day to the time.

(2) In Ariz., Calif., Ga., Ky., Miss., S. Dak., Tex., and Wis., notes, acceptances, etc., falling due on Saturday, are payable the same day. In all other states they are payable the next succeeding business day.

(3) In all states, notes, acceptances, etc., falling due on a holiday or on Sunday, are due and payable the next succeeding business day.

(4) In discounting paper falling due on Saturday, Sunday, or a legal holiday, bankers charge discount for the time till the "next succeeding business day" (except where payable on the same or the preceding day).

(5) Answers to problems in bank discount in this book are prepared in accordance with the most common method used by bankers. It is suggested that the student follow the custom of his own state.

446. Days of grace are three days allowed by law in a few states, for the payment of notes and drafts, beyond the expiration of the time specified in the paper. The practice of allowing three days of grace, which was formerly very common, has now been abolished in most states.

447. The solution of problems in bank discount is based on the principles of simple interest, or of percentage. The *value* of the note at maturity corresponds to the *principal* in *interest*, or to the *base* in *percentage;* the *rate of discount* to the *rate per cent* in *interest* and in *percentage;* the *bank discount* to the *interest* or the *percentage;* and the *proceeds* to the *difference.*

448. To find the date of maturity and the term of discount.

1. Find the date of maturity and the term of discount of a 3 months' note dated Apr. 14, and discounted May 3.

Apr. 14 + 3 mo. = July 14, date of maturity.
From May 3 to July 14 = 72 da., term of discount.

The date of maturity is found by counting forward 3 mo. from Apr. 14, which gives July 14. The term of discount is found by counting all the days from the date of discount, May 3, to the date of maturity, July 14. There are 28 da. remaining in May, 30 da. in June, and 14 da. in July, the sum of which is 72 da. (See Art. 259, page 132.)

2. Find the date of maturity and the term of discount of a 60 days' sight draft, dated Aug. 18, accepted Aug. 21, and discounted Sept. 4.

Aug. 21 + 60 da. = Oct. 20, date of maturity.
From Sept. 4 to Oct. 20 = 46 da., term of discount.

A draft drawn at 60 days' sight does not mature until 60 da. after it is accepted, hence, the date of maturity in this draft is 60 da. after Aug. 21, which is Oct. 20. Exact days must be counted when the time is expressed in days. (See Art. 265, page 135.) The term of discount is the exact number of days from Sept. 4 to Oct. 20, which is 46 da.

449. Copy the following and fill in the dates of maturity and the terms of discount:

	DATE OF PAPER	TIME	DATE OF ACCEPTANCE	DUE DATE	DATE OF DISCOUNT	TERM OF DISCOUNT
1.	Jan. 15	4 months			Mar. 10	
2.	Feb. 23	90 days			Mar. 1	
3.	June 10	3 months			July 18	
4.	May 8	90 days after sight	May 14		May 26	
5.	Oct. 7	2 months			Oct. 21	
6.	Aug. 11	30 days after sight	Sept. 1		Sept. 10	
7.	July 8	60 days			Aug. 14	
8.	May 1	45 days			May 14	
9.	Apr. 3	2 months after date	May 1		May 2	
10.	Feb. 18	90 days			Mar. 31	

450. To find the bank discount and the proceeds of a note or draft.

Find the date of maturity, the term of discount, the bank discount, and the proceeds of a 90-da. note of $1500, dated May 1 and discounted May 16, at 6%.

May 1 + 90 days = July 30, date of maturity.
From May 16 to July 30 = 75 days, term of discount.

$15|00 = discount for 60 days.
3|75 = discount for 15 days.
$18|75 = discount for 75 days.

$1500 – $18.75 = $1481.25, proceeds.

The date of maturity and the terms of discount are found as in the preceding exercise. The bank discount is the simple interest on the value of the note for the term of discount, 75 days, which is $18.75. The proceeds is the difference between the bank discount and the value of the note; $1500 – $18.75 = $1481.25.

451. Find the date of maturity, the term of discount, the bank discount, and the proceeds of the following notes and drafts:

	FACE	TIME	DATE OF PAPER	DATE OF ACCEPTANCE	DATE OF DISCOUNT	RATE OF DISCOUNT
1.	$1800.00	4 months	Aug. 1		Aug. 11	6%
2.	2000.00	3 months	July 11		Aug. 1	6%
3.	1640.00	90 days	May 13		June 4	6%
4.	1150.00	60 days after sight	Mar. 18	Mar. 24	Apr. 7	5%
5.	1990.00	30 days' sight	Apr. 27	May 3	May 6	5%
6.	2870.00	5 months	June 19		Aug. 1	6%
7.	3675.00	2 months	July 17		Aug. 1	7%
8.	9400.00	60 days' sight	Oct. 11	Oct. 18	Nov. 1	6%
9.	8478.00	30 days' sight	May 28	June 4	June 11	4½%
10.	7200.00	3 months	Aug. 13		Aug. 19	4%
11.	5900.00	1 month	Jan. 31		Feb. 4	5½%
12.	8212.00	2 months	July 1		Aug. 14	6%
13.	4750.00	4 months	Oct. 6		Nov. 6	7½%
14.	5125.00	6 months	June 30		Oct. 18	6%
15.	6000.00	90 days after date	Sept. 5	Sept. 25	Oct. 11	6%
16.	5400.00	60 days after sight	Aug. 4	Aug. 11	Aug. 23	8%
17.	4500.00	30 days' sight	July 5	July 11	July 11	7½%
18.	3100.00	5 mo. after date	Jan. 12	Mar. 11	Mar. 13	6%
19.	412.50	3 months	Apr. 12		May 1	9%
20.	368.75	90 days	May 10		June 11	10%

452. To find the proceeds of a note or draft when collection is charged.

Find the proceeds of a 90-da. note of \$3800 dated June 3, 1924, if discounted July 1, 1924, at 6%; collection $\frac{1}{10}\%$.

June 3 + 90 da. = Sept. 1, due date.

From July 1 to Sept. 1 = 62 da., term of discount.

\$38|00 = discount for 60 da.
1|27 = discount for 2 da.
\$39|27 = discount for 62 da.

$\frac{1}{10}\%$ of \$3800 = \$3.80, collection.
\$39.27 + \$3.80 = \$43.07, the bank's total charge.
\$3800 – \$43.07 = \$3756.93, the proceeds.

The collection charge is reckoned on the value of the note; $\frac{1}{10}\%$ of \$3800 is \$3.80. The bank's total charge is the sum of the bank discount, \$39.27, and the collection charge, \$3.80, or \$43.07. \$3800 – \$43.07 = \$3756.93, proceeds.

453. To find the proceeds of an interest-bearing note.

Find the proceeds of a 4 months' 6% interest-bearing note of \$3200, dated May 15, 1924, and discounted June 10, 1924 at 6%; collection $\frac{1}{8}\%$.

May 15 + 4 mo. = Sept. 15, due date.

From June 10 to Sept. 15 = 97 da., term of discount.

\$32|00 = int. for 2 mo.
64|00 = int. for 4 mo.

32|64 = dis. for 60 da.
16|32 = dis. for 30 da.
3|264 = dis. for 6 da.
|544 = dis. for 1 da.
\$52|768 = dis for 97 da.

\$3200 + \$64 = \$3264, value of the note of maturity.

$\frac{1}{8}\%$ of \$3264 = \$4.08, collection.
\$52.77 + \$4.08 = \$56.85, bank's total charge.
\$3264 – \$56.85 = \$3207.15, proceeds.

The due date and the term of discount are first found, as explained on page 243, Art. 448. Next the interest on the face of the note, \$3200, is found for the *entire time* the note has to run, 4 mo. Then the interest, \$64, is added to the face of the note, \$3200, which gives \$3264, the value of the note at maturity. The discount is then reckoned for 97 da. on the full value of the note, and amounts to \$52.77. The collection charge is reckoned on the value of the note at maturity, \$3264. $\frac{1}{8}\%$ of \$3264 equals \$4.08. The bank's total charge is equal to the sum of the bank discount and the collection, which is \$56.85. \$3264 – \$56.85 = \$3207.15, proceeds of note.

454. DISCOUNT REGISTER

No.	FOR WHOM DISCOUNTED	RATE	INDORSER	COLLATERAL	DATE	TIME
1.	A. C. Montgomery	6%			Jan. 10	4 mo.
2.	D. C. Tarbell . .	6%	A. G. Ward		Mar. 15	3 mo.
3.	M. L. Hess . . .	6%	W. C. Dowd		June 8	90 da.
4.	G. H. Dexter . .	6%		$2500 U.S. 3s	Oct. 17	60 da.
5.	S. M. Hamilton .	6%	G. O. Hall		Aug. 12	60 da.
6.	J. S. Lyman . .	5%			Oct. 11	6 mo.
7.	L. L. Holmes . .	4%			June 12	90 da.
8.	A. H. Beeman . .	4½%		$5000 N.Y.C. 4s	Apr. 20	75 da.
9.	L. S. Pierce . . .	6%	A. Sohmer		May 10	40 da.

Above and on the opposite page is shown a form of a bank discount register. Rule a similar form and fill in the missing items.

455. Find the proceeds of the following notes:

	FACE	TIME	DATE	INTEREST	DATE OF DISCOUNT	RATE OF DISCOUNT	COLLECTION
1.	$1400	2 mo.	Aug. 1	6%	Aug. 10	6%	⅛%
2.	1550	3 mo.	May 15	5%	June 1	5%	
3.	2160	90 da.	Feb. 15	8%	Mar. 17	7%	⅛%
4.	560	60 da.	Dec. 10	6%	Dec. 20	8%	¼%
5.	940	5 mo.	July 1	6%	Sept. 10	5%	¼%
6.	1120	4 mo.	Oct. 20	5%	Dec. 1.	6%	
7.	4382.75	90 da.	Nov. 17		Nov. 29	6%	$\frac{1}{10}$%
8.	526.80	2 mo.	Dec. 8		Dec. 13	5%	¼%
9.	2600	3 mo.	Mar. 8		Mar. 13	6%	
10.	557.68	2 mo.	Dec. 4		Dec. 13	5%	¼%
11.	943.75	4 mo.	Oct. 1		Oct. 2	6%	
12.	321.60	30 da.	Jan. 31		Feb. 4	7½%	$\frac{1}{10}$%
13.	416.50	60 da.	Mar. 31		Apr. 13	4½%	¼%
14.	531.60	3 mo.	Nov. 30		Jan. 3	5%	¼%

DISCOUNT REGISTER

FACE	RATE OF INT.	INT.	AMOUNT	DUE	DATE OF DIS.	DA.	DIS.	RATE OF EXCH.	EXCH.	PROCEEDS
$5000					March 8			$\frac{1}{20}$%		
$4500					Apr. 13			$\frac{1}{10}$%		
$2500					July 1			$\frac{1}{8}$%		
$1600					Nov. 10			$\frac{1}{4}$%		
$1748					Sept. 1			—		
$1372					Nov. 4			$\frac{1}{4}$%		
$2875	6 %				June 27			$\frac{1}{10}$%		
$3125	7½%				May 5			$\frac{1}{8}$%		
$4700	8 %				May 14			—		

15.

$4728.40 ROCHESTER, N. Y., June 3, 19

Two months after date we promise to pay to the order of ------------

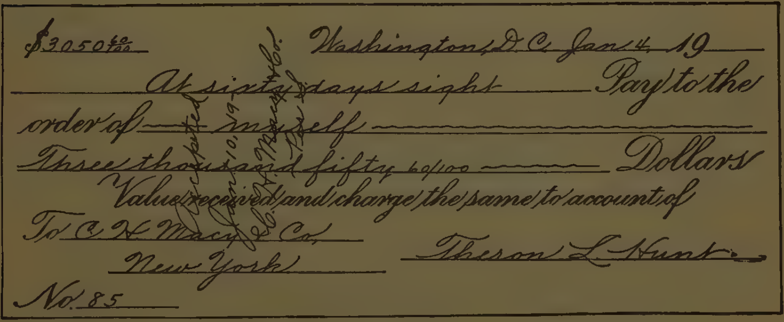
------------------------C. J. HERMANS & Co.--------------------

Forty-seven hundred twenty-eight--------------------- $\frac{40}{100}$ Dollars, with interest at 6%. Value received.

F. F. JONES & Co.

Discounted June 8 at 5%; collection $\frac{1}{8}$%.

16.



$3050 $\frac{60}{100}$ Washington, D. C. Jan. 4, 19

At sixty days sight Pay to the

order of myself

Three thousand fifty 60/100 Dollars

Value received and charge the same to account of

To C. H. Macy & Co.

New York

Theron L. Hunt.

No. 85

Accepted Jan. 10, 19

C. H. Macy & Co.

Per B.

Discounted Jan. 13 at 6%; collection $\frac{1}{8}$%.

PROBLEMS IN INTEREST

456. **1.** In what time will any sum of money double itself at 5%? at 6%? at 8%? at 9%?

2. At what rate per cent of interest will any sum of money double itself in 25 yr.? in 14$\frac{2}{7}$ yr.? in 10 yr.? in 8 yr. 4 mo.?

3. What is the difference between the ordinary interest and the accurate interest at 6% on $10,000 for 144 da.

4. A merchant's cash price of an article is $35; his price at 4 months' credit is $37.50. If money is worth 6% per annum, how much has he added to cover cost of collection and bad debts?

5. A banker pays 3% interest on a deposit of $45,000. He loans it 3 mo. at 6%; 4 mo. at 5$\frac{1}{2}$%; and for 48 da. at 7%. If it lies idle the remainder of the year, does he gain or lose, and how much?

6. Aug. 1, 1924, a wholesaler sold a bill of goods amounting to $1875.28, terms net 30 da. If the bill was not paid until Jan. 8, 1925, what amount was due, interest being 6%? (Exact time.)

7. On July 8, 1924, a man borrowed money at 6% interest and bought 3000 bu. of wheat at $1.08 a bushel. He sold the wheat Aug. 17, 1924, at $1.21 a bushel and returned with interest the money he borrowed. What was his gain?

8. A piece of property valued at $8000 is sold on the following terms: 20% of the selling price is to be paid in cash; for the remainder five notes falling due in one, two, three, four, and five years, respectively, with interest at 6%, are given. If each note is paid at maturity, what is the total amount paid for the property?

9. What amount can a man afford to pay for a house and lot that rents for $65 a month so that his investment shall net him 6% interest, if he pays for taxes, insurance, repairs, and all other expenses, $360 a year?

10. A young man had $1600 saved. He purchased a house and lot for $4000, borrowing $2400 at a saving bank, giving as security a mortgage on the house and lot. The house rented for $37.50 a month. He paid 5$\frac{1}{2}$% interest on the mortgage; $\frac{1}{8}$% premium on an insurance policy of $3000; 1.8% tax on a valuation of $3500; $14 water rent; and $50 for repairs. What rate of interest did he receive on his investment?

11. A real estate broker offers me a house and lot for $7500, guaranteeing an 8% investment. The property is assessed for $5000 on which a tax of $1\frac{3}{4}$% has to be paid. Insurance and other expenses will amount to $212.50 a year. At what price must the property rent per month to make good the broker's guarantee?

12. A merchant bought a bill of goods amounting to $13,875; terms 3 mo., 5% cash. He accepted the cash terms. How much did he save if money was worth 6%?

13. An invoice of merchandise was bought on the following terms: 4 mo. or 5% cash. The cash discount is equivalent to what rate of interest per annum on the gross amount? on the net amount?

14. On an invoice of $4378.60, a merchant is offered 4 mo. credit or a discount of 5% for cash. Not having the ready money, he accepts the credit terms. What rate per cent of interest does he pay on the net amount of the bill? How much would he have saved if he had borrowed the money at 6% and paid cash?

15. Lane and Son bought a bill of merchandise valued at $9542.50, terms 3/10, n/60. The bill was dated May 13. Not having ready cash to meet the bill May 23, they borrowed at their bank on a demand note with interest at 6%, a sum sufficient to pay the bill. Assuming that they paid their note at the bank the following July 12, find how much they gained by early payment.

16. Halsey and Hanson received an invoice of merchandise dated July 5, terms 3/10, n/90. The value of the invoice was $16,932.75 On July 15, they had only $9000 available cash to meet the bill. To raise the rest of the money they wrote their note at 80 days from date, for a sum which, when discounted at their bank, would yield sufficient proceeds to supplement the $9000 to meet the invoice on that day. Find the gain by early payment, assuming that money was worth 6% to the firm.

17. A man on June 7 gave a demand note of $4750 to a bank for a loan of that amount. On July 12, he paid the note with interest by writing a check for $4773.09. What rate of interest did the bank charge him?

TAXES

457. Who pays the salary of the governor, the legislators, the mayor, the policemen. and the teachers, in your state and city, or town?

How is the money raised for such purposes?

Ask the assessors of your town to explain to you the method of raising money for the above-named purposes.

458. Money charged against persons and their property for the payment of public expenses is called a **tax.**

459. Taxes are of two kinds—direct and indirect.

A **direct tax** is a tax levied on a person, his property, or his business.

460. A tax on a person is called a **poll tax.**

461. A tax on property, real or personal, is called a **property tax.**

462. A tax on a person's business is called a **license fee.**

463. An **indirect tax** is a tax (called a duty) on imported goods or a tax (called excise or internal revenue) on the manufacture of tobacco products. The tax need not be paid on tobacco that is exported.

HOW TAXES ARE APPORTIONED

464. Suppose that in the state of X there are 4 counties, whose property values are as follows:

County A	$15,000,000
County B	25,000,000
County C	27,000,000
County D	33,000,000
Total	$100,000,000

Suppose, also, that County A is divided into 3 towns, whose values are:

Town E	$5,000,000
Town F	4,000,000
Town G	6,000,000
Total	$15,000,000

The amount to be raised in the state for state expenses, exclusive of the revenue received from licenses, permits, etc., is, say, $100,000. This amount is apportioned among the several counties in the ratio of their valuation. The valuation of County A is 15% of that of the entire state; hence, County A must raise 15% of $100,000, or $15,000, for its share of the state tax.

The county tax is $45,000, to which the state tax of $15,000 is added, making a total of $60,000 to be raised in County A for state and county expenses. This $60,000 is apportioned among the towns in the ratio of their valuation. The value of Town E is $\frac{1}{3}$ that of County A; hence, Town E is charged with $\frac{1}{3}$ of $60,000, or $20,000, its share of state and county taxes. To this amount is added the town tax of $70,000, which makes $90,000 to be raised in the town of E. To find the *rate of tax*, $90,000 is divided by the valuation of the town, $5,000,000, which gives .018 = $1\frac{4}{5}$%. Each person owning taxable property in the town is then charged with a tax equal to $1\frac{4}{5}$% of the valuation placed on his property.

465. The persons appointed to make an estimate of the value of each person's property, and to apportion the taxes in proportion to the value of each person's property, are called **assessors.**

466. The assessors make what is called an **assessment roll.** It is a list showing the names of property owners, a brief description of the property owned, its assessed valuation, and the tax thereon.

467. The person who collects the tax from the property owners is called the **collector.**

468. Solutions of problems in taxes depend upon the principles of percentage.

The **assessed valuation** is the *base.*

The **tax rate** is the *rate.*

The **tax** is the *percentage.*

469. The tax rate is frequently expressed as $1.80 per $100, instead of 1.8%.

EXERCISES

470. 1. A tax of $.003 on a dollar is how how much on $100? on $1000?

2. What is the tax on $4500 at the rate of $.65 per $100?

3. At $8.50 per $1000, what is the tax on $7800?

4. Find the tax on $15,400 at 1.365%.

Find the tax on the following:

5. $13,500 at .0065.
6. $34,500 at .01467.
7. $44,800 at .00895.
8. $18,500 at .0102.
9. $125,000 at .00882.
10. $135,500 at .004.
11. $500,000 at .0115.
12. $240,000 at $1.64 per $100.
13. $15,900 at $1.845 per $100.
14. $32,600 at $13.26 per $1000.
15. $16,400 at $11.15 per $1000.
16. $69,500 at $.9875 per $100.
17. $51,600 at $1.065 per $100.
18. $12,600 at $12.853 per $1000.

Oftentimes the collector is allowed a commission on all taxes collected, instead of a salary.

471. 1. Find the total tax paid by a man whose property is assessed for $13,800, at 1.25%, the collector's commission being 1%.

1.25% of $13,800 = $172.50, tax.
1% of $172.50 = 1.73, collector's commission.
Total tax, $174.23.

The tax is 1¼% of $13,800, or $172.50. The collector's commission of 1% is reckoned on the amount of tax, and then added to the tax, and charged against the property owner.

2. The town of Wellsville has an assessed valuation of $2,118,000. The total amount of tax to be collected is $18,917. There are 910 polls at $1 each, and $2200 highway tax not to be charged on the valuation. Find the rate of taxation and the amount of tax paid by a farmer whose property is assessed for $6400, who pays for 1 poll, and $3 highway tax.

$910 + $2200 = $3110, to be deducted from total tax.
$18,917 − $3110 = $15,807, tax levied on property.
$15,807 ÷ $2,118,000 = $.007463 = 7.463 mills on $1.
$6400 × .007463 = $47.76, property tax.
$47.76 + $1 + $3 = $51.76, total tax.

To find the amount of the tax to be levied on the property of the town, deduct the sum of the poll tax and the highway tax, which leaves $15,807. The tax on the property divided by the value of the property, gives the rate, which is 7.463 mills to the dollar. The property tax on a farm valued at $6400 is found by multiplying $6400 by .007463, which gives $47.76, to which is added $1, poll tax, and $3, highway tax, making a total tax of $51.76.

PROBLEMS

472. 1. The assessed valuation of real estate, and the tax rate in the several boroughs of New York City for 1923 were as follows:

Borough	Valuation	Tax Rate	Amount of Tax
Manhattan	$6,177,890,668	.0274	
Bronx	926,682,418	.0274	
Brooklyn	2,536,590,061	.0274	
Queens	804,004,439	.0274	
Richmond	150,897,987	.0276	

Find the amount of tax for each borough.

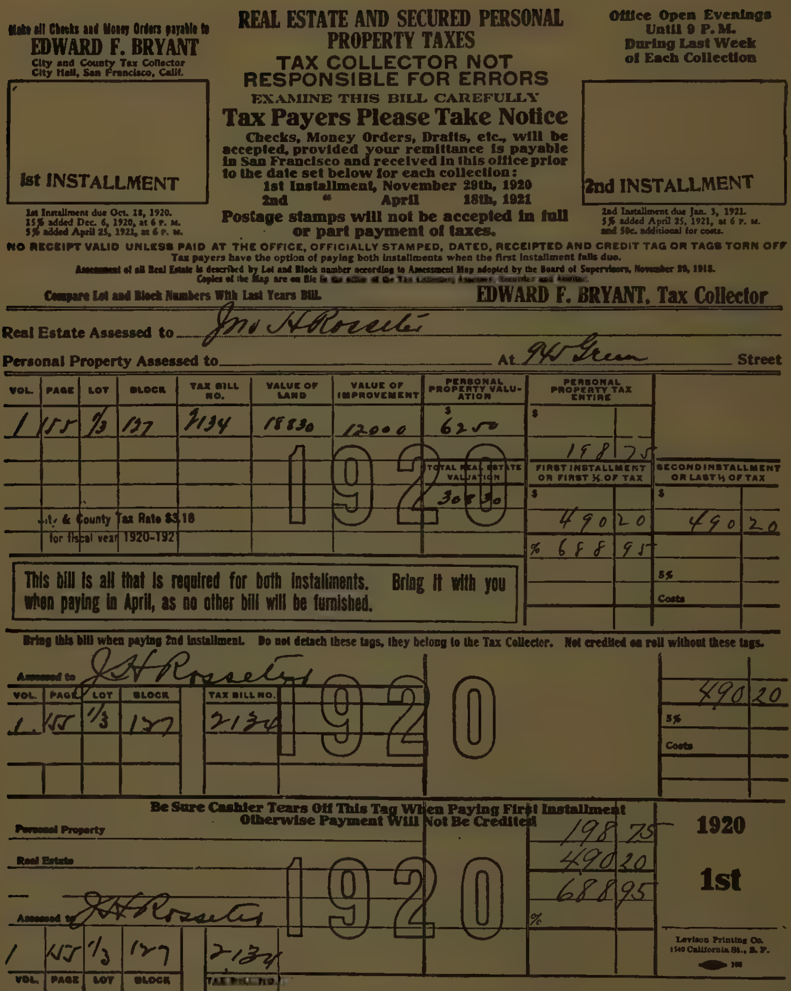
2. Find the per cent. of the total valuation in each borough.

3. The assessed valuation of the real and personal property in a certain town is $1,040,000. The total amount of tax to be collected is $7136. There are 206 polls at $1 each, and $1600, highway tax. Find the rate of taxation, and the amount of tax paid by a man whose property is assessed at $10,500, and who pays for 1 poll and a highway tax of $8.

In St. Louis, Mo., the tax rate for 1920 was, for state purposes, 18¢; for schools, 78¢; and for city purposes, $1.59 per $100. Taxes are due Sept. 1. Taxes become delinquent after Dec. 31. The penalty on all delinquent taxes is 1% a month from date of delinquency till paid, provided the taxes are paid before the Saturday preceding the first Monday in March. After said date all costs of collection are added.

Under the above conditions find the state, school, and city tax for the year 1920 on each of the following properties, if taxes were paid on the dates given:

	Description of Property	Valuation	Date Tax was Paid
4.	4-story apartment house	$25,000	Sept. 25, 1920
5.	Lot 75′ × 100′	5,500	Feb. 18, 1921
6.	12-story office building	1,200,000	Sept. 29, 1920
7.	2-story frame dwelling	5,000	Dec. 30, 1920



Make all Checks and Money Orders payable to
EDWARD F. BRYANT
City and County Tax Collector
City Hall, San Francisco, Calif.

REAL ESTATE AND SECURED PERSONAL PROPERTY TAXES

TAX COLLECTOR NOT RESPONSIBLE FOR ERRORS

EXAMINE THIS BILL CAREFULLY

Tax Payers Please Take Notice

Checks, Money Orders, Drafts, etc., will be accepted, provided your remittance is payable in San Francisco and received in this office prior to the date set below for each collection:
1st Installment, November 29th, 1920
2nd " April 18th, 1921

Postage stamps will not be accepted in full or part payment of taxes.

Office Open Evenings
Until 9 P. M.
During Last Week
of Each Collection

1st INSTALLMENT

1st Installment due Oct. 18, 1920.
15% added Dec. 6, 1920, at 6 P. M.
5% added April 25, 1921, at 6 P. M.

2nd INSTALLMENT

2nd Installment due Jan. 3, 1921.
5% added April 25, 1921, at 6 P. M.
and 50c. additional for costs.

NO RECEIPT VALID UNLESS PAID AT THE OFFICE, OFFICIALLY STAMPED, DATED, RECEIPTED AND CREDIT TAG OR TAGS TORN OFF

Tax payers have the option of paying both installments when the first installment falls due.

Assessment of all Real Estate is described by Lot and Block number according to Assessment Map adopted by the Board of Supervisors, November 28, 1918.
Copies of the Map are on file in the office of the Tax Collector, Assessor, Recorder and Auditor.

Compare Lot and Block Numbers With Last Years Bill.

EDWARD F. BRYANT, Tax Collector

Real Estate Assessed to Jno H Rossetti

Personal Property Assessed to ______ At 94 Green Street

VOL.	PAGE	LOT	BLOCK	TAX BILL NO.	VALUE OF LAND	VALUE OF IMPROVEMENT	PERSONAL PROPERTY VALUATION	PERSONAL PROPERTY TAX ENTIRE
1	155	1/3	127	2134	18830	12000	$ 6250	$ 198 75

1920

City & County Tax Rate $3.18
for fiscal year 1920-192

TOTAL REAL ESTATE VALUATION 30830

FIRST INSTALLMENT OR FIRST ½ OF TAX	SECOND INSTALLMENT OR LAST ½ OF TAX
$ 490 20	$ 490 20
% 688 95	
	5%
	Costs

This bill is all that is required for both installments. Bring it with you when paying in April, as no other bill will be furnished.

Bring this bill when paying 2nd installment. Do not detach these tags, they belong to the Tax Collector. Not credited on roll without these tags.

Assessed to J H Rossetti

VOL.	PAGE	LOT	BLOCK	TAX BILL NO.
1	155	1/3	127	2134

1920

490 20
5%
Costs

Be Sure Cashier Tears Off This Tag When Paying First Installment
Otherwise Payment Will Not Be Credited

Personal Property 198 75
Real Estate 490 20
688 95
%

1920

1st

Assessed to J H Rossetti

VOL.	PAGE	LOT	BLOCK	TAX BILL NO.
1	155	1/3	127	2134

Levison Printing Co.
1540 California St., S. F.

Above is a form of tax bill used in San Francisco, Cal. Note that all the personal property tax is due and payable at the same time the first half of the real estate tax is due.

Using the information given in the tax bill on this page, find the tax paid in San Francisco on the following properties:

	Real Estate Valuation	Personal Property Valuation	First Installment Paid	Second Installment Paid
8.	$1960		Nov. 20, 1920	April 10, 1921
9.	22500	5500	April 12, 1921	April 12, 1921
10.	13600	7700	Dec. 15, 1920	May 18, 1921
11.	5500	2300	June 1, 1921	June 1, 1921
12.	3200		Dec. 1, 1920	April 25, 1921

The following table shows how the tax rate is apportioned in Chicago, Ill.:

TAX RATES FOR 1921

	Total Rate	State	County	Forest Preserve	Sanitary	City	School	Town	Park	Lincoln Park Bond
West	5.57	.40	.52	.06	.17	2.29	1.62		.51	
South	5.39	.40	.52	.06	.17	2.29	1.62		.33	
North . . .	5.89	.40	.52	.06	.17	2.29	1.62	.06	.71	.06
Hyde Park . .	5.39	.40	.52	.06	.17	2.29	1.62		.33	
Lake	5.39	.40	.52	.06	.17	2.29	1.62		.33	
Lake View . .	5.89	.40	.52	.06	.17	2.29	1.62	.03	.74	.06
Rogers Pk. . .	5.06	.40	.52	.06	.17	2.29	1.62			
Jefferson . . .	5.06	.40	.52	.06	.17	2.29	1.62			

The rates in the above table apply to the $100 assessed valuation, which is one third of the full value.

Taxes for 1921 are collectible in 1922.

On general taxes a penalty of 1% is charged after May 1, 2% after June 1, 3% after July 1, and so on, until paid. In addition to this, advertising and copying costs of 19¢ for each lot or part of a lot, and 29¢ for each tract of land, are added to unpaid general taxes and special assessments, early in May.

Using the rates and information given above, find the tax for each purpose and the total tax on the following properties:

	Description of Property	Assessed Valuation	Town	Date of Payment
13.	Office building	$150,000	North Town	April 20, 1922
14.	Three-story brown stone dwelling	90,000	Hyde Park	May 20, 1922
15.	Vacant lot 100′ x 100′ . .	15,000	Jefferson	July 30, 1922
16.	A tract of land containing 17 acres	17,000	Lake	June 25, 1922

In Denver, Colo., taxes are paid in two payments of 50% each. Taxes are due Jan. 1 of each year. The first half is delinquent and draws interest at 1% a month on and after March 1. The second half is delinquent and draws interest at 1¼% a month on and after Aug. 1.

If the state tax is 21 ¢, and the city tax is $1.38 per $100, find the total tax paid on the following:

17. Store valued at $45,000, both installments paid July 20.

18. Dwelling valued at $12,500, first installment paid Feb. 27; second installment paid Dec. 1.

19. Hotel valued at $500,000, first installment paid July 20; second installment paid Oct. 10.

FIRE INSURANCE

473. Insurance is an agreement by one party, for a consideration, to indemnify another party for losses arising from certain stipulated causes.

474. Fire insurance treats of agreements to indemnify the insured for loss by fire.

475. "Loss by fire" is held to cover not only loss of property actually burned, but also loss or damage resulting from the use of chemicals or water used in extinguishing the fire, and from smoke. Fire caused by lightning comes within the meaning of the term "loss by fire."

476. The fire insurance policies of all companies in Connecticut, New Jersey, New York, and Pennsylvania, are uniform, and contain the "New York Standard (80%) Average Clause," which reads in part as follows: "This Company shall not be liable for a greater proportion of any loss or damage to the property described herein than the sum hereby insured bears to eighty per centum (80%) of the actual cash value of said property at the time such loss shall happen." These policies also contain a "Waiver Clause," which reads: "In case of loss if the value of the property described herein does not exceed $2500, the 80% average or co-insurance clause shall be waived."

In other states there is the ordinary policy, which stipulates the amount of loss for which the insurance company is liable. Under the ordinary policy the company pays any loss not exceeding the face of the policy.

477. In some states the policy contains an average clause which states that only such a part of the loss will be paid as the face of the policy bears to the value of the property insured.

478. If more than one company insures the same property, each company pays only its pro rata share of any loss.

479. In the **valued policy,** the value is stipulated and the company agrees to pay on that specified amount. The **open policy** is used to cover goods in storage and elsewhere, the amount of the policy varying as the quantity of goods is increased or diminished by additions or withdrawals. The addition and withdrawal of goods are recorded in an "Open Policy Book" retained by the company. Upon each receipt of goods, their nature and value are recorded in the policy book, and the premium charged is based on the annual rate.

In case of withdrawal in less than 1 yr. the company returns the unearned premium.

480. A policy may be canceled at any time, either by the insurer or by the insured, on 5 days' notice. If the insurance company cancels the policy, it will return to the insured such a part of the premium as the unexpired time of the policy is of the entire term of the policy. If, however, the insured cancels the policy, the company will return to him only the amount by which the premium paid is in excess of the premium reckoned at the short rate. The short rate is a correspondingly higher rate for a short period of time.

Insurance companies frequently issue a policy for 3 yr. for a premium equal to two and one half times the annual premium, and for 5 yr. for four annual premiums.

481. The principles of percentage apply in insurance.

The **value of the policy** is the *base.*

The **rate of premium** is the *rate.*

The **premium** is the *percentage.*

482. To find the premium.

If the property is insured for \$18,000 at 20¢ per \$100 per annum, what is the annual premium?

\$18,000 = 180 hundreds of dollars.
180 × \$.20 = \$36, the annual premium.

Since the premium rate is on \$100, first find the number of hundreds of dollars by pointing off 2 places in the \$18,000, which gives 180 hundreds dollars. If the premium on \$100 is \$.20, on 180 hundreds it is 180 times \$.20, or \$36.

PROBLEMS

483. Find the premium on each of the following policies:

	Face of Policy	Rate of Insurance		Face of Policy	Rate of Insurance
1.	\$13,200	22¢ per \$100	**8.**	24,600	$1\frac{1}{8}\%$, less 10%
2.	9,600	$37\frac{1}{2}$¢ per \$100	**9.**	55,000	$\frac{7}{8}\%$, less 10% and 5%
3.	16,400	45¢ per \$100	**10.**	48,000	55¢ per \$100, less 10%
4.	27,500	65¢ per \$100	**11.**	20,000	$\frac{3}{8}\%$
5.	8,500	$\frac{1}{4}\%$	**12.**	24,500	$27\frac{1}{2}$¢ per \$100
6.	11,500	$\frac{5}{8}\%$, less 10%	**13.**	38,000	$\frac{4}{5}\%$, less 20%
7.	17,800	$\frac{3}{4}\%$, less 15%	**14.**	75,000	44¢ per \$100, less 10%

484. To find the amount paid by the insurer.

1. Property valued at \$25,000 is insured for \$15,000 at $\frac{3}{8}\%$ per annum. Fire and water cause a loss of \$12,000. What amount would be paid by the insurance company (*a*) under an ordinary policy? (*b*) under a co-insurance clause policy? (*c*) under the New York Standard (80%) Average Clause Policy?

(*a*) \$12,000, amount paid under an ordinary policy.

(*b*) $\frac{15000}{25000} = \frac{3}{5}$.

$\frac{3}{5}$ of \$12,000 = \$7200, amount paid under a co-insurance policy.

(*c*) 80% of \$25,000 = \$20,000.
$\frac{15000}{20000} = \frac{3}{4}$.
$\frac{3}{4}$ of \$12,000 = \$9000, amount paid under New York policy

(*a*) Under an ordinary policy the company pays the full amount of loss up to the amount of the policy; hence, \$12,000 would be paid in this case.

(*b*) Under a co-insurance, or average clause policy, the part of the loss paid is the ratio of the face of the policy, \$15,000, to the value of the property \$25,000, or $\frac{15000}{25000}$, which is equal to $\frac{3}{5}$. Hence in this case the company would pay $\frac{3}{5}$ of \$12,000, or \$7200.

(*c*) Under the 80% average clause policy, the company pays such a part of the loss as the ratio of the face of the policy, \$15,000, is to 80% of the value of the property, 80% of \$25,000 = \$20,000. The ratio of \$15,000 to \$20,000 is $\frac{15000}{20000} = \frac{3}{4}$. Hence the company would pay $\frac{3}{4}$ of \$12,000, or \$9000.

2. A stock of merchandise is insured in Company A for \$5000, in Company B for \$7000, and in Company C for \$8000. In case of damage amounting to \$10,000, how much will each company pay?

\$5000 + \$7000 + \$8000 = \$20,000, total amount of insurance.
$\frac{5000}{20000} = \frac{1}{4}$; $\frac{1}{4}$ of \$10,000 = \$2500, paid by Company A.
$\frac{7000}{20000} = \frac{7}{20}$, $\frac{7}{20}$ of \$10,000 = \$3500, paid by Company B.
$\frac{8000}{20000} = \frac{2}{5}$; $\frac{2}{5}$ of \$10,000 = \$4000, paid by Company C.

The total insurance is \$20,000. Each company pays such part of the loss as its risk is of the total risk. Hence, Company A pays $\frac{1}{4}$ of the loss, or \$2500; Company B pays $\frac{7}{20}$ of the loss, or \$3500; and Company C pays $\frac{2}{5}$ of the loss, or \$4000.

PROBLEMS

485. 1. A dwelling valued at \$9600 was insured for $\frac{2}{3}$ of its value at $\frac{3}{4}$% and contents valued at \$3200, for $\frac{3}{4}$ of its value at $\frac{4}{5}$%. Fire causes a total loss of the building and a loss of \$1400 on the contents. Under an ordinary policy, how much will the insurance company pay?

2. Property insured under an ordinary policy for \$15,000 at 30¢ per \$100 per annum is damaged by fire to the amount of \$8000. What is the insurance company's net loss if they have held the insurance 15 yr., simple interest 5%.

3. A store and its contents were insured in Company X for \$22,500 at 60¢ per \$100; in Company Y for \$30,000 at $\frac{5}{8}$%; and in Company Z for \$40,000 at 65¢ per \$100. The property was damaged by fire and water to the amount of $40,000. What was each company's net loss, they having held the insurance 7 yr., if money is worth 5%? (Use simple interest.)

4. A man has his furniture insured for $\frac{2}{3}$ of its value, in a policy containing an average clause. In case of a loss of $2400, how much should the company pay?

5. A house valued at $15,000, and contents valued at $7500, are insured for $\frac{2}{3}$ of their value. Fire causes a loss of $4000 on the house and $1500 on the contents. Find the amount payable under an 80% average policy.

486. The following table shows the short rates adopted by the "Western Union":

Standard Short Rate Scale for computing Premiums for Terms less than One Year, as adopted by the "Western Union"

RULE. Take the percentage indicated in scale opposite the number of days risk is to run, on the premium for one year at given rate, and the result will be the premium earned in case of cancellation, or to be charged in case of short risks.

1 Day	2 per cent annual prem.	55 Days	29 per ct. an'l prem.
2 Days	4 per cent annual prem.	60 Days	30 per ct. an'l prem.
3 Days	5 per cent annual prem.	65 Days	33 per ct. an'l prem.
4 Days	6 per cent annual prem.	70 Days	36 per ct. an'l prem.
5 Days	7 per cent annual prem.	75 Days	37 per ct. an'l prem.
6 Days	8 per cent annual prem.	80 Days	38 per ct. an'l prem.
7 Days	9 per cent annual prem.	85 Days	39 per ct. an'l prem.
8 Days	9 per cent annual prem.	90 Days or 3 mo.	40 per ct. an'l prem.
9 Days	10 per cent annual prem.	105 Days	45 per ct. an'l prem.
10 Days	10 per cent annual prem.	120 Days or 4 mo.	50 per ct. an'l prem.
11 Days	11 per cent annual prem.	135 Days	55 per ct. an'l prem.
12 Days	12 per cent annual prem.	150 Days or 5 mo.	60 per ct. an'l prem.
13 Days	13 per cent annual prem.	165 Days	65 per ct. an'l prem.
14 Days	13 per cent annual prem.	180 Days or 6 mo.	70 per ct. an'l prem.
15 Days	14 per cent annual prem.	195 Days	73 per ct. an'l prem.
16 Days	14 per cent annual prem.	210 Days or 7 mo.	75 per ct. an'l prem.
17 Days	15 per cent annual prem.	225 Days	78 per ct. an'l prem.
18 Days	16 per cent annual prem.	240 Days or 8 mo.	80 per ct. an'l prem.
19 Days	16 per cent annual prem.	255 Days	83 per ct. an'l prem.
20 Days	17 per cent annual prem.	270 Days or 9 mo.	85 per ct. an'l prem.
25 Days	19 per cent annual prem.	285 Days	88 per ct. an'l prem.
30 Days	20 per cent annual prem.	300 Days or 10 mo.	90 per ct. an'l prem.
35 Days	23 per cent annual prem.	315 Days	93 per ct. an'l prem.
40 Days	26 per cent annual prem.	330 Days or 11 mo.	95 per ct. an'l prem.
45 Days	27 per cent annual prem.	360 Days or 12 mo.	100 per ct. an'l prem.
50 Days	28 per cent annual prem.		

487. To find the cost of insurance at short rates, or the amount of premium to be returned if policy is canceled.

1. Find the cost of insuring a stock of merchandise for $15,000 for 3 mo. if the annual rate is $\frac{7}{8}$%.

$\frac{7}{8}$% of $15,000 = $131.25, annual premium.
40% of $131.25 = $52.50, premium for 3 mo.

By the short rate scale, page 260, the premium for 3 mo. is 40% of the annual premium. The annual premium is $131.25; hence, the premium for 3 mo. is 40% of $131.25, or $52.50.

2. Property valued at $24,000 is insured for $\frac{3}{4}$ of its value for 1 yr. at 60¢ per $100. How much of the premium should be returned if the policy is canceled at the expiration of 8 mo. (*a*) by the insured? (*b*) by the insurance company?

(*a*) $\frac{3}{4}$ of $24,000 = $18,000.
180 × $.60 = $108, annual premium.
20% of $108 = $21.60, amount to be returned if insured cancels the policy.

(*b*) 4 mo.: 12 mo.:: $? : $108.

$$\frac{4 \times \$108}{12} = \$36,$$ amount to be returned if insurer cancels the policy.

(*a*) By the short rate scale the rate for 8 mo. is 80% of the annual rate. That is, the company will return 20% of the premium paid. The annual premium is $108; hence, the amount returned is 20% of $108, or $21.60.

(*b*) When the insurer cancels the policy, the amount returned bears the same ratio to the entire premium paid as the unexpired time bears to the entire time; hence, the proportion 4 mo. : 12 mo.:: $?: $108, the solution of which gives $36, the amount to be returned.

PROBLEMS

488. 1. A policy of insurance for $10,000 at $\frac{3}{4}$% per annum was dated Apr. 7, 1924. 5 mo. later it was canceled by the insured. How much of the premium was returned?

2. May 10, 1924, I took out a policy of insurance on my furniture for $2400 at 40¢ per $100 per annum. Feb. 19, 1925, I canceled the policy. How much of the premium should be returned to me?

3. A stock of merchandise was insured for 1 yr. for $25,000 at ⅞%. At the end of 7 mo. the insurance company canceled the policy. How much of the premium was returned? (Solve by proportion.)

4. A merchant insured his store and contents for 1 yr. for $16,500 at ⅗%. At the end of 5 mo. and 15 da. the insurer canceled the policy. What was the return premium? (Solve by proportion).

5. A policy of $32,000 at ⅝% for 1 yr. was canceled by the insured at the expiration of 8 mo. Find the amount of the return premium. If the company had canceled the policy, what amount of premium would have been returned?

6. The Aetna Insurance Company insured an office building for $45,000 at ½% for 1 yr. Find the return premium if the policy was canceled at the end of 6 mo. (*a*) by the company; (*b*) by the insured.

7. Find the cost of insuring a stock of goods for $12,000 for 6 mo. at 75¢ per $100 per annum.

REVIEW PROBLEMS

489. 1. Find the cost of insuring a house for $8000 for 3 yr. under a 3-year policy if the annual premium is ¼%.

2. At 60¢ per $100 per annum, what is the cost of insuring a shipment of merchandise for $6500 for 30 da.?

3. Property worth $75,000 is insured as follows: In the Hartford Insurance Company for $20,000 at 1¼%, and in the Albany Insurance Company for $30,000 at 1⅛%. There is a $30,000 loss by fire during the first term of insurance. Find each company's net loss under (*a*) an ordinary policy; (*b*) an average clause policy.

Find the amount to be paid in each of the following cases, supposing each policy contains the Standard 80% Average Clause:

	VALUE OF PROPERTY	FACE OF POLICY	DAMAGE BY FIRE	PAID BY INSURANCE CO.
4.	$20,000	$12,000	$ 8,000	?
5.	28,000	15,000	15,000	?
6.	12,000	5,000	12,000	?
7.	16,500	10,000	3,000	?
8.	45,000	20,000	25,000	?

9. An open policy of insurance was issued on merchandise stored, or to be stored, in a warehouse, the premium to be 80¢ per $100 per annum. Goods withdrawn inside of 1 yr. were to be charged the short rate. Find the total premium paid, and the total return premium on the following receipts and withdrawals:

Receipts			Withdrawals		
Date	Amount		Date	Amount	
Apr 1, 1924	Silk,	$ 8,400	Oct. 1, 1924	Woolens,	$5000
Apr. 15, 1924	Furs,	12,800	Oct. 10, 1924	Furs,	9000
May 10, 1924	Woolens,	11,500	Oct. 20, 1924	Silk,	8400
June 1, 1924	Gloves,	6,500	Nov. 10, 1924	Gloves,	4500
June 20, 1924	Hosiery,	5,600	Dec. 1, 1924	Woolens,	6500

POSTAL SAVINGS BANKS

490. The **postal savings** system was established in the United States January 3, 1911, at 48 second-class post offices, one in each state and territory. By Jan. 31, 1913, there were 12,823 offices where patrons of the system might deposit their funds.

491. Deposits are evidenced by postal-savings certificates issued in fixed denominations of $1, $2, $5, $10, $50, $100, $200, and $500, each bearing the name of the depositor, the number of his account, the date of issue, the name of the depository office, and the date on which interest begins.

492. Interest is allowed on all deposits at the rate of 2 per cent per annum, computed on each savings certificate separately, and payable annually. No interest is paid on money which remains on deposit for a fraction of a year only.

493. Deposits bear interest from the first day of the month next following that in which deposited.

494. Compound interest is not allowed on an outstanding certificate, but a depositor may withdraw interest payable and include it in a new deposit, which bears interest at the regular rate.

495. A depositor is permitted to exchange his deposits for registered or coupon United States postal savings bonds, issued in denominations of $20, $100, and $500, bearing interest at the rate of 2½ per cent per annum, payable semiannually. The exchange may be made under date of Jan. 1 and July 1 of each year.

PROBLEMS

496. **1.** On Feb. 28, 1911 (after 2 months' operation), the number of banks was 48, the total number of depositors was 3664, and the amount on deposit was $133,869. Dec. 30, 1911, there were 5132 depositories, 162,697 depositors, and $10,614,676 on deposit. Show the per cent of increase in the number of depositories and of depositors, and in the amount of deposits as compared with Feb. 28, 1911.

2. A stenographer bought $15 worth of postal savings certificates each month for three years, beginning Dec. 20, 1920. She drew her interest as it became due. On what date was she first able to draw interest? How much did she receive at that time? Prepare a table showing the amount of interest she could draw each month to and including January 1, 1924.

3. Beginning May 20, 1920, a man made a monthly deposit of $25 in a postal savings bank. On Jan. 1, 1922, he exchanged his total deposits for a postal savings bond. Assuming that he made no deposits after Jan. 1, 1922, and that he drew his interest as it became due, how much interest did he receive on his deposits and bond to and including July 1, 1924?

4. On the last business day of each month, a man deposits $10 in a postal savings bank, for five years. If no interest has been drawn, what amount will he then have to his credit?

EXAMINATIONS

SPEED TEST

497. Minimum time thirty minutes; maximum, one hour. Deduct one credit for each minute beyond minimum time.

1. Find the interest in the following cases:

$500	for 2 yr. 2 mo. 6 da.	at 6%.
$300	for 5 yr. 6 mo. 18 da.	at 6%.
$930	for 6 yr. 8 mo.	at 5%.
$750	for 4 yr. 6 mo.	at 3½%.

2. Find the proceeds of the following discounted notes:

$350	90 da.	6%.
$750	75 da.	6%.
$960	45 da.	4%.
$150	72 da.	5%.

3. Find balance due July 1, 1924, on the following (United States Rule):

Face of note, $3000; date, July 1, 1923; Interest, 6%.
Payments, Nov. 1, 1923, $1000.
Apr. 1, 1924, $1500.

4. Find the accurate interest in the following cases:

$3650 for 125 da. at 5%.
$7300 for 116 da. at 6%.
$1095 for 17 da. at 4%.
$ 550 for 146 da. at 3½%.

5. Find the proceeds of each of the following notes:

$515 due in 6 mo. at 6%.
$2020 due in 72 da. at 5%.
$ 858.50 due in 60 da. at 6%.
$1540.25 due in 90 da. at 4%.

6. Find the total insurance on the following properties:

Store worth $25,000 at ⅘% premium.
Mdse. worth $40,000 at 1¼% premium.
House worth $15,000 at 60¢ per $100 premium.
Furniture worth $5000 at 80¢ per $100 premium.

7. Find the total tax on the following:

Farm valued at $2400 at .0175.
House and lot valued at $18,000 at .01875
City lot valued at $5500 at $1.60 per $100.
Store valued at $30,000 at $1.7246 per $100.

8. At what price each shall these articles be sold to gain the rates given?

COST PER DOZEN	RATE OF GAIN
$18	20%
$24	40%
$15	33⅓%
$14.40	25%

9–10. Add vertically and horizontally and find grand total:

16832	21265	17234	18875
15293	17294	16872	5570
13846	17286	9237	18765
16450	18642	27436	18778
8287	18643	21299	17694

WRITTEN TEST

498. **1.** In a certain county containing 25,482 taxable inhabitants, a tax of $103,294.60 is assessed for town, county, and state purposes: a part of this sum is raised by a tax of 30¢ on each poll; the entire valuation of the property on the assessment roll is $38,260,000. What is the per cent tax, and what is a person's tax who pays for 1 poll and whose property is valued at $9470?

2. Find the interest on $738.72 for 1 yr. 3 mo. 22 da., at 5%.

3. I bought a horse for $360 and sold it for a note at 60 da. for $418. I discounted the note at 6% the day it was made. What was my gain?

4. A store was insured for $12,000 at ¾% and the goods for $15,000 at 1¼%. What was the entire premium?

5. A wholesale grocer sold 200 barrels of molasses, each containing 31 gal., 2 qt., at $.60 a gallon, receiving in payment a note at 90 days, with interest at 6%. What would be the proceeds of the note if discounted at a bank the day it was written, at 7½% per annum?

6. In a given town the real estate is assessed at $1,250,000, and the personal property at $150,000. The tax to be raised therefrom amounts to $19,250. Find the tax rate. How much tax does a man pay whose property is valued at $12,000, if it is assessed at 80% of its value?

7. A man deposited $15,000 in a bank June 1, 1924, receiving a certificate of deposit which stated that 3% interest would be paid if the money was left on deposit 6 mo. or more. The bank loaned the money at 5%. Dec. 20, 1924, the man asked for his money and interest. If the bank paid exact interest and charged ordinary interest, find the gain to the bank, the money having been loaned all the time.

8. A merchant bought a bill of goods invoiced at $5475.76, less 10% and 5%, terms net 60 da. or 5% 10 da. Not having the ready money, he borrowed it at 6% interest and accepted the 10-da. terms. Find his gain by so doing.

9. A stock of household furniture valued at $1800 was insured for 3 years from April 1, 1924, at 70¢ per $100. July 1, 1925, the company canceled the policy. Find the return premium.

10. A factory was insured under a policy containing the standard 80% average clause, as follows: Building for $22,000 at 1¼%; machinery for $25,000 at 1⅛%; and stock for $15,000 at 1⅜%. The building was worth $30,000, the machinery $35,000, and the stock $20,000. Fire broke out causing a damage as follows: to the building, $8000; machinery, $6000; and to the stock, $12,000. Find the amounts paid by the company.

STOCKS

499. **Stock, or capital stock,** is the name given to the capital of a corporation.

500. A **corporation** is an imaginary person composed of several real persons. It is formed only by the consent of the state, and hence, has only those rights and powers that are granted to it by the state as set forth in its charter. The list of powers, rights, and duties are stated in writing and issued to the corporation. This written instrument is called **a charter.**

501. The capital stock of a corporation is divided into shares, usually $100 each, though shares may be of various values. Reading Railroad shares are $50 par value. Mining stocks are at all prices from 10 ¢ to $100.

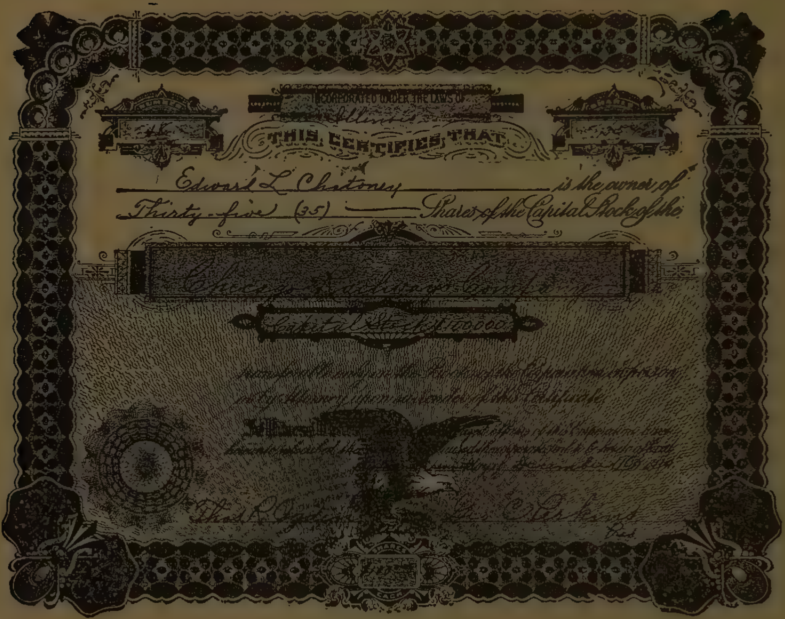
502. A **stockholder** is any person who owns one or more shares of stock. As evidence of ownership of stock, each stockholder receives a written statement called a **stock certificate,** showing the number of shares he owns and their par value.

503. Stock is either common or preferred. **Common stock** is the kind most usually issued by a corporation, and carries no guaranty of a specified dividend. Preferred stock is stock on which dividends are due and payable before dividends may be paid on common stock. Preferred stock is often issued in times of emergency to provide working capital or to pay debts.

504. The **par value** of stock is the value stated on the stock certificate. The **market value** is the price for which the stock can be sold.

505. The market value of stock depends chiefly upon the rate of dividends paid or earned by the company. If the dividends are small, the market value is below par (less than face value, or at a discount), and if the dividends are large, the market value is above par (or at a premium).

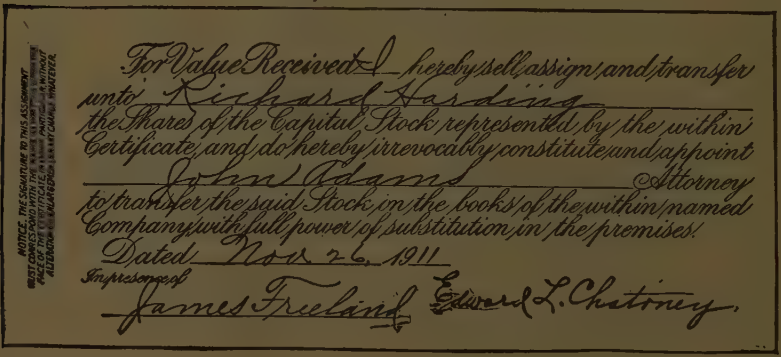
506. ILLUSTRATION OF A STOCK CERTIFICATE



INCORPORATED UNDER THE LAWS OF
Illinois
THIS CERTIFIES THAT
Edward L. Chatoney is the owner of
Thirty-five (35) Shares of the Capital Stock of the
Chicago Railway Company
Capital Stock $100000

507. Shares of stock are transferred by assignment. The assignee, then, has all the rights and privileges of the original stockholder.

508. FORM OF ASSIGNMENT



NOTICE. THE SIGNATURE TO THIS ASSIGNMENT MUST CORRESPOND WITH THE [illegible] FACE OF THE CERTIFICATE [illegible] PARTICULAR, WITHOUT ALTERATION [illegible] ENLARGEMENT [illegible] ANY CHANGE WHATEVER.

For Value Received I hereby sell, assign, and transfer unto Richard Harding the Shares of the Capital Stock represented by the within Certificate, and do hereby irrevocably constitute and appoint John Adams Attorney to transfer the said Stock on the books of the within named Company with full power of substitution in the premises.

Dated Nov 26, 1911

In presence of
James Freeland

Edward L. Chatroney

509. The following prices of certain stocks were printed in the *New York Times*, July, 1923.

New York Stock Exchange

Stock Exchange Sales and Quotations July 17, 1923

Sales	First	High	Low	Last	Net Change
800 Am. Car & Foundry	151½	153	151¼	153	+1
200 Am. Cotton Oil	4½	4½	4¼	4¼	----
1,900 Am. Smelting & Ref.	57⅞	57⅞	56¼	56½	−1½
400 Am. Sugar Refining	62⅝	62¾	62¼	62¼	− ¼
900 Am. Tel. & Tel.	122⅜	122⅜	122⅜	122⅜	----
1,900 Atch. Top. & S. F.	98⅞	99½	98⅞	99½	----
300 Ches. & Ohio	59	59	59	59	−1
400 Chi. Mil. & St. Paul	18⅞	19⅛	18¾	18¾	----
300 Chi. & N. W.	70⅜	70⅜	70	70¼	+ ¼
100 Colorado Southern	28	28	28	28	----
1,600 Consolidated Gas	59⅝	60	59⅝	60	+ ¼
700 Delaware & Hudson	105	105	104½	104¾	− ¼
500 Eastman Kodak, pfd	108½	108½	107¾	108	− ⅜
400 Erie	11¼	11¼	11¼	11¼	+ ⅛
1,100 Erie, 1st pfd.	18¼	18¾	18¼	18¾	+ ⅛
100 Illinois Central	109	109	109	109	+1
500 Lehigh Valley	59¼	59¼	59	59¼	+ ¼
300 Maxwell Motors Cl. A	39	39	38½	38½	−1
1,200 Missouri Pacific, pfd.	30⅜	31⅜	30⅜	31¼	+ ⅜
400 National Lead	118	118	117	117	−1¾
4,800 New York Central	97⅝	98⅛	97	97¾	----
2,100 Northern Pacific	65¼	66⅛	64⅜	66⅛	+ ¾
100 Otis Elevator	116	116	116	116	+ ½
1,200 Pennsylvania R. R.	43⅞	43⅞	43¾	43¾	− ¼
1,100 Reading	71	71⅜	70¾	71¼	----
1,100 Southern Pacific	86⅜	86⅝	86¼	86¼	− ⅛
76,100 Studebaker Co.	102⅛	103½	101⅞	103¼	+ ½
100 United Fruit	166	166	166	166	+1
17,100 U. S. Steel	90¾	90⅞	89⅝	90⅞	----
500 U. S. Steel, pfd.	117⅞	118	117⅞	118	----
1,000 Western Union Tel.	104	106½	104	106½	+2½
1,000 Woolworth (F. W.) Co.	234	236¾	234	236¾	+6¼

510. The profits of a corporation, after deductions are made for sinking fund, reserve for bad debts, depreciation, etc., may or may not be divided among the stockholders according to the decision of the board of directors. Such profits, when voted by the directors, are called *dividends*. Dividends are reckoned as a per cent of the par value of the issued stock of the company. Dividends on preferred stock (if any) are voted first. The balance of the profits may be voted as dividends on the common stock.

511. An **assessment** is a certain per cent of the par value of the stock levied against the stockholders to pay losses, etc.

512. Stocks are bought and sold by **stockbrokers** who make a business of buying and selling stocks for others. For their services they charge a commission, or brokerage. On some exchanges it is ⅛% of the par value of the stock bought or sold. On the New York Stock Exchange the rate of commission for buying and selling stocks is as follows:

On stock selling at

$1 per share and above but under $10	Not less than 7½¢ per share.
$10 per share and above but under $25	Not less than 12½¢ per share.
$25 per share and above but under $50	Not less than 15¢ per share.
$50 per share and above but under $75	Not less than 17½¢ per share.
$75 per share and above but under $100	Not less than 20¢ per share.
$100 per share and above but under $200	Not less than 25¢ per share.
$200 per share and over	Not less than 30¢ per share plus 5¢ per share for each $50 or fraction thereof beginning at $250.

513. Stock quotations on all exchanges, except Montreal and Toronto, are in dollars (or cents) per share.

514. There is a Federal tax of $2 per 100 shares of stock of $100 par value on all sales of stock, the tax in all cases to be paid by the seller. In New York State there is a similar tax of $2 per 100 shares on all sales of stock. This tax is paid by the seller. If the par value of the stock is other than $100 the tax bears the same ratio to $2 per 100 shares that the par value of the stock bears to $100.

EXPLANATION OF STOCK EXCHANGE TERMS

515. 1. **Bear.** A broker who operates for declining prices. He has sold "short" (see 5) and wishes to cover his short sale at a lower price.

2. Bull. A broker who operates for rising prices. He has bought stocks (is "long"—see 4) and wishes to sell them at higher prices.

3. Collaterals. Stocks or bonds deposited as security for a loan.

4. Long. A trader who buys stocks for a rise in price is long.

5. Short. When a trader has sold more stock than he owns, he is said to be short.

516. To find the cost of a purchase, or the proceeds of a sale, of stock.

Unless otherwise indicated, brokerage at the rates charged on the New York Stock Exchange, and the Federal transfer tax of $2 per hundred shares will be reckoned on problems in stocks in this text.

Examples.

1. Find the cost of 1250 shares of American Cotton Oil at the lowest price quoted on page 269.

$\$4.25 + \$.07\frac{1}{2} = \$4.32\frac{1}{2}$, gross cost of 1 share.
$1250 \times \$4.32\frac{1}{2} = \$5406.25$, cost.

2. Find the proceeds of a sale of 400 shares U. S. Steel, pfd. at the highest price quoted on page 269.

$400 \times \$118 = \$47{,}200$, gross proceeds of sale.
$400 \times \$.25 = \$100$, commission.
$4.00 \times \$2 = \$8$, transfer tax.
$\$47{,}200 - (\$100 + \$8) = \$47{,}092$, proceeds of sale.

517. Using the quotations on page 269, making allowance for brokerage, and for Federal transfer tax on all sales, find the results as indicated in the following:

Find, at the lowest price, the cost of:

1. 350 Am. Smelting & Ref.
2. 125 Am. Tel. & Tel.
3. 375 Chi. & N. W.
4. 175 Consol. Gas.
5. 225 Del & Hudson
6. 325 Lehigh Valley
7. 560 Nat. Lead
8. 750 New York Central
9. 600 Northern Pacific
10. 800 U. S. Steel

Find, at the highest price, the proceeds of:

11. 200 Am. Car & Foundry
12. 275 Studebaker Co.
13. 175 Missouri Pacific, pfd.
14. 450 Reading (Par $50)
15. 375 United Fruit
16. 650 Otis Elevator

17. 500 F. W. Woolworth & Co.
18. 475 Northern Pacific
19. 1250 Western Union Tel.
20. 1375 Erie

Find the gain or the loss on the following stocks if bought at the lowest and sold at the highest price quoted on page 269, allowing for brokerage each way:

21. 150 Am. Car and Foundry
22. 200 Atch., Top. & S. F.
23. 300 Chi., Mil. & St. P.
24. 450 Lehigh Valley (Par $50)
25. 750 Erie, 1st pfd.
26. 575 Eastman Kodak, pfd.
27. 875 F. W. Woolworth & Co.
28. 500 Maxwell Motors
29. 315 Illinois Central
30. 625 Penn. R.R. (Par $50)

518. To find the number of shares.

How many shares of Maxwell Motors, at the last price quoted on page 269, including brokerage, can be bought for $9276?

$38.50 + $.15 = $38.65, gross cost of 1 share.
$9276 ÷ $38.65 = 240, no. shares.

The gross cost of 1 share is the sum of the price and the brokerage. Dividing the total investment by the cost of 1 share gives the number of shares.

For prices of stocks and brokerage charges, see pages 269 and 270.

PROBLEMS

Use the "first" price in these problems.

519. **1.** How many shares of Am. Sugar Refining stock can be bought for $11052.80?

2. An investment of $7280 will buy how many shares of Erie?

3. A man sold 220 shares of Eastman Kodak, pfd., and invested the proceeds in Delaware & Hudson. How many shares did he buy, and how much cash had he left? (Reckon in whole shares.)

4. To provide himself with $27,500 a man sold 175 National Lead, and a sufficient quantity of Otis Elevator for the balance of the funds needed. How many shares of Otis Elevator did he sell, and how much additional money over and above the $27,500 did he receive?

5. A broker bought on his own account 1000 shares of F. W. Woolworth & Company at the first price, and when it advanced to the highest price (page 269) he sold enough of the stock to make a profit of $1638. How many shares did he sell?

520. To find the dividend on stocks.

1. A man invested \$42,490 in Am. Car and Foundry at the first price. If the stock paid 12% dividends, how much was his annual income?

\$151.50 + \$.25 = \$151.75, gross cost of 1 share.
\$42,490 ÷ \$151.75 = 280, no. shares.
Value of 280 shares = \$28,000, par value of stock.
12% of \$28,000 = \$3360, annual income.

Since the dividends are always based on the par value of the stock, it is necessary, first, to find the par value of the stock. The number of shares is found as in Art. 518. At \$100 a share, the par value of the stock is 280 times \$100, or \$28,000. 12% of the par value gives the annual income, \$3360.

2. The capital stock of a corporation is \$5,000,000. The gross earnings are \$1,250,000; the operating expenses are \$900,000. \$50,000 is carried to the surplus fund. What per cent dividend can be declared? If A owns 250 shares, how much will he receive?

\$900,000 + \$50,000 = \$950,000, to deduct from gross earnings.
\$1,250,000 − \$950,000 = \$300,000, total dividend.
\$300,000 ÷ \$5,000,000 = .06 = 6%, rate of dividend.
Value of 250 shares = \$25,000, par value of A's stock.
6% of \$25,000 = \$1500, A's dividend.

From the gross earnings there are deducted the operating expenses, and the amount set aside for the surplus fund, which leaves \$300,000 as the total dividend. Dividing by the capital stock, \$5,000,000, gives 6%, the rate of dividend. The par value of A's stock is \$25,000. 6% of \$25,000 gives \$1500, A's dividend.

PROBLEMS

521. 1. An investment of \$63,261 was made through a broker in F. W. Woolworth & Company at the low price. The stock paid 8% dividends. Find the income from the investment.

2. A man drew \$16,682 from a bank in which he was receiving $3\frac{1}{2}$% interest annually, and invested it in Pennsylvania Railroad stock at the low price. If the stock paid 3% dividends, was his income increased or diminished, and how much per annum?

3. Which is better, and how much per annum, to invest \$33,250 in United Fruit at the low price, paying 8% dividends, or to deposit it in savings banks paying 4% compounded semiannually?

4. A railroad company has a capital stock of $42,400,000. The gross income for 1 yr. was $44,829,555.20; the total expenses were $38,363,381.79. Find the largest whole per cent dividend that could be declared, and the balance left over for the reserve fund. What amount of dividend would be received by a man who owned 1500 shares of the stock?

522. To find the investment required to produce a given income.

How much must be invested in United States Steel, pfd. at the last price (page 269) to yield an annual income of $1750 if the stock pays 7%?

7% of $100 = $7, income from 1 share.
$1750 ÷ $7 = 250, no. of shares.
250 × $118.25 = $29562.50, investment.

7% stock pays 7% of its par value. 7% of $100 is $7, income from 1 share. To produce $1750 requires 250 shares. 1 share costs $118.25 (including brokerage). 250 shares cost $29,562.50.

523. **1.** What investment in New York Central stock paying 7% dividends, at the low price, will yield an income of $2450?

2. When the Northern Pacific pays 5% dividends, how much must be invested in its stock at the first price to yield $2250 annually?

3. An investment in Illinois Central stock paying 7% at the last price pays $2625 annually. Find the amount of the investment.

4. You desire an income of $3500. You have an option on New York Central 7% stock, and on Northern Pacific 5% stock, at the low prices quoted on page 269. Which stock requires the smaller investment?

524. To find what per cent an investment pays.

When Western Union Telegraph stock at the low price pays 7% dividends, what rate per cent does the stock pay on the investment?

7% of $100 = $7, income from 1 share.
$104 + $.25 = $104.25, cost of 1 share.
$7 ÷ $104.25 = .0671; that is 6.71% on the investment.

Since the income from 1 share is $7, and the cost of 1 share is $104.25, the rate of income is found by dividing the income by the cost, which gives 6.71%.

525. 1. If 6% stock is bought at 120, no brokerage, what per cent on the investment does the stock pay?

2. When a stock costs 210, and pays quarterly dividends of 3%, what rate does it pay on the investment (no brokerage)?

3. Which is better, and how much per cent, to invest, at the last price, in Pennsylvania Railroad stock paying 3% dividends, or in Consolidated Gas stock, paying 5% dividends?

4. You are offered American Telephone and Telegraph stock, paying 9% dividends, or New York Central stock, paying 7% dividends. Reckoning at the low price, which is the better investment, and how much per cent better?

5. A broker owns 600 shares of U. S. Steel pfd., which pays 7% dividends. He sells it at 120 and buys with the proceeds Atchison, Topeka and Santa Fe at 96. If the Atchison, Topeka and Santa Fe stock pays 6% dividends, is his income increased or diminished, and what per cent? (Make no allowance for transfer tax.)

526. To find the investment to yield a given rate per cent.

At what price will a 4% stock pay 5% on the investment?

$$4\% \text{ of } \$100 = \$4, \text{ income from 1 share.}$$

$$\$4 \div .05 = \$80, \text{ cost of 1 share.}$$

Since this is to be a 5% investment, the dividend, $4, must be equal to 5% of the cost of the share. Hence, $4 ÷ .05 gives $80, the cost of 1 share.

527. 1. A man desires a 5% investment. What price can he afford to pay for 6% stock?

2. A 7% stock pays only 4% on the investment. What is the market value, brokerage $\frac{1}{8}$%?

3. What is the market value of stock paying dividends of $4\frac{1}{2}$%, if an investment in them, including brokerage, pays 5%?

4. Find the cost, including brokerage, of 350 shares of stock paying 6% dividends, if the rate on the investment is $4\frac{4}{5}$%.

5. A broker has an option on 500 shares of 5% stock at a price to yield 8% on the investment, or a sufficient quantity of stock paying 2% semiannually at 50, to require the same investment. He accepts the latter stock. Is his income more or less, and how much, than it would have been on the other stock, if money is worth 6% per annum?

PROBLEMS

528. **1.** A man purchased 600 shares of stock paying annual dividends of 8% at $227.50 a share, brokerage $\frac{1}{8}$%. What is the rate of interest on the investment?

2. A corporation has a capital stock of $250,000 divided into 1500 shares of common stock and 1000 shares of preferred stock, the latter paying 7% dividends. In 1923, the company earned a net profit of $29,250. What is the largest whole per cent dividend that can be declared on the common stock after deducting $12,000 for the sinking fund?

3. The M. & W. Railroad Company with a capital stock of $3,750,000, has a mortgage debt of $125,000. Gross earnings for a year were $621,000, and operating expenses were $275,000. After paying expenses, and 6% interest on the debt, $1000 was added to the surplus. The balance of the profits was distributed as dividends. How much should a man receive who owned 25 shares?

4. A man received $1560 as the annual 12% dividend on stock that he owned. He afterward sold 35 shares of the stock at 142$\frac{1}{4}$, and the remainder of the stock at 143. Making allowance for brokerage, and for both the Federal and the New York State transfer tax, find the net proceeds of each sale.

5. A corporation with a capital stock of $100,000 has a net profit of $16,875.50 for the fiscal year just ended. The directors vote to pay a dividend of 7$\frac{1}{2}$%, and to put the remainder of the profit into the surplus fund. (*a*) What is the amount of the dividend declared? (*b*) How much of the profit will be put into the surplus fund? (*c*) What is the amount of dividend that A. L. Jones will receive if he owns 230 shares of the stock?

6. One year ago a man bought 20 shares of a certain stock at par ($100), brokerage $\frac{1}{8}$%. He has since received 4 quarterly dividends each of $1.75 per share. To-day he sells the stock at 126$\frac{1}{2}$, brokerage $\frac{1}{8}$%. (*a*) How much was his total profit? (*b*) What rate of interest did he realize on his investment for the year?

7. The capital stock of a corporation is $50,000, of which $25,000 is 6% preferred stock, and the balance common stock. The gross income for 6 months was $11,612, and the expenses were $7147. What rate of dividend can be paid on the common stock?

BONDS

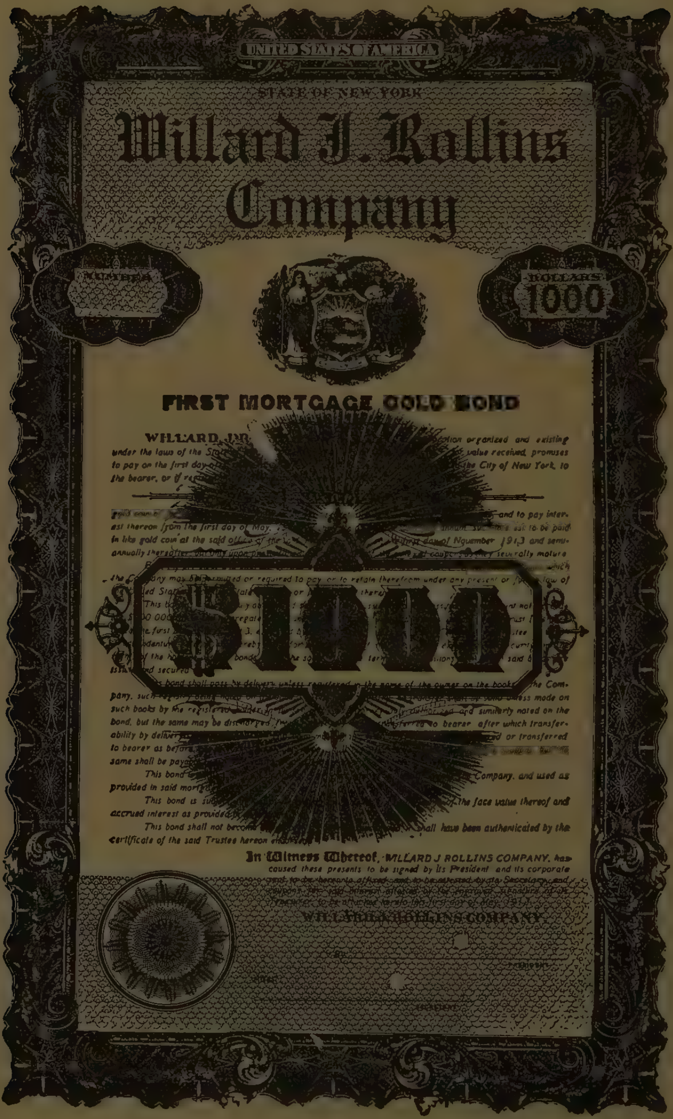
529. A **bond** is a secured obligation, promising to pay, for value received, a sum of money at a certain time, with interest, payable at stated intervals and at a fixed rate.

530. Bonds are known by the name of the government, municipality, or corporation issuing them, by the rate of interest they bear, the date of maturity, the purpose for which they were issued, and by the type of security back of them. For example, U. S. 2% Panama Canal loan of 1918-1938, means United States bonds paying 2% interest, due in 1938, payment of the bond being optional on any interest date after 1918. City of New York $4\frac{1}{2}$'s, 1957, means bonds of the City of New York bearing $4\frac{1}{2}$% interest, due in 1957. Chicago, Milwaukee & St. Paul General Mortgage 4's, 1989, means bonds bearing 4% interest, issued by the Chicago, Milwaukee & St. Paul Railway, secured by a general mortgage on the property of the company, and due in 1989.

Bonds have many names, some of which are First Mortgage 5's, General Mortgage 4's, Prior Lien $4\frac{1}{2}$'s, Refunding and Extensions 5's, etc. If issued by a private corporation, they are generally secured by mortgage on the property of the company.

English Consols (consolidated stock) are government bonds of England bearing $2\frac{1}{2}$% interest payable quarterly, Jan. 5, April 5, July 5, and Oct. 5. These bonds have no definite date of ma-

531. ILLUSTRATION OF A BOND



UNITED STATES OF AMERICA

STATE OF NEW YORK

Willard J. Rollins Company

NUMBER

DOLLARS 1000

FIRST MORTGAGE GOLD BOND

WILLARD J. R[illegible] ...ation organized and existing under the laws of the Sta[illegible] value received, promises to pay on the first day o[illegible] the City of New York, to the bearer, or if re[illegible]

gold coin of [illegible] and to pay interest thereon from the first day of May, [illegible] to be paid in like gold coin at the said office of the [illegible] first day of November 1913 and semi-annually thereafter but only upon pre[illegible] coupons as they severally mature [illegible] which the Company may be permitted or required to pay or to retain therefrom under any present or f[illegible] law of [illegible]

$1000

[illegible]

This bond shall pass by delivery unless registered in the name of the owner on the books of the Company, such registry being noted [illegible] unless made on such books by the registered holder [illegible] authorized and similarly noted on the bond, but the same may be discharged [illegible] transferred to bearer after which transferability by deliver[illegible] or transferred to bearer as before [illegible] same shall be paya[illegible]

This bond [illegible] Company, and used as provided in said mortg[illegible]

This bond is su[illegible] the face value thereof and accrued interest as provided [illegible]

This bond shall not become [illegible] shall have been authenticated by the certificate of the said Trustee hereon endorsed.

In Witness Whereof, WILLARD J ROLLINS COMPANY, has caused these presents to be signed by its President and its corporate seal to be hereunto affixed and to be attested by its Secretary, and coupons for said interest attested by the engraved signature of its Treasurer to be attached hereto this first day of May, 1913.

WILLARD J. ROLLINS COMPANY,

By ____________ President

Attest ____________ Secretary

turity, but could not be paid before April 5, 1923. They are one of the so-called perpetual loans.

French Rentes are government bonds of France, and bear interest at 3% payable quarterly. These loans are also perpetual loans.

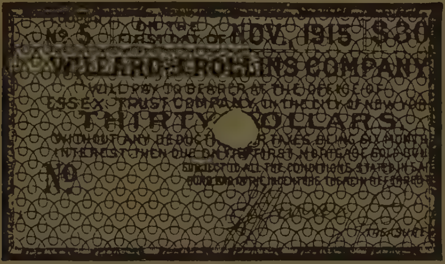
These bonds are issued in small denominations in order that persons with small means may purchase them.

532. There are two general classes of bonds—coupon and registered.

533. Coupon bonds have interest coupons attached. The coupons are made payable to bearer, and are dated at regular intervals (usually semiannually) and are, in effect, negotiable notes promising to pay the interest on the bond as it becomes due.

534. Registered bonds take their name from the fact that the name of the owner is registered with an agent of the maker of the bond. On "fully registered" ("registered as to principal and interest") bonds, interest is paid by check or draft mailed to the registered holders. Bonds "registered as to principal only" carry interest coupons like an ordinary coupon bond.

535. ILLUSTRATION OF A COUPON



No 5 ON THE FIRST DAY OF NOV. 1915 $30

WILLARD J. ROLLINS COMPANY

WILL PAY TO BEARER AT THE OFFICE OF

ESSEX TRUST COMPANY IN THE CITY OF NEW YORK

THIRTY DOLLARS

WITHOUT ANY DEDUCTION [illegible] TAXES, BEING SIX MONTHS

INTEREST THEN DUE [illegible] FIRST MORTGAGE GOLD [illegible]

SUBJECT TO ALL THE CONDITIONS STATED IN SAID [illegible]

No

TREASURER

536. The terms, *par value, market value, above par, premium, below par*, and *discount*, have the same signification as they have when applied to stocks.

537. STOCKS AND BONDS COMPARED

STOCKS	BONDS
1. Owner is part owner of corporation.	1. Owner is a creditor of the corporation.
2. Owner generally has a vote in the election of directors.	2. Owner generally has no vote.

STOCKS	BONDS
3. Dividends not due until declared by the directors.	3. Dividends (interest) are payable regularly.
4. Dividends depend on earning power of the corporation.	4. Interest is fixed.
5. Purchased, many times, for speculation.	5. Purchased, generally, for investment.

538. Bond quotations are not subject to sudden fluctuations, as are stock quotations. The price of bonds depend chiefly upon four things: 1st, the security back of the bonds; 2d, the time the bond has to run; 3d, interest rates of money; and 4th, the earning power of the issuing corporation.

NOTES. Bonds are redeemable at maturity at face value unless otherwise specified.

If interest payments on bonds are not made when due the company is said to default in the interest. To default in the bonds is to fail to redeem them at maturity.

539. Following is a partial list of bonds sold on the New York Exchange during the week ended July 21, 1923.

NAME OF BOND	INT. RATE	DATE OF MATURITY	PRICE
Am. Tel. & Tel. (Collateral)	5%	1946	97½
Armour & Company	4½%	1939	84¼
Bethlehem Steel (Sinking Fund)	6%	1948	98¼
Canadian Northern	7%	1940	112¾
Chi. M. & St. Paul	4½%	1989	81
Delaware and Hudson	5½%	1937	108¼
General Electric Debentures	5%	1952	100½
International Mer. Marine	6%	1941	80½
Kansas City, Southern	5%	1950	85½
Northern States Power (Refunding)	5%	1941	90
Southern Railway	6½%	1956	101¾
Union Pacific (1st Refunding)	4%	2008	84¼

The brokerage charge for buying or selling bonds on the New York Stock Exchange is as follows:

Not less than $2 per $1000 par value.

NOTE. The above is the charge on most bonds on the Exchange. The exceptions are not of sufficient importance to warrant their inclusion in this work.

540. To find the cost of bonds.

Find the cost of $25,000 Northern States Power 5's, at price quoted on page 280, on July 23, interest dates being April and October 1.

90% of $25,000 = $22,500, cost, flat.
From April 1 to July 23 = 3 mo. 22 da.
Interest on $25,000 at 5% for 3 mo. 22 da. = $388.89.
$22,500 + $388.89 = $22,888.89, cost.

The cost includes the interest on the face of the bonds since the last preceding interest date. At 90, the bonds would cost 90% of $25,000, or $22,500. The last interest date was April 1. The interest since that time to July 23 is $388.89. The cost is the sum of the "flat" price and the interest, or $22,888.89.

541. Using the prices quoted on page 280, find the cost, including interest, of the following bonds:

		Int. Dates	Date of Purchase
1.	6,000 Am. Tel & Tel. Collateral 5's	June—Dec.	Oct. 18
2.	8,000 Canadian Northern 7's	June—Dec.	Sept. 20
3.	12,000 Delaware & Hudson $5\frac{1}{2}$'s	May—Nov.	Nov. 15
4.	15,000 Chi. Milwaukee & St. Paul $4\frac{1}{2}$'s	Jan.—July	Jan. 4
5.	25,000 General Electric Debenture 5's	Mar.—Sept.	April 3
6.	50,000 Southern Railway $6\frac{1}{2}$'s	Apr.—Oct.	Aug. 25
7.	75,000 Union Pacific 1st Refunding 4's	Mar.—Sept.	June 10
8.	35,000 Kansas City, Southern 5's	Jan.—July	June 30
9.	45,000 Am. Tel & Tel. Collateral 5's	June—Dec.	July 18
10.	90,000 International Mercantile Marine 6's	Apr.—Oct.	Aug. 11

BOND TABLES

542. Bankers use bond tables to determine the rate of interest an investment in bonds will pay, or the price to pay to secure a given rate on the investment. The following is a reproduction of one page from the "Bond Tables" published by N. W. Harris & Co., Bankers, New York and Boston, and is printed with their permission:

HOW TO USE BOND TABLES

20 YEARS—Interest Payable Semi-annually.

Net per Annum.	BONDS BEARING INTEREST AT THE RATE OF						
	7%	6%	5%	4½%	4%	3½%	3%
4	141.03	127.36	113.68	106.84	100.00	93.16	86.32
4.10	139.32	125.76	112.20	105.42	98.64	91.86	85.09
4⅛	138.90	125.37	111.84	105.07	98.31	91.54	84.78
4.20	137.63	124.19	110.75	104.03	97.31	90.59	83.87
4¼	136.80	123.42	110.04	103.35	96.65	89.96	83.27
4.30	135.98	122.65	109.33	102.66	96.00	89.34	82.68
4⅜	134.75	121.51	108.27	101.65	95.04	88.42	81.80
4.40	134.35	121.14	107.93	101.32	94.72	88.11	81.51
4½	132.74	119.65	106.55	100.00	93.45	86.90	80.35
4.60	131.16	118.18	105.19	98.70	92.21	85.72	79.22
4⅝	130.77	117.82	104.86	98.38	91.90	85.42	78.94
4.70	129.61	116.74	103.86	97.43	90.99	84.55	78.11
4¾	128.84	116.02	103.20	96.80	90.39	83.98	77.57
4.80	128.08	115.32	102.55	96.17	89.79	83.40	77.02
4⅞	126.95	114.27	101.59	95.24	88.90	82.56	76.22
4.90	126.58	113.92	101.27	94.94	88.61	82.28	75.95
5	125.10	112.55	100.00	93.72	87.45	81.17	74.90
5.10	123.65	111.20	98.76	92.53	86.31	80.09	73.86
5⅛	123.29	110.87	98.45	92.24	86.03	79.82	73.61
5.20	122.22	109.87	97.53	91.36	85.19	79.02	72.85
5¼	121.51	109.22	96.93	90.78	84.64	78.49	72.34
5.30	120.81	108.57	96.33	90.21	84.09	77.97	71.85
5⅜	119.77	107.60	95.44	89.36	83.27	77.19	71.11
5.40	119.42	107.28	95.14	89.07	83.01	76.94	70.87
5½	118.06	106.02	93.98	87.96	81.94	75.92	69.90
5⅝	116.38	104.47	92.55	86.59	80.64	74.68	68.72
5¾	114.74	102.95	91.15	85.26	79.36	73.46	67.57
5⅞	113.13	101.46	89.78	83.95	78.11	72.27	66.43
6	111.56	100.00	88.44	82.66	76.89	71.11	65.33
6¼	108.50	97.17	85.84	80.18	74.51	68.85	63.19
6½	105.55	94.45	83.34	77.79	72.24	66.69	61.14
6¾	102.72	91.83	80.95	75.50	70.06	64.62	59.17
7	100.00	89.32	78.64	73.31	67.97	62.63	57.29

543. 1. If a 4% bond maturing in 20 yr. costs 80.64, how much will it net the purchaser?

Find the 4% column in the table and follow down the column until 80.64 is reached. Then follow the horizontal line in which 80.64 is found to the extreme left-hand column, and there find 5⅝ in the column marked "net per annum." The bond will net 5⅝% on the cost.

2. If a man wishes a 4½% investment, what price can he afford to pay for a 6% bond maturing in 20 yr.?

In the left-hand column find 4½, and follow the horizontal line in which 4½ is, to the right until the column having 6% at its top is reached. The number found in both the horizontal line and in the vertical column, viz., 119.65 is the price he can pay.

PROBLEMS

544. Using the table on this page, find the price at which bonds maturing in 20 yr. can be bought to produce as follows:

1. 5% bond to yield 4%.
2. 4½% bond to yield 5%.
3. 6% bond to yield 4.6%.
4. 3½% bond to yield 4½%.
5. 5% bond to yield 5½%.
6. 7% bond to yield 5⅜%.

Find the rate on the investment on bonds maturing in 20 yr. and bought as follows:

7. 4½% bonds bought at 96.80.
8. 4% bonds bought at 88.90.
9. 6% bonds bought at 117.82.
10. 5% bonds bought at 101.27
11. 3½% bonds bought at 71.11.
12. 7% bonds bought at 126.95.

EXCHANGE

545. The system by which merchants in different places discharge their debts to each other without the transmission of money is called **exchange.**

546. Exchange is of two kinds, viz.: Inland or Domestic, and Foreign.

DOMESTIC EXCHANGE

547. Domestic exchange treats of the payment of debts between merchants in the same country, without the sending of money.

For instance, if a merchant of New York owes a manufacturer of St. Louis for a bill of merchandise, he can pay his debt without sending money, in any one of the following ways:

1. By his personal check.
2. By a bank draft.
3. By a commercial draft on one of his debtors.
4. By a postal money order.
5. By an express money order.
6. By a telegraphic money order.

If he sends his check, the manufacturer in St. Louis will deposit it in his bank. In due time the check will be returned to the bank on which it is drawn, whereupon the amount will be charged against the merchant's account.

If the merchant sends a bank draft, he must buy and pay cash in advance for it. The draft is then sent in the same manner as a check.

NOTE. Bankers in the City of New York seldom draw drafts on other cities. They sell cashier's or manager's checks, drawn on their own bank.

If the merchant draws and sends a draft on one of his debtors, the manufacturer must present the draft to the drawee for payment, or he must have his bank collect it for him, and place the proceeds to his credit. Or, if it is a time draft, the bank will buy (discount) it, after acceptance by the drawee, and credit him at once with the proceeds. The bank will then collect the amount of the draft from the drawee at maturity, to reimburse itself for the credit given the indorser.

548. New York, Chicago, and San Francisco are the principal **exchange centers** in the United States.

549. An exchange center is a recognized money center. In it are located the correspondent banks of the banks in the surrounding cities and towns.

550. A **postal money order** is a written order given by the postmaster in one place to the postmaster in another place, to pay a specified sum of money to a certain person or to his order.

551. To obtain a postal money order, one has to make a written application stating:

1. The amount of the order wanted.
2. The name and address of the payee.
3. The name and address of the purchaser of the money order.

552. The largest amount for which a single postal money order can be obtained is $100. The rates charged are as follows:

$2.50 or less	3¢	$30.01 to $40.00	15¢
2.51 to $5.00	5¢	40.01 to 50.00	18¢
5.01 to 10.00	8¢	50.01 to 60.00	20¢
10.01 to 20.00	10¢	60.01 to 75.00	25¢
20.01 to 30.00	12¢	75.01 to 100.00	30¢

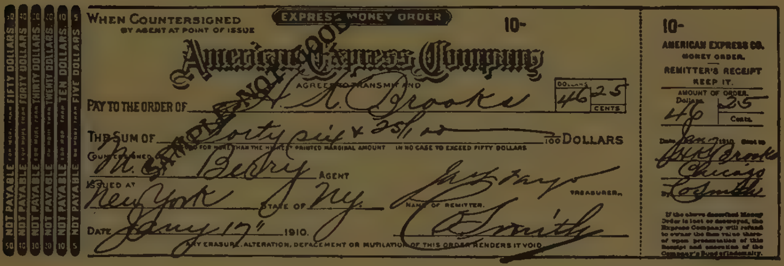
553. More than one indorsement on a postal money order is prohibited by law, except indorsements by banks.

554. Postal money orders may be presented for payment at the post office on which they are drawn, or at any bank. If not presented for payment within one year from their date, or if lost, the Post Office Department at Washington will upon request and presentation of evidence of loss, issue a duplicate.

555. An **express money order** is a written order of an agent of an express company directing another agent of the company to pay a certain sum of money to a specified person or to his order.

855. Express money orders are similar to postal money orders. The rates charged for them are the same. Express money orders may be indorsed and transferred any number of times, in the same manner as checks and drafts.

FORM OF EXPRESS MONEY ORDER



NOT PAYABLE FOR MORE THAN FIFTY DOLLARS 50
NOT PAYABLE FOR MORE THAN FORTY DOLLARS 40
NOT PAYABLE FOR MORE THAN THIRTY DOLLARS 30
NOT PAYABLE FOR MORE THAN TWENTY DOLLARS 20
NOT PAYABLE FOR MORE THAN TEN DOLLARS 10
NOT PAYABLE FOR MORE THAN FIVE DOLLARS 5

WHEN COUNTERSIGNED BY AGENT AT POINT OF ISSUE

EXPRESS MONEY ORDER

10-

American Express Company AGREES TO TRANSMIT AND

PAY TO THE ORDER OF H. H. Brooks DOLLARS 46 25 CENTS

THE SUM OF Forty six & 25/100 DOLLARS

FOR MORE THAN THE HIGHEST PRINTED MARGINAL AMOUNT IN NO CASE TO EXCEED FIFTY DOLLARS

COUNTERSIGNED M. E. Berry AGENT

ISSUED AT New York STATE OF N.Y.

TREASURER.

NAME OF REMITTER. C. Smith

DATE Jany 17th 1910.

ANY ERASURE, ALTERATION, DEFACEMENT OR MUTILATION OF THIS ORDER RENDERS IT VOID

10-

AMERICAN EXPRESS CO.
MONEY ORDER.
REMITTER'S RECEIPT
KEEP IT.
AMOUNT OF ORDER.
Dollars 46 .25 Cents.
Date Jan 17 1910 Sent to H. H. Brooks
Chicago
By C. Smith

856. A telepraph money order is an order by telegraph sent by an agent of a telegraph company at one place to a telegraph agent at another place, directing him to pay a specified sum of money to the person named in the telegram, with or without identification, as directed by the sender.

857. The cost of sending a telegraphic money order consists of three parts, as follows:

1. A toll charge, which is the regular charge for a fifteen word telegram. See toll charges on next page.

2. A transfer charge, as follows:

For $25 or less	$.25
Over $25 and not over $50	$.35
Over $50 and not over $75	$.60
Over $75 and not over $100	$.85

After the first $100, up to and including $3000, add $.25 for each $100 or fractional part thereof. For amounts above $3000, $.20 is charged for each $100 or fractional part thereof.

3. A tax of $.05 on any toll charge of $.50 or less, and a tax of $.10 on any toll charge over $.50.

858. The regular charge for a telegram varies according to distance. There is a fixed charge for 10 words or less for each of the several zones or districts into which the country is divided. For each word in excess of 10 words there is an additional charge. In the following schedule of rates, the first number is the toll charge in cents for 10 words, and the second number is the charge in cents for each additional word.

TOLL CHARGES FOR TELEGRAMS

30 − 2.5	36 − 2.5	42 − 2.5	48 − 3.5	60 − 3.5	72 − 5	96 − 6	120 − 8.5

560. Domestic rates of exchange are practically at par throughout the United States. A charge made by a bank in San Francisco, for instance, for a draft on New York, is a service or commission charge, and not a premium charge.

561. To find the cost of a draft or money order.

1. Find the cost of a draft of $3500 on Chicago if the exchange charge is $\frac{1}{10}$%.

$\frac{1}{10}$% of $3500 = $3.50.
$3500 + $3.50 = $3503.50, cost.

2. Find the cost of a telegraphic money order of $3875, if the toll charge is at the rate of 48 − 3.5.

The first $100 costs		$.85
The next $2900 costs 29 × $.25	=	7.25
The next $875 costs 9 × $.20	=	1.80
The toll charge is $.48 + (5 × $.035)	=	.66
The tax is		.10
		$10.66

$3875 + $10.66 = $3885.66, total cost.

PROBLEMS

562. 1. Find the cost of a draft of $2672.50 on New Orleans if exchange is at $\frac{1}{4}$%.

2. Find the cost of a telegraphic money order of $1965, if the toll charge is at the rate of 60 − 3.5.

3. A traveling salesman telegraphs for expense money. His employer sends him a telegraphic money order for $250. The telegraph rate is 30 – 2.5. Find the total cost of the order.

4. Compare the cost of sending $175 by postal money order, express money order, and by telegraphic money order, the toll rate being 72 – 5.

5. If the exchange on a draft is $\frac{1}{2}$%, which is the cheaper way to make payment of $500, by draft, or by telegraphic money order, the toll rate being 30 – 2.5?

6. A man pays the following bills: $39.75 by express money order; $78.50 by postal money order; $382.50 by telegraphic money order (toll rate 42 – 2.5); and $1837.50 by draft, exchange $\frac{1}{8}$%. Find the total cost of all.

7. Find the cost of a telegraphic money order of $6475, the toll rate being 120 – 8.5.

563. To find the proceeds of a draft.

1. Find the proceeds of a sight draft of $2840, collection and exchange being $\frac{1}{8}$%.

$\frac{1}{8}$% of $2840 = $3.55, collection and exchange.
$2840 – $3.55 = $2836.45, proceeds.

Since the bank charges $\frac{1}{8}$% for collection, the charge amounts to $\frac{1}{8}$% of $2840, or $3.55. The proceeds is the difference between $2840 and $3.55, or $2836.45.

2. Find the proceeds of a 60-da. commercial draft of $1500, if sold on its date at $\frac{1}{2}$% discount, money being worth 6%.

$15.00 = the interest for 60 da.
$\frac{1}{2}$% of $1500 = $7.50, discount.
$15 + $7.50 = $22.50, total discount.
$1500 – $22.50 = $1477.50, proceeds.

This solution is identical with that of bank discount. The interest (bank discount) is found for the unexpired time of the draft (term of discount). The $\frac{1}{2}$% discount is equivalent to a collection charge, and is reckoned on the face value of the draft. The total discount is the sum of the two discounts, $15 and $7.50, or $22.50. The proceeds is the face of the draft less the total discount.

564. Find the proceeds of the following sight drafts:

	Face	Collection and Exchange		Face	Collection and Exchange
1.	$ 784.00	½%	**11.**	$3981.43	¼%
2.	1260.00	¼%	**12.**	4289.64	⅛%
3.	2500.00	⅛%	**13.**	1562.50	¼%
4.	3675.00	$\frac{1}{10}$%	**14.**	5283.25	$\frac{1}{10}$%
5.	3875.00	½%	**15.**	6874.90	$\frac{1}{20}$%
6.	4272.50	¼%	**16.**	9743.85	$\frac{1}{20}$%
7.	1761.40	⅛%	**17.**	8498.75	$\frac{1}{10}$%
8.	2364.82	¼%	**18.**	4964.80	⅛%
9.	4152.87	$\frac{1}{20}$%	**19.**	3829.34	¼%
10.	7387.95	⅛%	**20.**	4382.86	½%

21. Find the proceeds of a 90-da. draft of $968.40 sold at ¼% discount, money being worth 6%.

22. A jobber purchased a bill of goods invoiced at $6483.28, less 5% for cash. To make immediate payment, he discounted a draft of $1864.50 due in 48 da. at ½% discount, and 6% interest; a note of $2487.25 due in 33 da., at ¼% discount and 6% interest. He sent a bank draft bought at $1.0025 for the balance. How much did he save by discounting the invoice and paying cash?

EXAMINATIONS

SPEED TEST

565. Minimum time thirty minutes; maximum, one hour. Deduct one credit for each minute required beyond the minimum time.

1. Find the market value in each of the following cases:

Par Value	Market Price	Market Value
$7,500	110	——
7,300	90	——
67,200	62½	——
475,200	131¼	——

2. What must be the asking price of a watch costing $24 in order to make a gain of 33⅓%, and allow the purchaser 20% discount?

3. Goods are invoiced at $800, less 20%, 5%, and $2\frac{1}{2}\%$. Find the net cost.

4. Find the cost of the following bill of lumber at $24 per M:

24 planks 2″ × 8″ × 14′
25 boards 1″ × 9″ × 16′
17 pieces 4″ × 6″ × 14′
66 boards 1″ × 8″ × 12

5. Find cost of each, and the total cost of the following drafts:

Face	Rate	Cost
$1,500	50¢ premium (per $1000)	——
1,875	$1.00 discount	——
3,500	$1.50 premium	——
5,000	10¢ discount	——
	Total	

6. What is the market value of a 6% stock paying 5% on the investment, brokerage $\frac{1}{8}\%$?

7. Find the par value of bonds necessary to produce income as follows:

Bonds	Income	Par Value
City of New York $4\frac{1}{2}$'s	$1,800	——
United States 3's	2,400	——
C. B. & Q. 4's	2,000	——
Sea. Air Line 5's	3,000	——

8. Add vertically, horizontally, and prove by finding the sum of the totals:

$16,157	$11,273	$10,559	——
48,287	2,380	16,272	——
473,892	8,721	145,468	——

9. Find gain or loss and per cent of gain or loss:

Cost	Selling Price
$2.50	$3.00
3.60	4.50
5.00	6.50
120.00	110.40

10. Find total interest on the following:

$1200 for 50 days at $4\frac{1}{2}\%$.
1275 for 90 days at 6%.
2500 for 72 days at 5%.
3600 for 13 days at 7%

WRITTEN TEST

566. 1. A sight draft on Chicago for $1771 is purchased at $\frac{5}{8}$% premium. Find the cost.

2. How many yards of carpet 27 in. wide will be required to carpet the floor of a room 24 ft. long and 18 ft. wide, if the strips run the long way of the room, and 6 in. are allowed on each strip for loss in matching?

3. What must be the list price of goods that cost $410 in order to make a profit of 20%, if they are to be sold at a discount of 20% and 10% from the list price?

4. On May 2, 1924, J. Williams bought the following goods less a discount of 5%, terms 2/10, n/60; 5 pieces velveteen, 42, 44, 43, 42, 45 yd. at 25¢ a yard. Williams sold the goods May 3 at 35¢ a yard, terms 1/10, n/30. He received cash for his sale on May 9, and paid for his purchase on May 10. Find his profit.

5. My broker buys for me 200 shares C. M. & St. P. at 18$\frac{3}{4}$, brokerage $\frac{1}{8}$%. Thirty days later, I instruct him to sell at 19$\frac{1}{8}$. Allowing $\frac{1}{8}$% for selling, find my gain or loss.

6. If the cost of erecting a $28,000 building is assessed on $450,000 capital stock, how much must a stockholder pay who owns 100 shares at $100 each?

7. On an asking price of $400, which is the more favorable for the buyer, and how much: a discount of 30%, 20%, and 12$\frac{1}{2}$%, or a discount of 40%, 12$\frac{1}{2}$%, and 10%?

8. On the Panama Canal Bonds, interest is payable quarterly—on the 1st of February, May, August, and November, the annual rate being 2%. Find the cost, including interest, of $50,000 of these bonds on the 10th of June at 99$\frac{3}{4}$.

9. A 4-months' note for $875, without interest, dated May 17, 1924, was discounted at a bank July 11, 1924, at 6%. Find the discount and the proceeds.

10. I sold 2978 bu. wheat at $1.05 a bushel, and invested the proceeds in sugar, reserving my commission of 5% for selling and 1$\frac{1}{2}$% for buying, and the expenses of shipping, $53.57. How much did I invest in sugar?

APPENDIX

INVOLUTION AND EVOLUTION

INVOLUTION

567. A **power** of a number is the product obtained by multiplying the number by itself. 16 is a power of 4, because 4 × 4 = 16.

568. Powers are named from the number of times a given factor is used to produce the power.

Thus, 36 is the *second* power (called *square*) of 6, because 6 is used twice as a factor to produce 36.

27 is the *third* power (called *cube*) of 3, because 3 is used three times as a factor to produce 27.

256 is the *fourth* power of 4, because 4 is used four times as a factor to produce 256, etc.

569. The number of times a given factor is to be used is shown by a small figure, called an **exponent,** written to the right of the factor.

Thus in 5^2, 2 is the exponent and shows that 5 is to be used twice as a factor;

$$5^2 = 5 \times 5 = 25.$$

In the same manner

$$4^3 = 4 \times 4 \times 4 = 64.$$

$$5^4 = 5 \times 5 \times 5 \times 5 = 625, \text{ etc.}$$

570. **Involution** is the process of finding the power of a number.

PROBLEMS

571. **1-20.** Find the square, the cube, and the fourth power of all the numbers from 1 to 20 inclusive.

21-40. Find the square, the cube, and the fourth power of all the two-place decimals .01 to .20 inclusive.

Find the square, the cube, and the fourth power of:

41. $\frac{1}{2}$
42. $\frac{1}{3}$
43. $\frac{2}{3}$
44. $\frac{1}{4}$
45. $\frac{3}{4}$
46. $\frac{1}{5}$
47. $\frac{2}{5}$
48. $\frac{3}{5}$
49. $\frac{4}{5}$
50. $\frac{5}{6}$
51. $\frac{7}{8}$
52. $\frac{5}{8}$
53. $\frac{9}{16}$
54. $\frac{5}{11}$
55. $\frac{4}{7}$

EVOLUTION

572. 1. What number multiplied by itself will produce 9? 16? 25? 36? 49? 64? 81?

2. What number used three times as a factor will produce 8? 27? 64? 125?

573. The **root** of a number is one of the equal factors of that number. Thus, 4 is a root of 16; 5 is a root of 125; etc.

574. The **square root** of a number is one of the *two* equal factors which produce the number. 5 is the square root of 25.

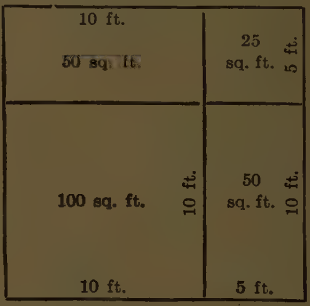
575. The **cube root** of a number is one of the *three* equal factors which produce the number. 6 is the cube root of 216.

576. **Evolution** is the process of finding the root of a number.

577. The **radical sign** ($\sqrt{\ }$) denotes that a root of a number is to be found.

$\sqrt{625}$ indicates that the square root is to be found.
$\sqrt[3]{343}$ indicates that the cube root is to be found.

578. The following diagram will illustrate the principle involved in finding the square root of a number.



The square is 15 ft. on a side, and is divided into four parts as follows:

1 square 10 ft. on a side
$= (10 \text{ ft.})^2 = 100$ sq. ft.

2 rectangles 10 ft. long and 5 ft. wide
$= 2 \times (10 \times 5)$ sq. ft. $= 100$ sq. ft.

1 square 5 ft. on a side
$= (5 \text{ ft.})^2 = 25$ sq. ft.

Adding the several results gives

$$[10^2 + 2 \times (10 \times 5) + 5^2] \text{ sq. ft.} = 225 \text{ sq. ft.}$$

This result may be expressed as follows:

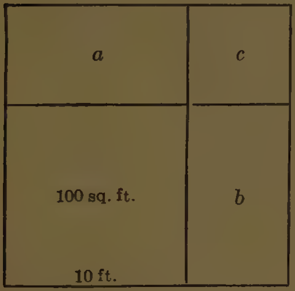
The square of a number of two figures is equal to the square of the tens plus twice the product of the tens by the units plus the square of the units.

By careful inspection and application of this principle, the square root of any number may be found.

579. **1.** Find the square root of 225.

$$\begin{array}{ll} & \sqrt{2'25} = 10 + 5 = 15. \\ 2 \times 10 = 20, \text{ trial divisor.} & \underline{1,00} \\ 20 + 5 = 25, \text{ complete divisor.} & 1,25 \\ & \underline{1,25} \end{array}$$

Beginning at units, divide the number into periods of two figures each. The largest perfect square in the left-hand period is 1. The square root of 1 is 1. Write, as part of the square root, 1 with a cipher to represent the remaining period in the number 225. The square of 10 is 100, which subtract from 225, leaving 125 as the new dividend.



The accompanying diagram illustrates the condition of the problem at this point.

The remaining 125 sq. ft. is represented by the two rectangles *a* and *b*, and by the square *c*. Each rectangle is 10 ft. long. Hence, both rectangles are 2 × 10 ft., or 20 ft. long. 20 ft. is the trial divisor as shown in the solution. 125 ÷ 20 = 6, but 6 is too large because, when it is added to the 20 to complete the divisor, it makes the divisor too large. Try 5 as the quotient figure. Complete the divisor by adding the 5 to the 20, making 25. The 5 is the width of the rectangles *a* and *b*, and is the length of one side of the square *c*. The complete divisor represents the total length of both rectangles *a* and *b*, and of the square *c*.

Now multiply the total length, 25, by the width, 5, and the result, 125, is just equal to the remaining area of the large square. Hence the square root of 225 is 15.

NOTE. The student should observe that

$$2 \times (10 \times 5) + 5^2 = [\,(2 \times 10) + 5] \times 5.$$

2. Find the square root of 1489.96.

$$\begin{array}{rl} & \sqrt{14'89'.96} = 38.6 \\ & \underline{9} \\ 2 \times 3 = 6^0 & 5\;89 \\ \underline{60 + 8 = 68} & \underline{5\;44} \\ 2 + 38 = 76^0 & 45\;96 \\ 760 + 6 = 766 & \underline{45\;96} \end{array}$$

The preceding solution and explanation were made in full that the principle involved might be thoroughly understood. The actual work of finding the square root is reduced to the minimum in this solution.

3. Find the square root of $\frac{169}{256}$.

$\sqrt{169} = 13$

$\sqrt{256} = 16$

$\sqrt{\frac{169}{256}} = \frac{13}{16}$.

To find the square root of a common fraction, find the square root of the numerator for the numerator of the root, and the square root of the denominator for the denominator of the root.

4. Find the square root of $\frac{3}{8}$.

$\frac{3}{8} = .375$ $\quad \sqrt{.37'50'00} = .612$

$$\begin{array}{rr} & 36 \\ 2 \times 6 = 12^0 & 1\,50 \\ 120 + 1 = 121 & 1\,21 \\ 2 + 61 = 122^0 & 29\,00 \\ 1220 + 2 = 1222 & 24\,44 \end{array}$$

When the numerator and denominator of the fraction are not perfect squares, reduce the fraction to its decimal form and find the square root of the resulting decimal. The degree of accuracy of the result is determined by the number of places the result is carried out.

PROBLEMS

580. Find the square root of the following:

1. 289	4. 15.452	7. .144	10. $\frac{3}{4}$
2. 961	5. 18.215	8. .576	11. $\frac{576}{961}$
3. 1764	6. 18.0625	9. .8925	12. $\frac{529}{625}$

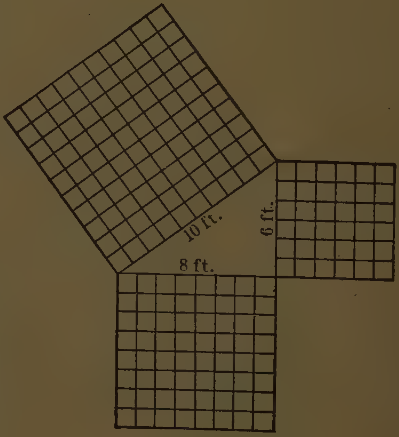
13. Find the hypotenuse of a right-angled triangle whose base and altitude are 8 ft. and 6 ft., respectively.

$8^2 = 64$

$6^2 = 36$

$8^2 + 6^2 = 64 + 36 = 100$

$\sqrt{100} = 10$, no. ft., hypotenuse.



The square on the base plus the square on the perpendicular equals the square on the hypotenuse. To find the side of a square area, extract the square root of the area. Hence, to find the hypotenuse of a right-angled triangle extract the square root of the sum of the squares of the base and the perpendicular.

PARTIAL PAYMENTS

581. When part of a note, draft, or other obligation is paid, such payment is called a **partial payment.**

Any partial payment of a note or a draft should be indorsed on the note or draft, thus:

Rec'd on this note

Apr. 1, 1924, $500.00
July 10, 1924, $675.50
Sept. 15, 1924, $853.75

UNITED STATES RULE

The United States Rule (so called because it has been approved by the United States Supreme Court) is based on the following principles:

1. *Any payment must first be applied to the paying of accrued interest. If the payment exceeds the amount of interest due, the principal sum is reduced by the amount of the excess.*

2. *Interest must not be charged upon interest.*

NOTE.—This rule is generally used when partial payments are made on interest-bearing notes having more than one year to run. The Merchants' Rule is generally used if the note has less than one year to run.

To find the balance due by the United States Rule.

$3000.00 NEW YORK, N.Y., Sept. 8, 1921.

Two years after date, I promise to pay to the order of A. C. Losey, Three thousand $\frac{00}{100}$ Dollars, with interest at 6%. Value received.

JOHN CARPENTER.

The following payments were indorsed on the above note: Jan. 20, 1922, $1200; Nov. 10, 1922, $75; and Apr. 4, 1923, $500. What was the balance due at maturity? (Use compound time.)

Principal sum due	$3000.00
Int. from Sept. 8, 1921, to Jan. 20, 1922 (4 mo. 12 da.) .	66.00
Amount due Jan. 20, 1922	3066.00
Payment, Jan. 20, 1922	1200.00
Balance due, to draw int. from Jan. 20, 1922 . . .	1866.00
Int. from Jan. 20, 1922, to Nov. 10, 1922 (9 mo. 20 da.)	90.19

Since the payment is less than the accrued interest, the balance due remains unchanged.

Int. on $1866 from Nov. 10, 1922, to Apr. 4, 1923 (4 mo. 24 da.)	44.78
Amount due Apr. 4, 1923	$2000.97
Payments, Nov. 10, 1922, and Apr. 4, 1923 ($75+$500)	575.00
Balance due, to draw int. from Apr. 4, 1923	1425.97
Int. from Apr. 4, 1923 to Sept. 8, 1923 (5 mo. 4 da.)	36.60
Balance due Sept. 8, 1923, date of maturity	$1462.57

The principal sum, $3000, draws interest from the date of the note, Sept. 8, 1921, to the date of the first payment, Jan. 20, 1922, a period of 4 mo. 12 da. The interest is $66, making a total amount due of $3066. Deducting the payment of $1200, leaves a balance of $1866 to draw interest from Jan. 20, 1922. From Jan. 20, 1922, to the date of the next payment, Nov. 10, 1922, is 9 mo. 20 da., and the interest accrued in that time is $90.19. But the payment made on Nov. 10, 1922, is only $75, not large enough to cancel the interest; hence the balance due, $1866, remains unchanged, and continues to draw interest to the date of the next payment, Apr. 4, 1923, or 4 mo. 24 da. The interest on $1866 for 4 mo. 24 da. is $44.78, making a total amount due of $2000.97. Deducting the sum of the two payments, $75 and $500, leaves $1425.97, the balance to draw interest till the date of maturity, Sept. 8, 1923. From Apr. 4, 1923, to Sept. 8, 1923, is 5 mo. 4 da., and the interest on $1425.97 for 5 mo. 4 da. is $36.60, which makes the amount due at maturity equal to $1462.57.

PROBLEMS

Find the balance due at maturity on each of the following notes, payments having been indorsed as indicated:

	DATE	TIME TO RUN	FACE	INT.	INDORSEMENTS
1.	May 1, 1921	3 yr.	$2500	6%	Dec. 1, 1921, $ 500 June 21, 1922, 300 Jan. 15, 1923, 1000
2.	Aug. 7, 1920	4 yr.	$5000	5%	Sept. 1, 1921, $1200 Aug. 15, 1922, 1500 Dec. 20, 1923, 1000
3.	Jan. 10, 1920	2 yr. 6 mo.	$4000	7%	Apr. 20, 1920, $ 500 July 30, 1920, 50 Dec. 10, 1921, 1000

DENOMINATE NUMBER TABLES

Linear Measure

582. Linear measure is used for measuring distance.

Table

12 inches = 1 foot
3 feet = 1 yard
5½ yards = 1 rod
320 rods = 1 mile

The unit of length is the yard.

Square Measure

583. Square measure is used to measure the areas of surfaces.

Table

144 square inches = 1 square foot
9 square feet = 1 square yard
30¼ square yards = 1 square rod
160 square rods = 1 acre
640 acres = 1 square mile

A square is 100 sq. ft.

Other Linear Measures

1 size, ⅓ in. Used by shoemakers.
1 hand, 4 in. Used in measuring the height of horses.
1 fathom, 6 ft. Used in measuring depths at sea.
1 knot, nautical or geographical mile — 1.152⅔ mi. or 6086 ft.

The knot is used in measuring distances at sea. It is equivalent to 1 min. of longitude at the equator.

Cubic Measure

584. Cubic measure is used to measure the volume of solids and the contents or capacity of hollow bodies.

Table

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard
24¾ cubic feet = 1 perch
128 cubic feet = 1 cord
1 cubic yard (of earth) = 1 load

A perch of stone or masonry is 16½ ft. (1 rd.) long, 1½ ft. wide, and 1 ft. high.
A cubic foot of water weighs 62½ lb. (Avoir.).
A ton of timothy hay in a well-settled mow is about 450 cu. ft.
A ton of clover hay in a well-settled mow is about 550 cu. ft.
A ton of stove coal is about 34½ cu. ft.

Liquid Measure

585. Liquid measure is used for measuring liquids, and in estimating the capacity of cisterns, tanks, reservoirs, etc.

TABLE

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

Apothecaries' Fluid Measure

586. Apothecaries' fluid measure is used by druggists in prescribing and compounding liquid medicines.

TABLE

60 minims (m.) = 1 fluid drachm (fʒ)
8 fluid drachms = 1 fluid ounce (f℥)
16 fluid ounces = 1 pint (O)
8 pints = 1 gallon (Cong.)

The gallon of this measure is the same as the wine gallon.

Technically, the barrel contains $31\frac{1}{2}$ gal. In practice, barrels are of various sizes, as are also hogsheads, pipes, butts, etc. The capacity of each is marked upon it.

The unit of liquid measure is the wine gallon of 231 cu. in. A gallon of water weighs about $8\frac{1}{3}$ lb. (Avoir.).

Dry Measure

587. Dry measure is used for measuring grain, fruit, vegetables, etc.

TABLE

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel

Liquid and Dry Measures Compared

	GALLON	QUART	PINT
Liquid	231 cu. in.	$57\frac{3}{4}$ cu. in.	$28\frac{7}{8}$ cu. in.
Dry	$268\frac{4}{5}$ cu. in. ($\frac{1}{8}$ peck)	$67\frac{1}{5}$ cu. in.	$33\frac{3}{5}$ cu. in.

The unit of dry measure is the Winchester bushel, which contains 2150.42 cu. in. It is a hollow cylinder $18\frac{1}{2}$ in. in diameter and 8 in. deep.

The Winchester bushel is used for measuring grain, sand, etc.

The heaped bushel of 2747.71 cu. in. is used for measuring fruits, vegetables, and other coarse articles.

Weight

588. Weight is the measure of the earth's attraction of matter on its surface.

589. The **Troy pound** is the standard unit of weight in the United States.

Troy Weight

590. Troy weight is used for weighing gold, silver, diamonds, and other precious minerals.

TABLE

24 grains = 1 pennyweight
20 pennyweights = 1 ounce
12 ounces = pound

Apothecaries' Weight

591. Apothecaries' weight is used by druggists and physicians in compounding and prescribing medicines.

TABLE

20 grains = 1 scruple
2 scruples = 1 dram
8 drams = 1 ounce
12 ounces = 1 pound

The term "carat" (or karat) has two meanings. In weighing diamonds, it is a denomination of weight, and is equal to 3.168 gr. Its second use is to denote the fineness of gold, and means $\frac{1}{24}$ part. Gold marked 18K (18 carats) is $\frac{18}{24}$ by weight pure gold, and $\frac{6}{24}$ alloy.

Drugs and chemicals are bought and sold wholesale by avoirdupois weight.

Avoirdupois Weight

592. Avoirdupois weight is used in weighing all sorts of coarse, heavy articles.

TABLE

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweight = 1 ton (T.)

Long Ton

593. The long ton (or gross ton) contains 2240 lb. It is used in the United States customhouse in determining the duty on merchandise taxed by the ton. Coal and iron are sold wholesale at the mines by the long ton.

TABLE

16 ounces = 1 pound
28 pounds = 1 quarter
4 quarters = 1 hundredweight
20 hundredweight = 1 ton

Comparison of Weights

	POUND	OUNCE	GRAIN
Troy	5760 gr.	480 gr.	1 gr.
Apothecaries'	5760 gr.	480 gr.	1 gr.
Avoirdupois	7000 gr.	437½ gr.	1 gr.

Other Common Measures

	POUNDS		POUNDS
Beef, Barrel	200	Nails, Keg	100
Butter, Firkin	56	Pork, Barrel	200
Flour, Barrel	196	Salt, Barrel	280

Circular or Angular Measure

594. Circular or angular measure is used for measuring angles or arcs of circles as applied to surveying, civil engineering, astronomical calculations, latitude, longitude, etc.

TABLE

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circle (cir.)

In astronomical calculations the circle is sometimes divided into signs, sextants, and quadrants.

In a circle there are:

12 signs of 30° each.
6 sextants of 60° each.
4 quadrants of 90° each.

Time

595. Time is the measure of duration.

TABLE

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
30 days = 1 month (mo.)

52 weeks = 1 year (yr.)
12 months = 1 year
365 days = 1 common year
366 days = 1 leap year
100 years = 1 century

The length of the year is determined by the time required for the earth to make one complete revolution around the sun. The exact time is 365 da. 5 hr. 48 min. 46 sec. (very nearly $365\frac{1}{4}$ da.). Hence we have a leap year once in 4 years, except in centennial years not divisible by 400.

English Money

596. English money is the legal currency of Great Britain.

TABLE

4 farthings (far.) = 1 penny (*d.*)
12 pence = 1 shilling (*s.*)
20 shillings = 1 pound (£)

French Money

597. French money is the legal currency of France.

TABLE

10 millimes (m.) = 1 centime (*c.*)
10 centimes = 1 decime (dc.)
10 decimes = 1 franc (F.)

German Money

598. German money is the legal currency of Germany.

TABLE

100 pfennig (pf.) = 1 mark (M.) The unit is the mark. Its value is $.238.

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